Atmospheric Seeing.

Sources.

A. Quirrenbach notes.

Adaptive Optics In Astronomy. chapter 2

F. Roddier et-al, CUP, ISBN 0-521-55375-x

Laser Beam Scintillation with Applications.

L.C. Andrews et-al, SPIE, ISBN 0-8194-4103-1

The atmosphere is always turbulent.

- Reynolds Number (dimensionless fluid scale) $R_e = VL/\nu$
- Laminar flow $R_e < 2300$
- Transitional $4000 < R_e \leq 2300$
- Turbulent $R_e \geq 4000$
- Viscosity of air $\nu \approx 1.5\text{e-}5\, m^2 s^{-1}$
- If we assume $V = 1\text{m}/s$ and $L = 1\text{m}$, then $R_e = 66000$
- Turbulence is an energy cascade, from large spatial scales to small scales.
- On smallest scales (the inner scale) the energy is dissipated as heat.

Turbulence does not imply seeing.

- Turbulence alone does not cause seeing.
  - Except for very high speed rotational flows.
- Need some source of refractive index variation, typically temperature or humidity.
- Important for understanding some mitigation strategies.

The inner and outer scales.

- Inertial regime between the inner and outer scale.
- Outer scale is the largest size scale of the turbulent structure. Related to the size of the structure that initiates the turbulence.
- Inner scale is the smallest scale where turbulent energy starts to dissipate due to viscous friction.
- The size of outer scale difficult to measure, and is clearly quite variable. Reported values vary between a few m and hundreds of m.
  - A small outer scale would have significant impact on the performance of large telescopes. Little evidence has turned up at existing large telescopes.
- The inner scale is somewhat variable depending on turbulence strength, typically 3 to 10mm.

Velocity structure function

- Structure function defined by $D_v(R_{1,} R_2) = \left\langle \left| v(R_1) - v(R_2) \right|^2 \right\rangle$
  - The velocity structure function may only depend on the rate of energy input per unit mass ε, and the kinematic viscosity ν (μ/ρ).
- Dimensions ε $Js^{-1} kg^{-1} = l^2 t^{-3}$, and ν= $l^2 t^{-1}$
- Functional form: $D_v(R_{1,} R_2) = \alpha \cdot f(\left| R_1 - R_2 \right| / \beta)$
  - α must have dimensions velocity squared $l^2 t^{-2}$
  - β must have dimensions of length l^1
  - Implies $\alpha = \nu^{1/2} \varepsilon^{1/2}$ and $\beta = \nu^{3/4} \varepsilon^{-1/4}$

Velocity structure function (continued)

- In the inertial regime *($L_o > l > l_o$)* the velocity structure function must be independent of the viscosity since dissipation is insignificant.
- This is only possible if $f = k\,(|R_1 - R_2|/\beta)^{2/3}$
- More conventionally written $D_v\,(R_1, R_2) = C_v^2 |R_1 - R_2|^{2/3}$
  - $C_v^2 = k\,\varepsilon^{2/3}$

Refractive index structure function

- Turbulence alone does not produce seeing.
  - Must mix fluids of different refractive index.
- Temperature structure function. $D_T(R_{1,} R_2) = C_T^2 \cdot \left| R_1 - R_2 \right|^{2/3}$
  - Has same functional dependence as velocity structure function, since temperature variations are entrained in the turbulence velocity field.
- Refractive index follows temperature. $N \equiv (n-1) \propto \rho$

$$D_n(R_{1,} R-2) \;=\; D_N(R_{1,} R_2) \;=\; C_N^2 \cdot \left| R_1 - R_2 \right|^{2/3}$$

$$C_N = (7.8 \cdot 10^{-5} P[mbar] / T^2[K]) \cdot C_T$$

Seeing strength.

- Index structure parameter C_N^2 often written C_n^2 .
- Weak turbulence $C_n^2 \sim 10^{-17} m^{2/3}$.
- Strong turbulence $C_n^2 \sim 10^{-13} m^{2/3}$.
- Index structure often given explicit altitude dependence $C_n^2(h)$.
  - Reality is more complex, requires path integral.

