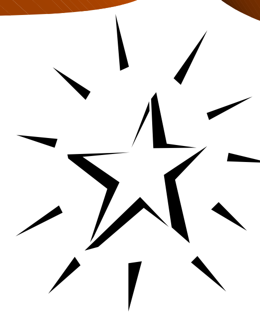
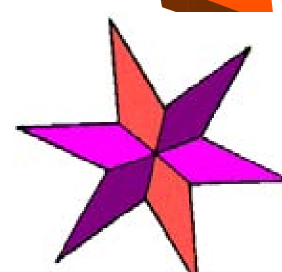
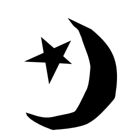
adaptive optics control systems

COLORING BOOK







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Slides and other papers
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We will use little words and little equations until you are more mature with control systems.

The big AO control systems are just like the little one we tell about in this coloring book.

Here we design Captain Billy’s autopilot for his little single-hulled tanker.

Captain Billy needs an autopilot so he can spend more time in his cabin with his friends.

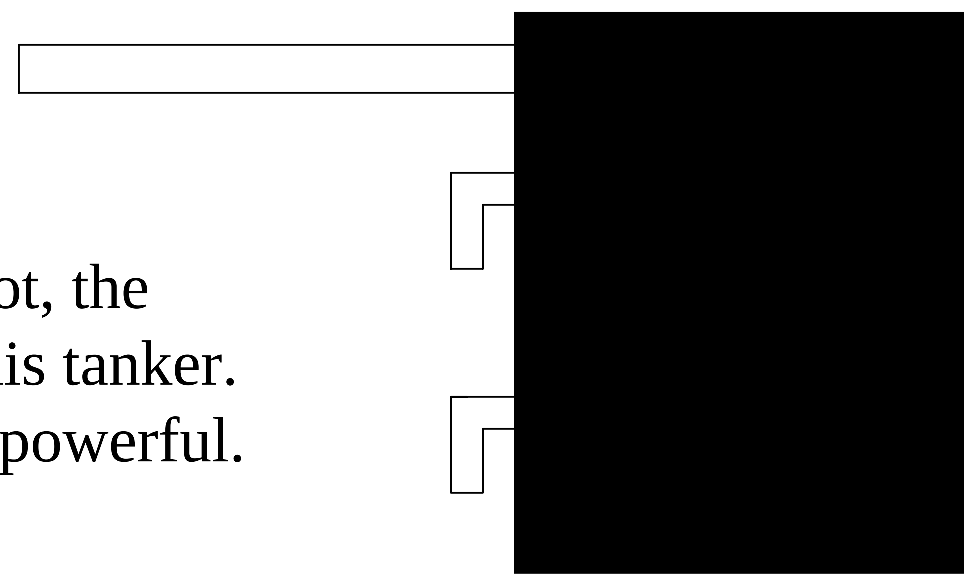
Captain Billy’s autopilot behaves just like a tip/tilt mirror locked in tip only mode, or just like an AO system with only one sensor and one actuator on the deformable mirror.

After you understand Captain Billy’s autopilot, we will use vectors and matrices to understand the big AO systems.

Control system components

Control systems are made up by connecting together components.

Actuators are one kind of component. Actuators are the mechanisms that do the work on the controlled process.



For Captain Billy's autopilot, the actuator is the rudder on his tanker. Color it black because it is powerful.

Other actuators

The motor turning the gears of the tip/tilt mirror is the actuator on the tip/tilt system.

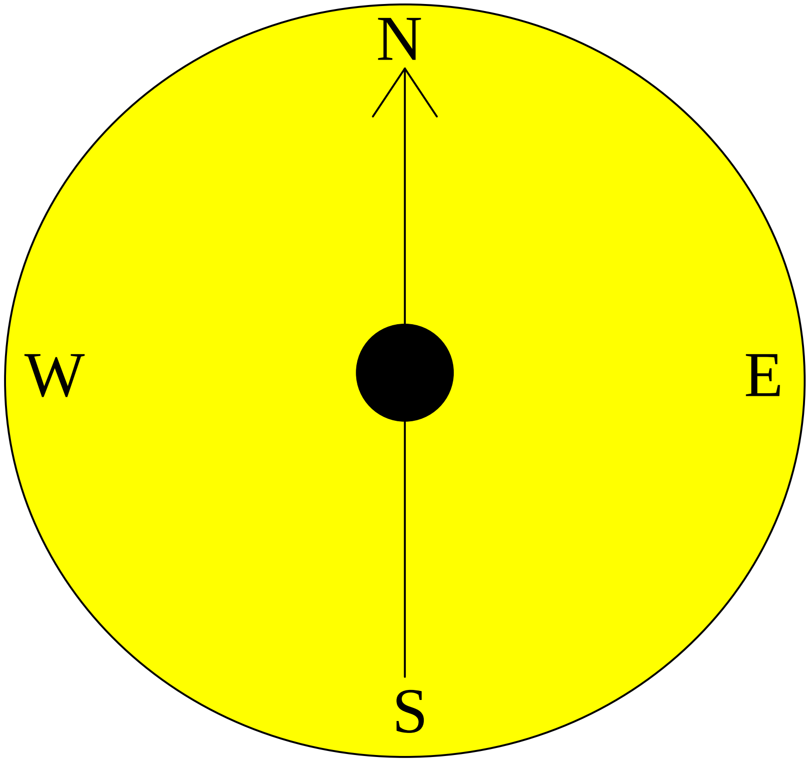
The finger poking the deformable mirror is an actuator in the AO control system.

Sensors are components, too

The sensors measure the actions of the controlled process.

The sensor on Captain Billy's autopilot is a compass. Color it yellow like the sun.

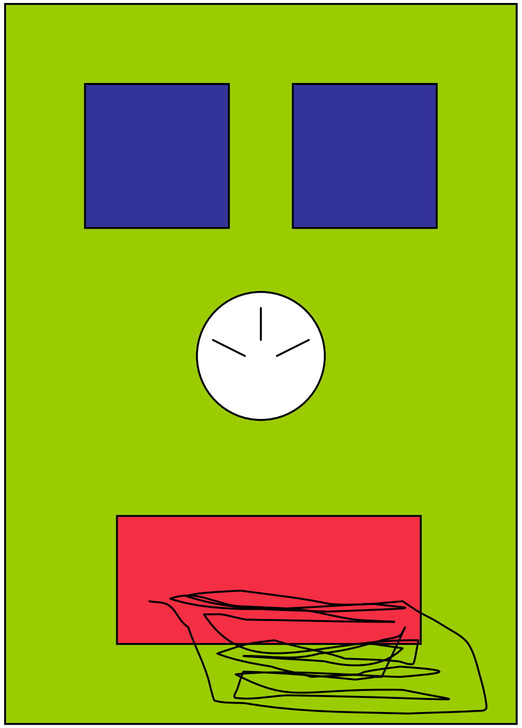
A sensor in both the tip/tilt and AO systems is a Shack-Hartmann local wavefront gradient sensor.