The phase structure function.

- How to define optical effect of seeing.
- Assume wavefront defined as surface of equal phase.
- Assume weak turbulence (or thin slice).
- Define phase structure function $D_{\phi}(r)=\langle|\phi(x)-\phi(x-r)|^2\rangle$
- Calculation of the phase structure function requires integrating the optical phase perturbations through a layer of atmosphere.
- For weak turbulence the phase can be integrated directly.

Phase coherence function

- Assume integration thickness is great enough to give Gaussian statistics (many turbulence cells).

$$\phi(x) = k \int_{h}^{h+\delta h} n(x, z)\, dz$$

- Then phase coherence at height *h* given by:

$$B_h(r) = \left\langle \exp i\left[\phi(x) - \phi(x+r)\right]\right\rangle$$

$$B_h(r) = \exp\left(-\frac{1}{2} D_\phi(r)\right)$$

Phase structure from C_n^2

- It can be shown that the structure function of phase fluctuations in a layer of thickness δh *is:*

$$D_\phi(r) = 2.914\, k^2 \delta h\, C_n^2 r^{5/3}$$

- For sufficiently weak turbulence the phase structure function can be integrated linearly through the total optical column. This leads to the definition of Fried's parameter given on the next page.
- Note that astronomers typically parameterize the integral by *h* but its more general to integrate along the optical path.

Definition of the Fried's Parameter.

- Phase structure function conveniently parameterized by Fried's parameter r_o .

$$D_\phi(r) = 6.88\left(\frac{r}{r_o}\right)^{5/3}$$

- Where r_o the Fried's parameter is defined by.

$$r_o = \left[0.423\, k^2 \int C_N^2(l)\, dl\right]^{(-3/5)}$$

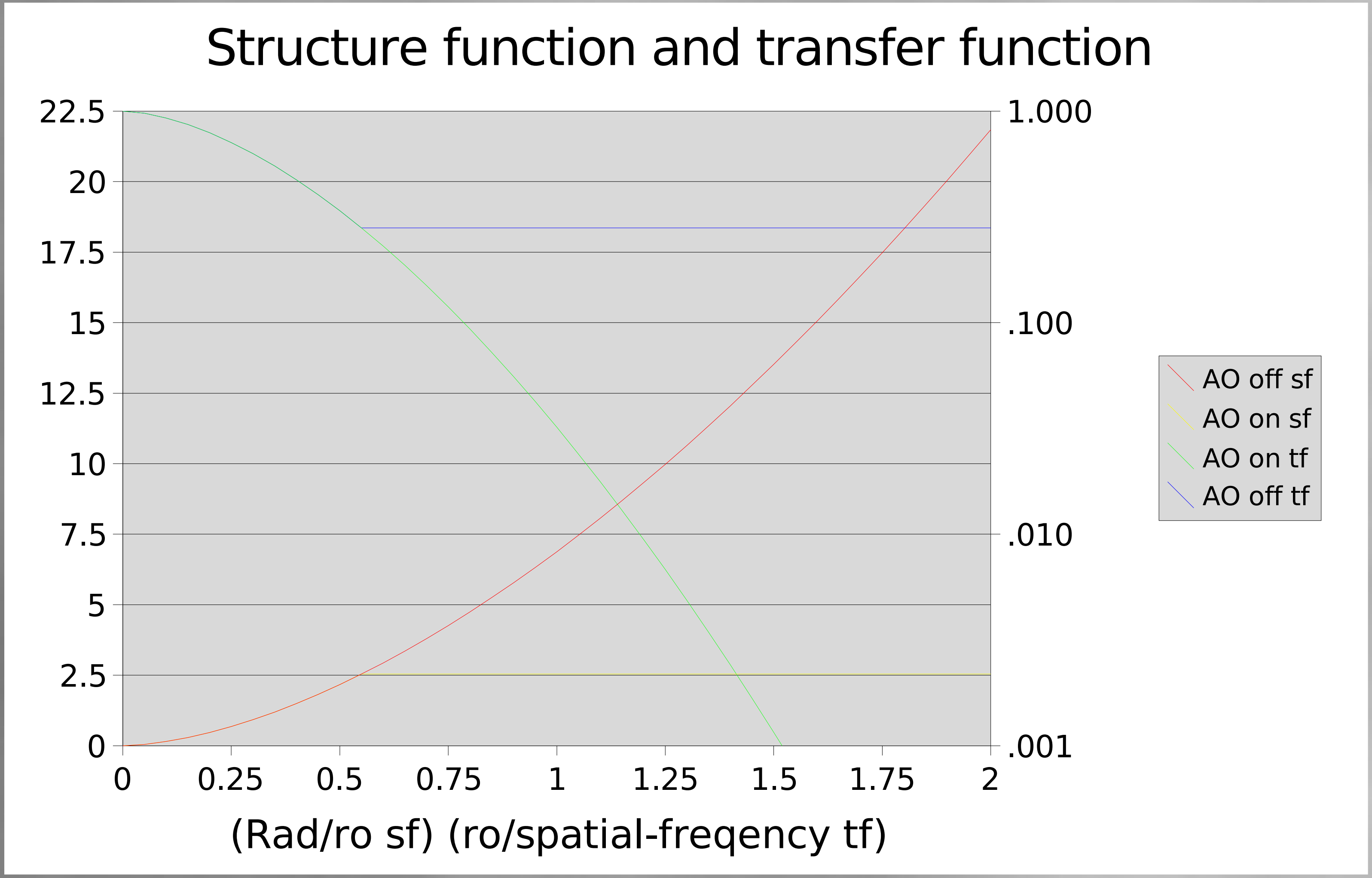
  - The RMS wavefront error within a circle of the Frieds parameter diameter is approximately 1 radian (1.03).
  - The FWHM of a seeing limited image is approx λ/r_o .

Effect of seeing on image formation.

- Typical r_0 for a good astronomical site is in the range of 0.1 to 1m at a wavelength 550nm.
- Long exposure image FWHM is approx $1.2\lambda/r_o$

Strehl ratio.

Specular and diffuse components (strehl > 0.05).



Other parameters of interest.

- Isoplanatic angle.
- Greenwood frequency.

Greenwood frequency.

- Taylor Hypothesis, or frozen phase screen.
  - Assume that the turbulence evolves much more slowly than the time it takes to advect past the aperture. Reasonable assumption for sub-sonic flows.
- Greenwood frequency.
  - Time over which the wavefront phase decorrelates by 1 radian RMS. For a single layer simply r_o/v

Stationarity of Frieds Parameter.

In computer simulations the Strehl ratio converges to a few percent within a second of simulated observing time.

In actual observations convergence is much slower.

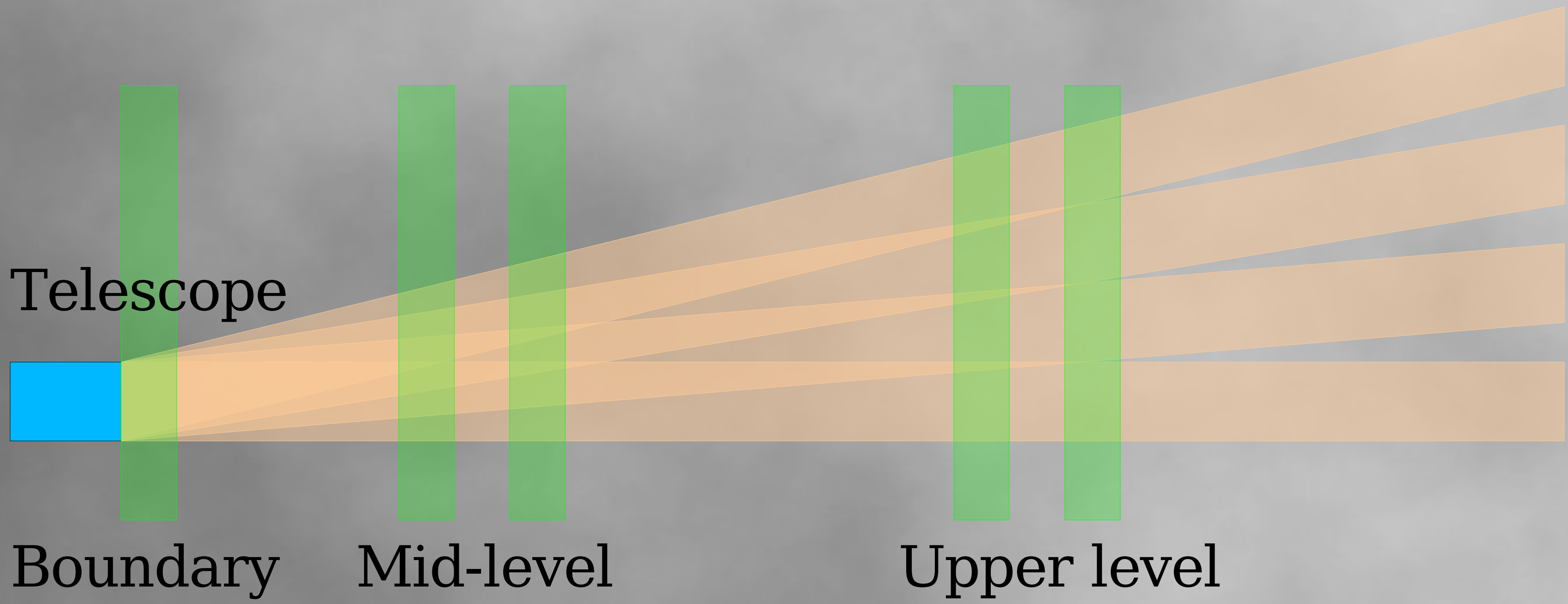
Real time measurements show that the effective Fried's parameter varies rapidly on times scales of a second or shorter.