Control Computer

The control computer takes the signal from the sensor and uses an algorithm, called the **CONTROL LAW,** to turn the sensor signal into a command to the actuators.

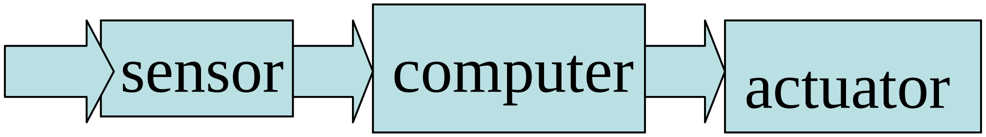
Color it green because it is smart.



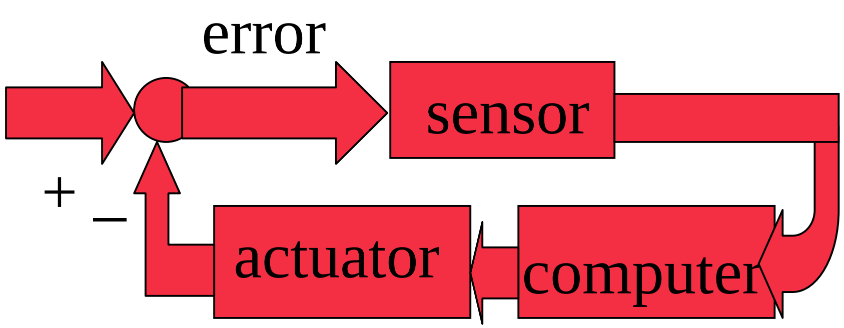
Captain Billy used to pilot his own tanker, but now we want to find a control law to program into the computer to tell it how to turn the compass (sensor) signal into a command to the rudder (actuator).

Open or closed loop?

There are two kinds of control law to choose from, open loop (feedforward) or closed loop (feedback).
The **open loop control** law uses the measurement directly to control the actuator.
The **closed loop control** law feeds back the measurement to compare the the desired process action with the measured process action. This comparison forms an error signal which is then used to control the actuator.

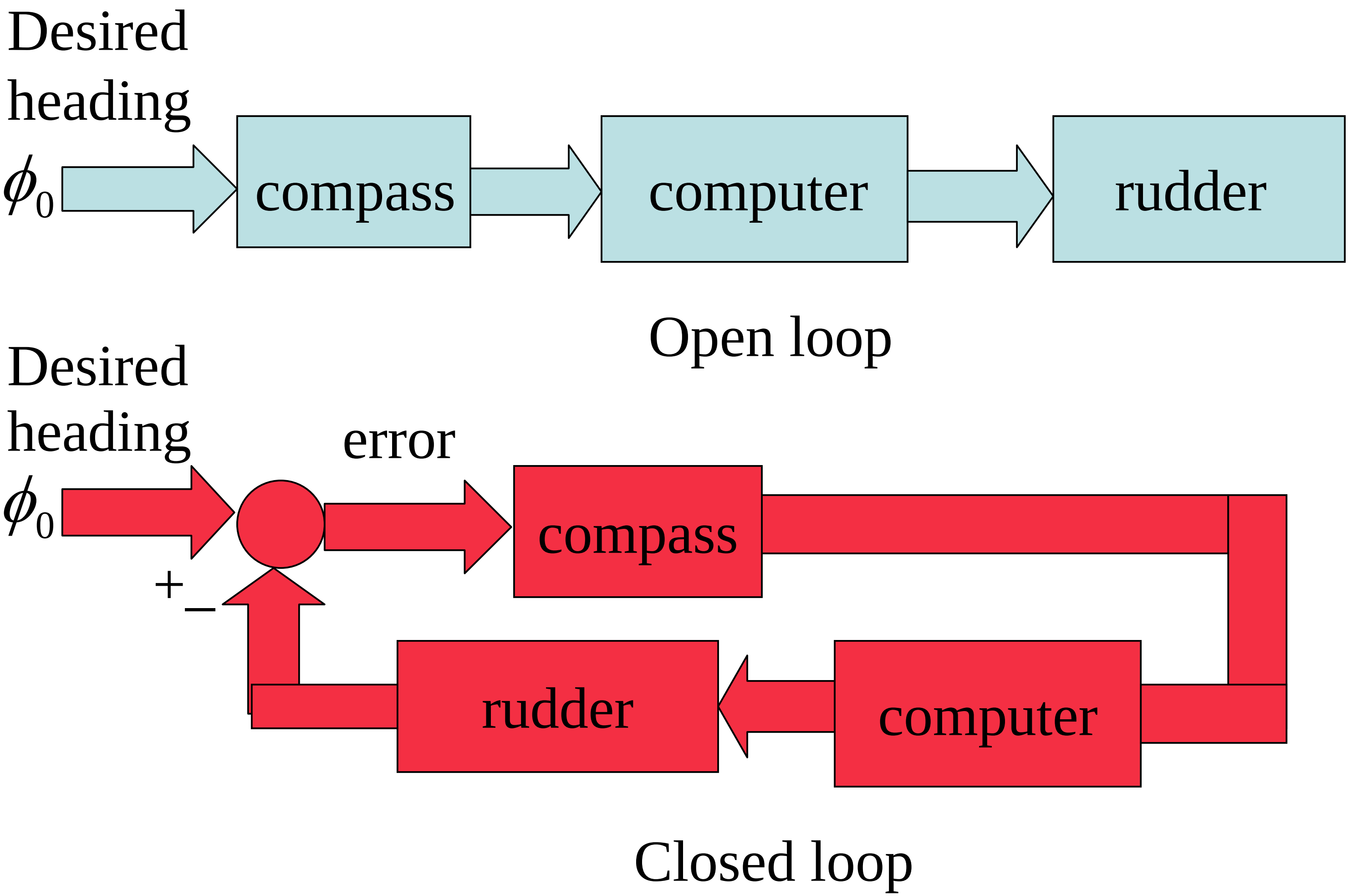


Open loop control
Color it blue for directness

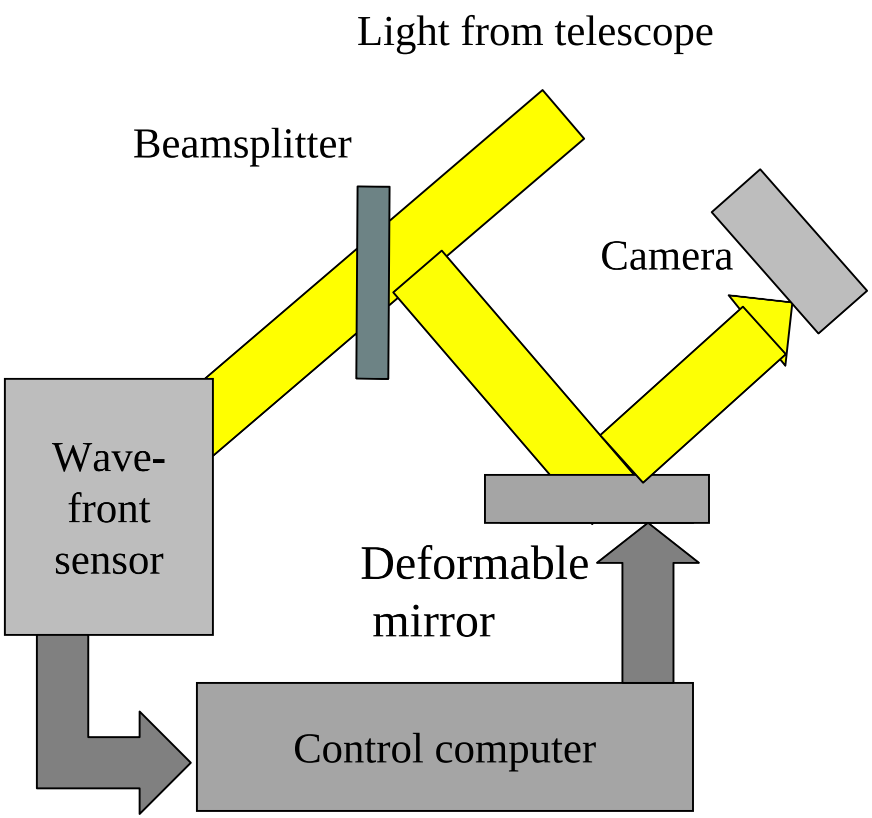


Closed loop controller - red

Captain Billy’s autopilot

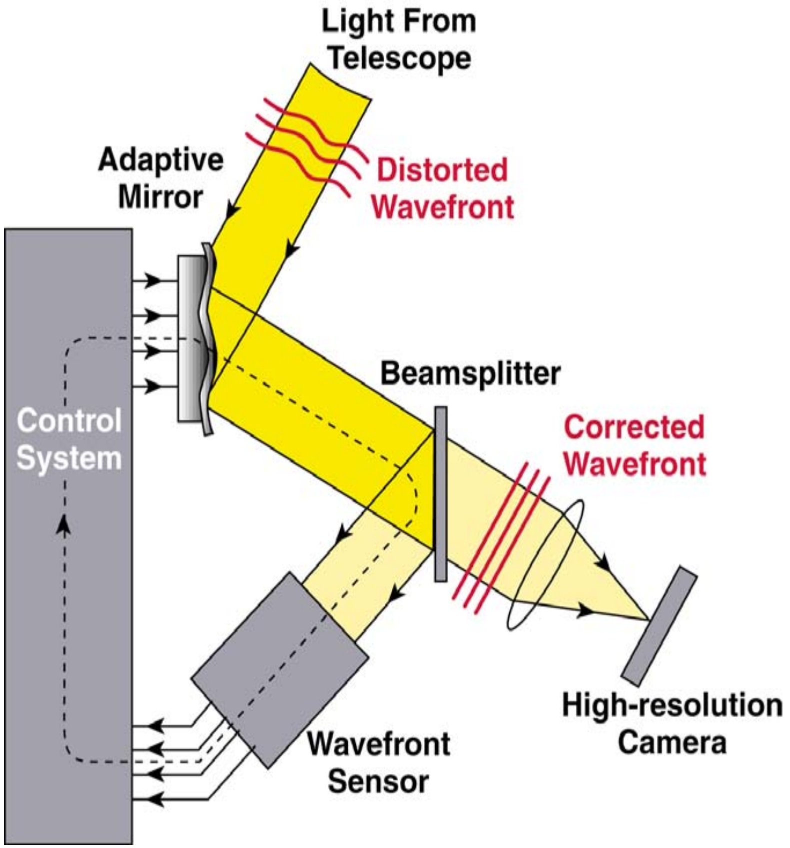


Adaptive optics systems



Open loop

Light hits sensor first



Closed loop

Light hits deformable mirror first

Open loop advantages/disadvantages

Advantages:

1. Simpler because error need not be computed.
2. Can not go unstable.

Disadvantages:

1. Must be very accurately calibrated.
2. Any inaccuracy or disturbance can greatly affect control of the system.

Closed loop advantages/disadvantages

Advantages:

1. Increased accuracy. For example, the ability to faithfully reproduce the input.
2. Reduced sensitivity of the ratio of the output to input due to variations in system characteristics (mismodelling).
3. Reduced effect of non-linearities and distortion.
4. Increased bandwidth, i. e. increased range of frequencies of the input over which the system will respond satisfactorily.

Disadvantages:

1. Increased complexity and cost.
2. The possibility of instability or objectionable oscillations.

Design of Captain Billy's autopilot

Mathematical model of tanker heading:

$$\phi(t) = Pc(t)$$

$\phi(t)$ = tanker heading at time t.
c(t) = commanded signal from the computer to the rudder.
P = gain of the command c to the resulting heading ϕ.

To find the gain P, do an experiment. Get in the tanker and move the rudder one degree.

$$\phi = Pc = P(1^o) = P^o$$

The response ϕ, in degrees, to the one degree input c, equals P.
Then if c = 3 degrees, we know ϕ = 3P degrees,
And if c = 5 degrees, we know ϕ = 5P degrees, etc.

Open loop design of autopilot

Mathematical model of tanker heading:

$$\phi(t) = Pc(t)$$

Open loop control law: $$c(t+T) = \frac{1}{P}\phi_0$$

Then $\phi(t+T) = \phi_0$, the desired heading.