Variations of a factor of 2 are typical with larger variations being common.

Makes characterization of AO performance difficult.

Origin of anisoplanitism.

- The beams defined by the telescope aperture and objects at various angles diverge linearly with distance from the telescope.



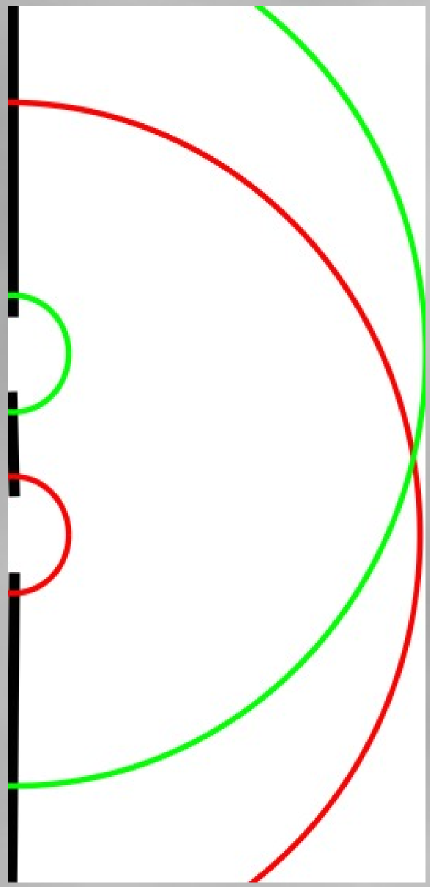
scaling

- Fried's parameter $r_o \propto \lambda^{6/5}$
- Image FWHM $\propto \lambda^{-1/5}$
- Greenwood and Isoplanatic angle scale like Fried's.

Caveat on Wavefronts

- The wavefront is a surface of equal time of flight. Sometimes defined as a surface of equal phase.
- Wavelength independent.
- Only well defined within coherence radius.
  - But still a useful construct for incoherent objects.
- The concept of a wavefront can break down, for instance when diffraction is important.
- Long atmospheric propagation paths strong turbulence.

Youngs slits.



Rytov variance.

For plane wave defined by $\sigma_l^2 = 1.23\, C_N^2\, k^{7/6}\, L^{11/6}$

Rytov variance above 1 indicate strong seeing.

Rytov variance below 0.1 are considered weak seeing.