The T time delay occurs because the computer must take one cycle time T to compute the product of 1/P times ϕ_0 .

Note if gain in the model is really Q, not P, then $\phi(t) = \frac{Q}{P}\phi_0$

Closed loop design of the autopilot

Call the error e(t), so $e(t) = \phi_0 - \phi(t)$

Take the feedback control law to be c(t) = Ke(t-T),

where K is a feedback gain to be determined.

Then from the mathematical model, ϕ(t) = PKe(t-T).

Substituting the definition of e(t) gives ϕ(t) = PK$\left[\phi_0 - \phi(t-T)\right]$

$$If\ \ \phi_0 = 0, \quad \phi(t) = -PK\phi(t-T)$$

$$at\ \ t = T, \quad \phi(T) = -PK\phi(0)$$

$$at\ \ t = 2T, \ \ \phi(2T) = -PK\phi(T) = (-PK)^2\,\phi(0)$$

$$at\ \ t = t, \quad \phi(t) = (-PK)^{t/T}\,\phi(0)$$

For the case $\phi_0 \neq 0,\ try\ \ \phi(t) = (-PK)^{t/T}\,\phi(0) + \alpha$

Where α is a constant to be determined

Closed loop design of the autopilot

Substituting this into ϕ(t) = PK $\left[\phi_0 - \phi(t-T)\right]$

$$\phi(t) = \left(-PK\right)^{t/T}\phi(0) + \alpha = PK\left[\phi_0 - \left(-PK\right)^{(t-T)/T}\phi(0) - \alpha\right]$$

$$\alpha = PK\left[\phi_0 - \alpha\right] \qquad \alpha = \frac{PK}{1+PK}\phi_0$$

Giving the solution as $$\phi(t) = \frac{PK}{1+PK}\phi_0 + (-PK)^{t/T}\phi(0)$$

Where ϕ(0) is the initial heading and time t takes the values t = T, 2T, 3T, …

For ϕ(0) = 0, try different values of the feedback gain K

$$K = \frac{1}{P} \Rightarrow \phi(t) = \frac{1}{2}\phi_0 \cdots K = \frac{10}{P} \Rightarrow \phi(t) = \frac{10}{11}\phi_0 \cdots K = \frac{100}{P} \Rightarrow \phi(t) = \frac{100}{101}\phi_0$$

Feedback design of the autopilot, cont.

So large K, say K = 100/P, gets us close to the desired heading ϕ_0 .

Also, if we miscalibrate and really have a gain Q in the model, for the feedback controller, suppose K=100/P,

$$\phi(t) = \frac{Q\frac{100}{P}}{1+Q\frac{100}{P}}\phi_0 = \frac{1}{1+\frac{P}{100Q}}\phi_0$$

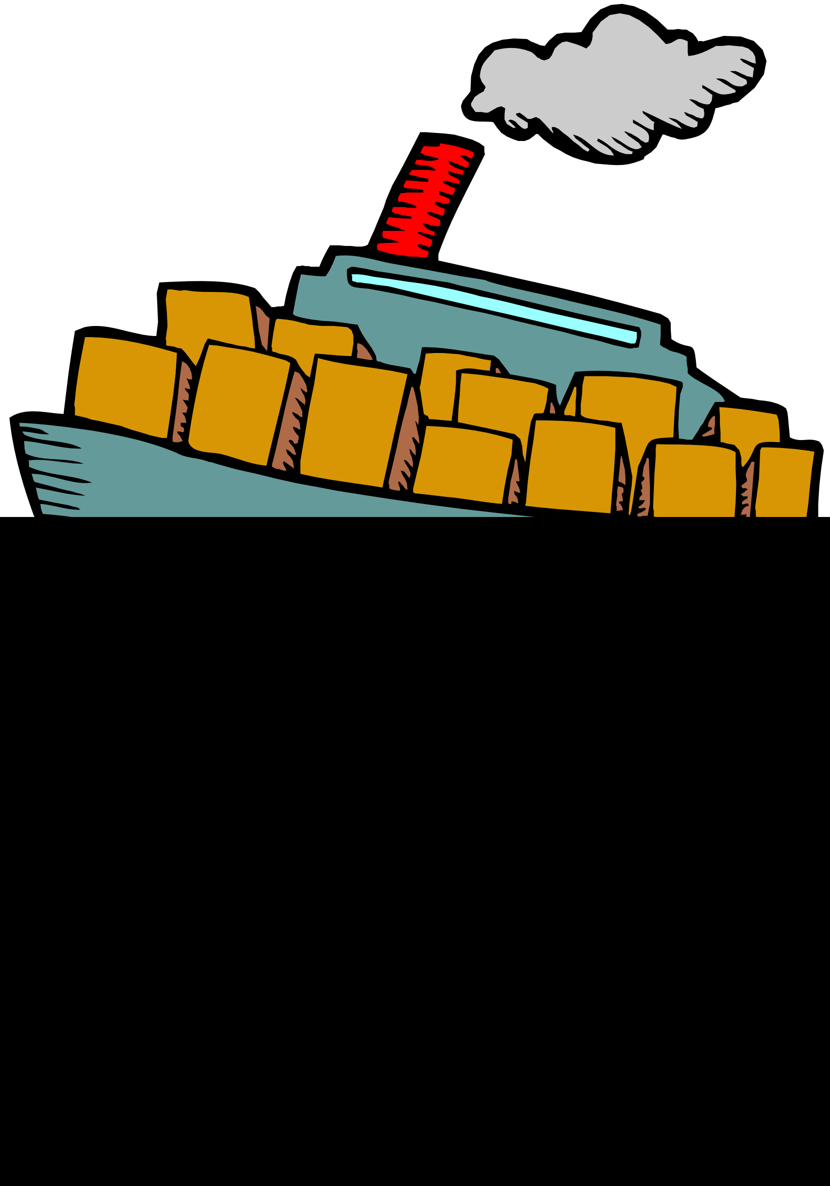
If Q is not too different from P we are still pretty close to the desired heading. This is much less sensitive to variations of Q away from P than the open loop that we computed before:

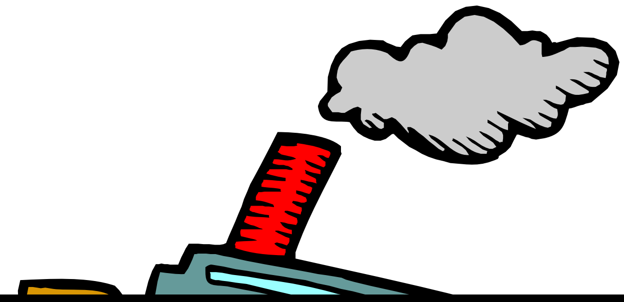
$$\phi(t) = \frac{Q}{P}\phi_0$$

The choice K=100/P seems good because we can follow ϕ_0 and the sensitivity to mismodelling is low.

SO LET'S TRY BUILDING THE AUTOPILOT WITH K=100/P

OIL SPILL





WHAT HAPPENED?

Answer: $\phi(0) \neq 0 \qquad If \quad K = 100/P \quad then$

$$\phi(t) = (-100)^{t/T}\,\phi(0) + \frac{100}{101}\,\phi_0$$

This term gets big fast as t = T, 2T, 3T, 4T…

This is called instability. You have experienced instability when someone is speaking into a microphone and the amplifier gain is turned up too high. Then some of the amplified sound gets fed back into the microphone, is amplified again, which gets fed back, etc., etc., to produce an annoying wail until someone turns the gain down.

You must consider the possible consequences of an
Inadvertent loss of control or instability when designing
A control system.

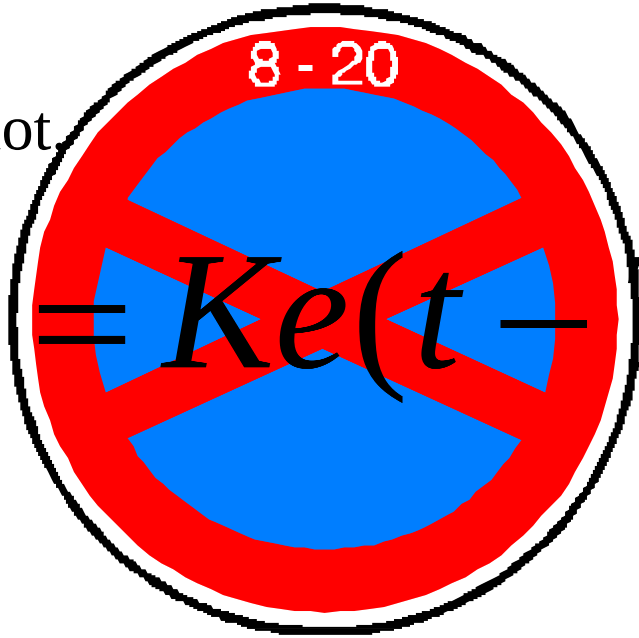
Closed loop systems give good performance when the feedback gain is high,

BUT

Closed loop systems go unstable when the Feedback gain is TOO high.

Let’s build Captain Billy a good autopilot.
Don’t use the dumb old feedback law

$$c(t) = Ke(t-T)$$



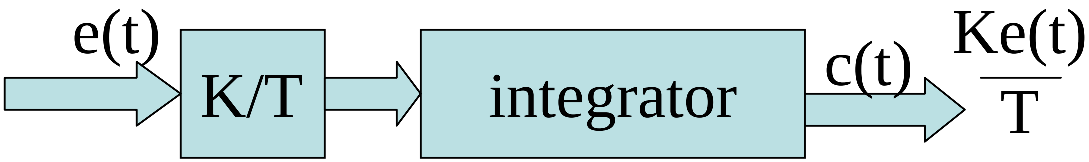
Instead, use an integrator as feedback law:

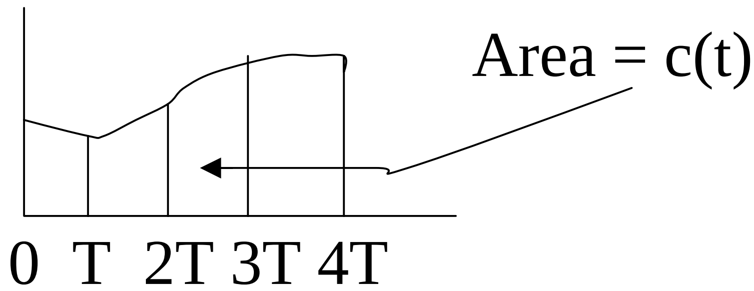
$$c(t) - c(t-T) = Ke(t-T)$$

Why is this called an integrator?

An integrator in discrete time

In continuous time, an integrator integrates up its input:





$$\Leftrightarrow \quad c(t) = \frac{K}{T}\int_0^t e(\tau)d\tau + c(0)$$

Differentiate to get $\frac{dc}{dt}(t) \approx \frac{c(t+T)-c(t)}{T} = \frac{K}{T}e(t)$

Then $c(t) = c(t-T) + Ke(t-T)$ With c(0) = 0

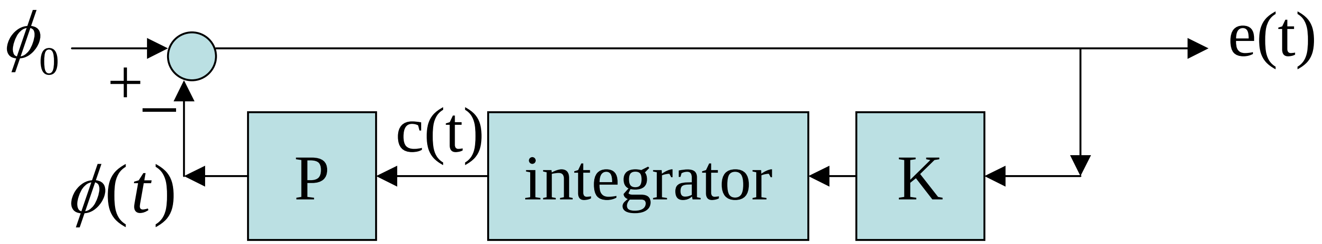
Solution in discrete time $c(t) = \frac{K}{T}T\sum_{\tau=0}^{\tau=(t/T)-1} e(\tau)$ With c(0) = 0

Analysis of feedback with integrator

Math model: $\phi(t) = Pc(t)$

Error definition: $e(t) = \phi_0 - \phi(t)$

Integrator: $c(t) = c(t-T) + Ke(t-T)$



$$\phi(t+T) = Pc(t+T) = P\left[c(t) + Ke(t)\right]$$

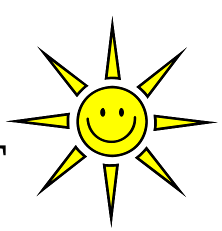
$$= \phi(t) + PK\left[\phi_0 - \phi(t)\right]$$

$$\phi(t+T) = (1-PK)\phi(t) + PK\phi_0$$

For t = 0, T, 2T,…

Solution: $\phi(t) = \phi_o + (1-PK)^{t/T}\phi(0)$

GOOD AUTOPILOT!