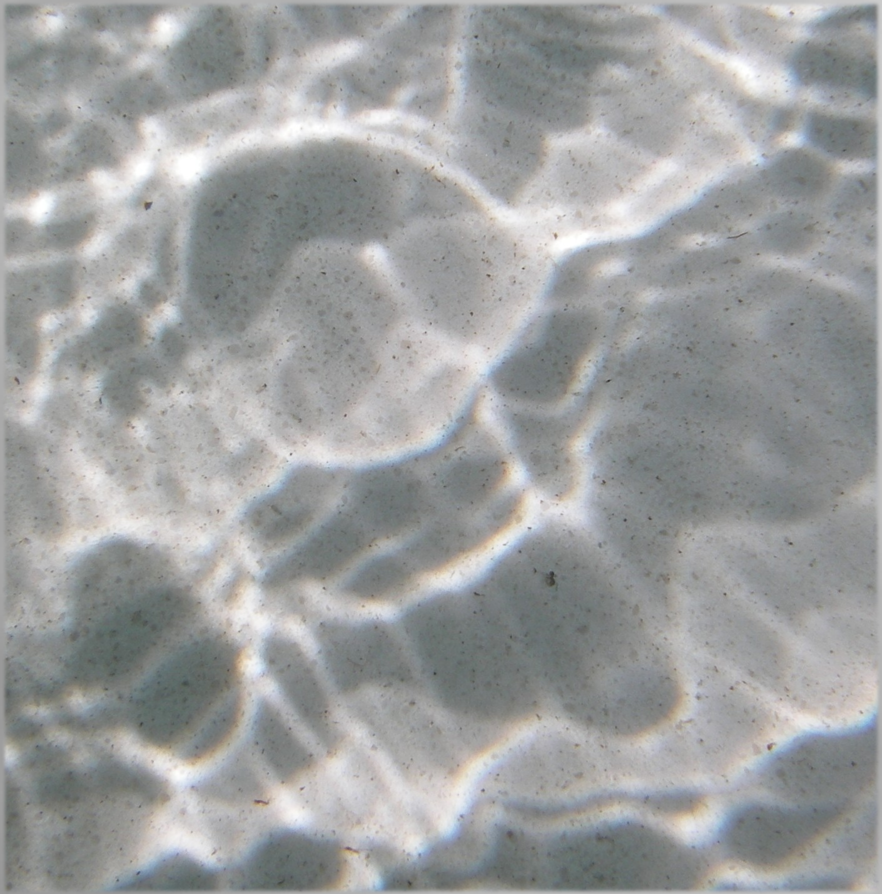
For moderate to strong seeing the concept of a wavefront is no longer useful (multipath).

For weak turbulence the Rytov variance is the variance of the light intensity in the telescope aperture. For non-uniform turbulence:

$$\sigma_l^2 = 2.26\, k^{7/6} \int C_N^2(l)\, l^{5/6}\, dl$$

Effects of strong turbuence.

- Shadow patterns on the bottom of a pool caused by surface waves.