If |1-PK| < 1, system is stable and $\phi(t) \to \phi_0$

If we set K=1/P then $\phi(t)$ goes to ϕ_0 in one time step T regardless of the value of ϕ_0

Sensitivity to Mismodeling- Integral Feedback

If the process gain P is really = Q, then

$$\phi(t) = \phi_0 + \left(1 - QK\right)^{t/T} \phi(0)$$

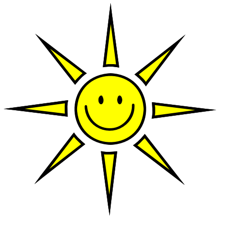
If |1-QK| < 1 system is stable and $\phi(t) \rightarrow \phi_0$

Setting K =1/P , for P ~ Q, means $\phi(t)$ still tends to ϕ_o

Still true for all values of ϕ_o or $\phi(0)$.

Therefore the control law is insensitive to small variations in Process gain P. Also it can be shown integral feedback is insensitive to small nonlinearities and distortion.

Really good autopilot!



Any Questions?

Do you understand pretty much what I have said so far?

Now you are mature enough in control systems to look at the Big AO system, and we can use the autopilot example as a guide.

Big kids use vectors and matrices to describe the AO system, which has m sensors and n actuators.

Let $\vec{\phi}_0$ = desired local wavefront slopes as seen by sensors, in volts from each sensor, an m-vector, constant in time.

Let $\vec{e}(t)$ = sensor signals after deformable mirror, volts, m-vector.

Let $\vec{\phi}(t)$ = local wavefront slopes produced by deformable mirror, as seen by the sensors, m-vector.

Let $\vec{c}(t)$ = n actuator commands, n-vector

AO math model: $\vec{\phi}(t) = P\vec{c}(t)$ where P is the mxn poke matrix.

Explanation of poke matrix

For the tanker, to find P in $\phi = Pc$, we did set c = one degree and measured the resulting ϕ.

For adaptive optics, we set one actuator at a time to one unit, like one degree, and measure the resulting phase $\vec{\phi}$ m-vector that is read by the m sensors, and repeat this measurement for Each one of the n actuators.

Explanation of poke matrix P, cont.

$$\vec{\phi} = P\vec{c}$$

$$\begin{pmatrix} \phi_1 \\ \vdots \\ \phi_m \end{pmatrix} = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & & \vdots \\ p_{m1} & \cdots & p_{mn} \end{bmatrix} \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$$

Command $c_1 = 1, c_2 = c_3 = \cdots c_n = 0$, measure the m local wavefront slopes $\phi_1, \phi_{2,} \cdots, \phi_m$ produced by the deformable mirror deflecting laser light put into the telescope.

$$\begin{pmatrix} \phi_1 \\ \vdots \\ \phi_m \end{pmatrix} = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & & \vdots \\ p_{m1} & \cdots & p_{mn} \end{bmatrix} \begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} p_{11} \\ \vdots \\ p_{m1} \end{pmatrix}$$

Then command $c_1 = 0, c_2 = 1, c_3 = \cdots = c_n = 0$, etc., in turn to $c_1 = c_2 = \cdots = c_{n-1} = 0, c_n = 1$ to get $\begin{pmatrix} p_{12} \\ \vdots \\ p_{m2} \end{pmatrix}, \cdots, \begin{pmatrix} p_{1n} \\ \vdots \\ p_{mn} \end{pmatrix}$

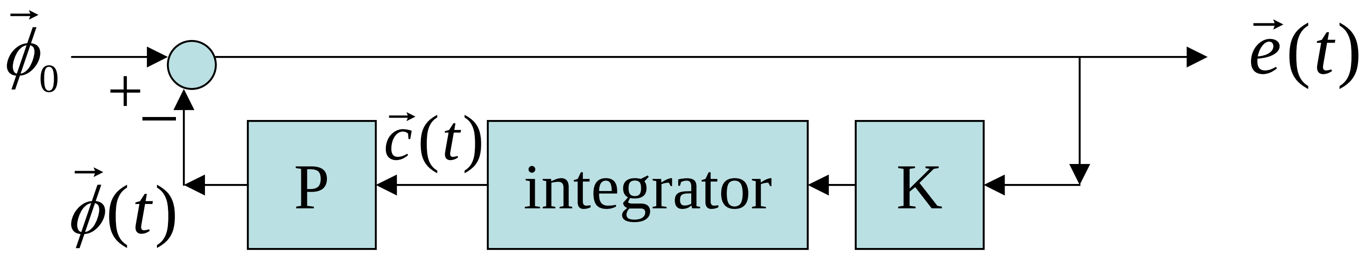
to fill out the poke matrix P.

Analysis of AO feedback with integrator

Math model: $\vec{\phi}(t) = P\vec{c}(t)$ c is n vector, P is mxn poke matrix

Error definition: $\vec{e}(t) = \vec{\phi}_0 - \vec{\phi}(t)$ e and ϕ are m vectors

Integrator: $\vec{c}(t) = \vec{c}(t-T) + K\vec{e}(t-T)$ K is nxm matrix



$$\vec{\phi}(t+T) = P\vec{c}(t+T) = P\left[\vec{c}(t) + K\vec{e}(t)\right]$$

$$= \vec{\phi}(t) + PK\left[\vec{\phi}_0 - \vec{\phi}(t)\right]$$

For t = 0, T, 2T,…

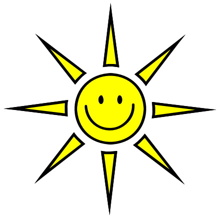
$$\vec{\phi}(t+T) = (I - PK)\vec{\phi}(t) + PK\vec{\phi}_0$$

I is unit matrix, λ is eigenvalue

Solution: $\vec{\phi}(t) = \vec{\phi}_o + (I - PK)^{t/T}\,\vec{\phi}(0)$

GOOD AO SYSTEM!