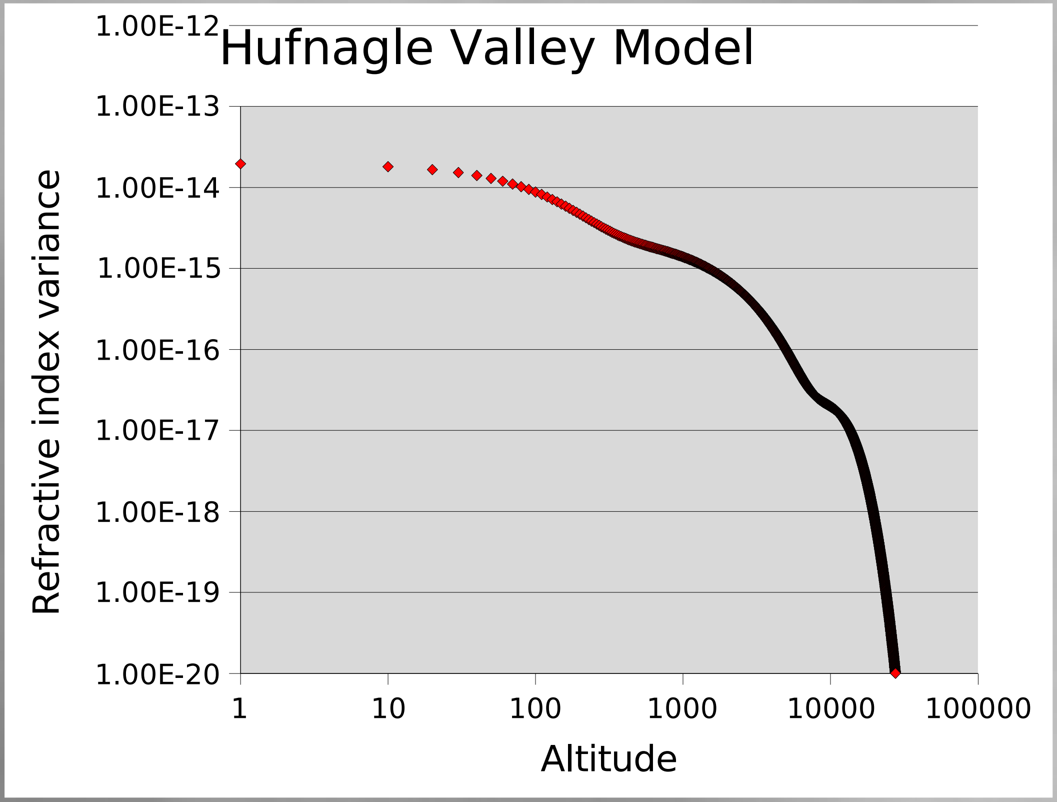
Strong atmospheric turbulence

- Laser propagation over a long horizontal distance.
- Different speckle scales indicate turbulence range and strength.

Hufnagle valley model.

Hufnagle valley model represents statistical average.

- Widely used but not believed to be accurate except over a statistical ensemble of atmospheres.