If all $|\lambda_i$ (I-PK)| < 1, system is stable and $\vec{\phi}$ (t)→ $\vec{\phi}_0$



If we set K= P^{-1} then $\vec{\phi}(t)$ goes to $\vec{\phi}_0$ in one time step, regardless of the value of $\vec{\phi}_0$ or $\vec{\phi}(0)$.

Some practical problems

1. The number of sensors = m is usually bigger than the number of actuators = n, so P is an mxn rectangular matrix and does not have an inverse, then K can't = P inverse.

Solution: We use the pseudo inverse K = $\left(P^T P\right)^{-1} P^T$

Note: If P were nxn and invertible, then $K = P^{-1} P^{T-1} P^T = P^{-1}$

But this also has a problem, because $P^T P$ is ill conditioned.

Then we use a singular value decomposition and throw out the smaller eigenvalues of $P^T P$.

Singular value decomposition

Given an nxn matrix M such that $M = M^T$ (M is symmetric)

and M is non-negative definite ($\vec{x}^T M\vec{x} \geq 0$ for all n vectors $\vec{x}$)

$$Let \quad \lambda_1^2, \lambda_2^2, \ldots, \lambda_n^2 = eigenvalues\, of\ M\ and\, \vec{m}_1, \vec{m}_2, \ldots, \vec{m}_m =$$

$$eigenvectors\, of\ M, \quad Then \quad \lambda_i^2 \vec{m}_i = M\vec{m}_i \quad for\, i = 1,2,\ldots,n.$$

$$Fact: \quad \lambda_i^2\ is\, real\, and\, \lambda_i^2 \geq 0\ and \quad \vec{m}_i^T \vec{m}_j = 0 \quad for\, all\, i \neq j$$

$$\left(\lambda_1^2 \vec{m}_1 \mid \lambda_2^2 \vec{m}_2 \mid \ldots \mid \lambda_n^2 \vec{m}_n\right) = \left(M\vec{m}_1 \mid M\vec{m}_2 \mid \ldots \mid M\vec{m}_n\right)$$

$$\left(\vec{m}_1 \mid \vec{m}_2 \mid \ldots \mid \vec{m}_n\right)\begin{pmatrix} \lambda_1^2 & \ldots & 0 \\ \vdots & & \vdots \\ 0 & \ldots & \lambda_n^2 \end{pmatrix} = M\left(\vec{m}_1 \mid \vec{m}_2 \mid \ldots \mid \vec{m}_n\right) and \begin{pmatrix} \vec{m}_1^T \\ \vdots \\ \vec{m}_n^T \end{pmatrix}\left(\vec{m}_1 \mid \ldots \backslash\, \vec{m}_n\right) = I$$

$$M = MI = M\left(\vec{m}_1 \mid \ldots \mid \vec{m}_n\right)\begin{pmatrix} \vec{m}_1^T \\ \vdots \\ \vec{m}_n^T \end{pmatrix} = \left(\lambda_1^2 \vec{m}_1 \mid \lambda_2^2 \vec{m}_2 \mid \ldots \mid \lambda_n^2 \vec{m}_n\right)\begin{pmatrix} \vec{m}_1^T \\ \vdots \\ \vec{m}_n^T \end{pmatrix}$$

$M = \lambda_1^2 \vec{m}_1 \vec{m}_1^T + \lambda_2^2 \vec{m}_2 \vec{m}_2^T + \cdots + \lambda_n^2 \vec{m}_n \vec{m}_n^T$ Singular value decomposition

Singular Value Decomposition

Let $\lambda_1^2, \lambda_2^2, \ldots, \lambda_m^2$ = eigenvalues of $P^T P$

Let $\vec{p}_1, \vec{p}_2, \ldots, \vec{p}_m$ be the corresponding eigenvectors

Note $(P^T P)^T = P^T P$ And $0 \leq y_1^2 + \ldots + y_n^2 = \vec{y}^T \vec{y}$ *so if* $\vec{y} = P\vec{x}$

$0 \leq \vec{y}^T \vec{y} = \vec{x}^T P^T P \vec{x}$ For every n vector $\vec{x}$

Therefore we can take $P^T P$ = M from the previous slide

Then $P^T P = \lambda_1^2 \vec{p}_1 \vec{p}_1^T + \lambda_2^2 \vec{p}_2 \vec{p}_2^T + \ldots + \lambda_m^2 \vec{p}_m \vec{p}_m^T$

and so $(P^T P)^{-1} = \lambda_1^{-2} \vec{p}_1 \vec{p}_1^T + \lambda_2^{-2} \vec{p}_2 \vec{p}_2^T + \ldots + \lambda_m^{-2} \vec{p}_m \vec{p}_m^T$

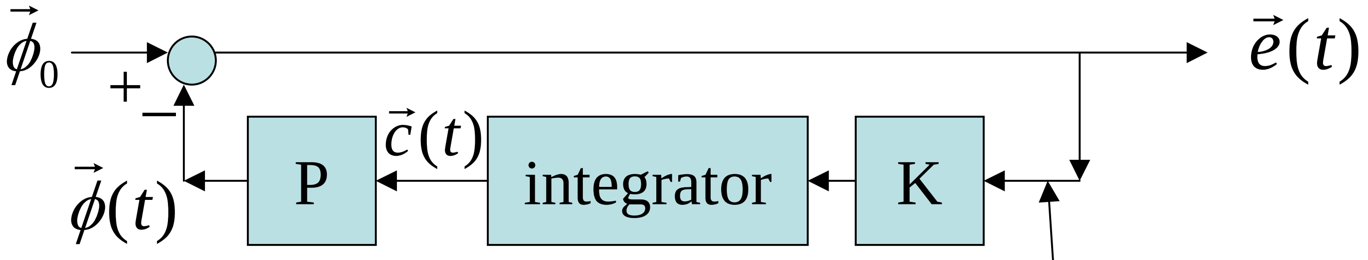
If any λ_i^2 is small for i = 1, 2, … , m, then λ_i^{-2} is big, which gives high feedback gain K, which can be TOO high, which leads to instability. So set small $\lambda_i^2 = 0$ in here, and take $0^{-2} = 0$

Problems, continued

Math model: $\vec{\phi}(t) = P\vec{c}(t)$ c is n vector, P is mxn poke matrix

Error definition: $\vec{e}(t) = \vec{\phi}_0 - \vec{\phi}(t)$ e and ϕ are m vectors

Integrator: $\vec{c}(t) = \vec{c}(t-T) + K\vec{e}(t-T)$ K is nxm matrix



2. Sometimes the system is tending to be unstable or oscillates too much, then insert a scalar gain γ button here for the operators to adjust between 0 and 1.