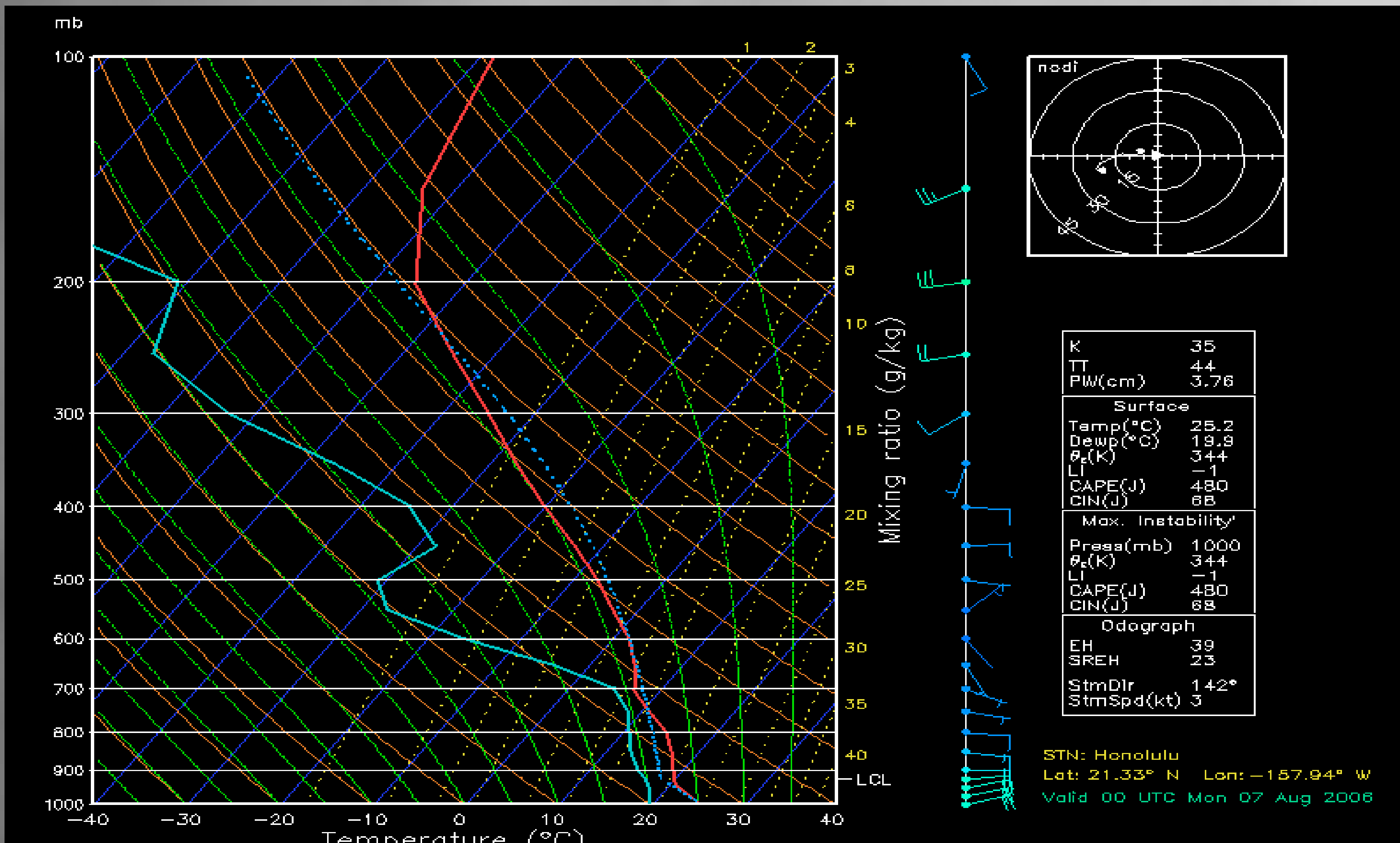


Seeing tends to occur in discrete layers

- Instrument.
- Telescope/dome.
- Boundary layer.
- Jet stream layers.
- Other atmospheric flows.
- Atmospheric layers often occur in pairs (Foy?).

Skew-T

Seeing layers associated with non-adiabatic temperature gradients



Amelioration of seeing.

- Dome and instrument seeing through careful thermal management.
- Forced air flow.
  - Hot mirrors, dome environments.
- Site selection.
- Evacuation
- Helium

Isoplanatic angle.

Cause of isoplanatic angle.

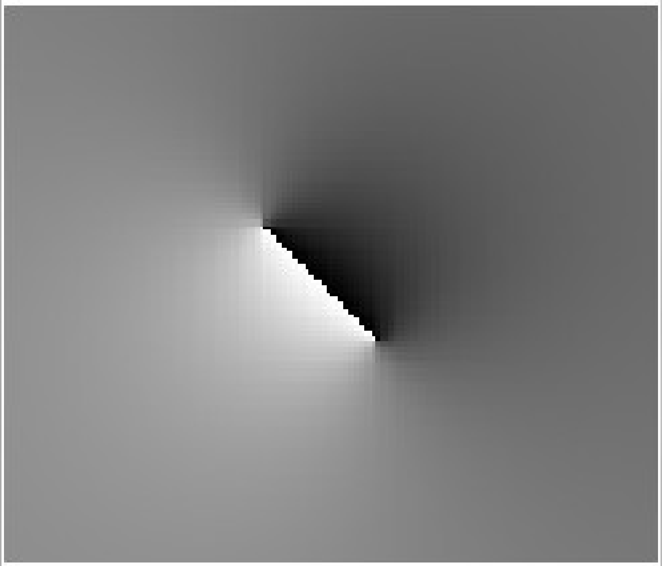
- As beams propagating in different directions through a series of turbulence layers, each beam will sample slightly different turbulence.
- Isoplanatic angle is hugely variable, running from a few arc-seconds up to an arc minute at MKO (in the IR).
- In extreme turbulence situations the isoplanatic angle can approach the diffraction limit of the telescope.

Severe turbulence.

Wavefront model breaks down.

- When interference effects become strong enough to cause zeros, positive and negative poles (screw dislocations occur). Location and number of poles is wavelength dependent.

Phase due to pole pair



Ref:

- Fried, Vaughn, Branch cuts in the phase function Applied Optics 15, 1992