$$\vec{c}(t) = \vec{c}(t-T) + \gamma K\vec{e}(t-T)$$

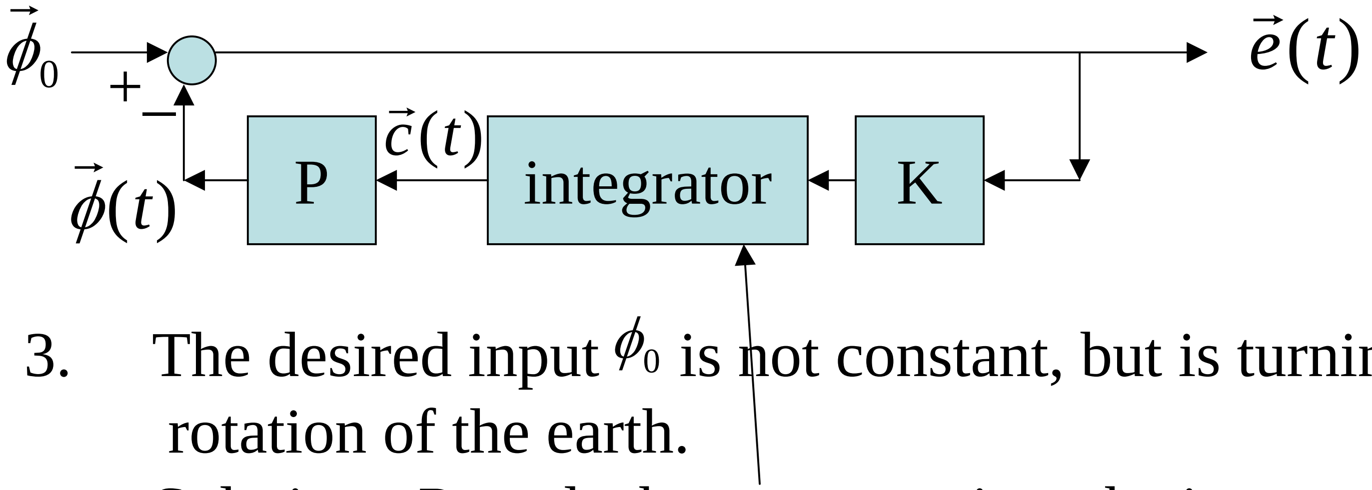
Decreasing γ has the effect of decreasing all the matrix gains in the closed loop, giving better stability but reduced performance.

Problems, continued

Math model: $\vec{\phi}(t) = P\vec{c}(t)$ c is n vector, P is mxn poke matrix

Error definition: $\vec{e}(t) = \vec{\phi}_0 - \vec{\phi}(t)$ e and ϕ are m vectors

Integrator: $\vec{c}(t) = \vec{c}(t-T) + K\vec{e}(t-T)$ K is nxm matrix



3. The desired input ϕ_0 is not constant, but is turning with the rotation of the earth.

Solution: Put a leak constant α into the integrator, where α is some number slightly less than one.

$$\vec{c}(t) = \alpha\vec{c}(t-T) + \gamma K\vec{e}(t-T)$$

For each observation, the operators adjust only α and γ.

Effect of a leak α in the integrator

For $\gamma = 1$, we have seen that $\vec{\phi}(t) \to \vec{\phi}_0 \; if \; \vec{\phi}_0$ is constant.
For γ slightly less than one, if $\vec{\phi}_o = \vec{\phi}_0(t)$ is time-varying,

$$\vec{\phi}(t+T) = P\vec{c}(t+T)$$

$$= P\left[\alpha\vec{c}(t) + \gamma K\vec{\phi}_0(t) - \gamma K\phi(t)\right]$$

$$\vec{\phi}(t+T) = (\alpha I - \gamma PK)\vec{\phi}(t) + \gamma PK\vec{\phi}_0(t)$$

$$Solution: \quad \vec{\phi}(t) = (\alpha I - \gamma PK)^{t/T}\vec{\phi}(0) + \sum_{\tau=0}^{\tau=t/T-1}\alpha^{t/T-\tau-1}\gamma K\vec{\phi}_0(\tau)$$

Past values of $\vec{\phi}_0(t)$ are exponentially discounted as powers of α, so only the most recent values of $\vec{\phi}_0(t)$ are followed. But if $\vec{\phi}_0$ is truly constant,

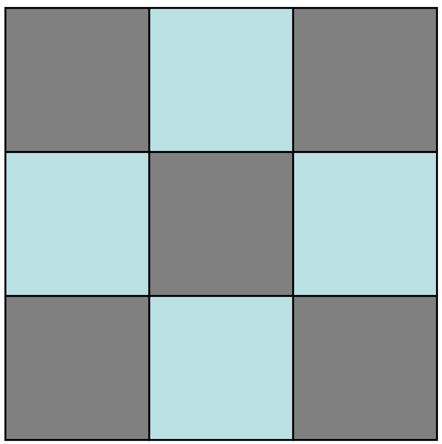
$$\vec{\phi}(t) = \left[(1-\alpha)I + \gamma PK\right]^{-1}\gamma PK\vec{\phi}_0 + (\alpha I - \gamma PK)^{t/T}\vec{\phi}(0)$$

For constant $\vec{\phi}_0, \quad \vec{\phi}(t) \to \left[(1-\alpha)I + \gamma PK\right]^{-1}\gamma PK\vec{\phi}_0, \quad not \to \vec{\phi}_0$

which is why α and γ are kept only slightly less than one.

Problems, continued

4. “Waffle modes” are a problem, too. If the sensors are located on a rectangular grid, wavefronts looking like waffle shapes



can not be sensed, and are called “unobservable”. The control system can programmed to reject these shapes, or the sensors can be located in different positions. The waffle control problem and definitions of unobservability and uncontrollability are in the papers on the CfAO web site.

I will not discuss waffle, etc., further today.

Summary

We have shown you how most AO feedback systems are designed.
The AO feedback controller consists of deformable mirrors, wavefront sensors, and a control computer.
The control law in the computer is an integrator with a feedback control gain matrix $K = \left(P^T P\right)^{-1} P^T$ which is the pseudo-inverse of the poke matrix P.
The pseudo-inverse must be regularized, usually by doing a singular value decomposition and omitting the smallest eigenvalues until satisfactory performance is obtained.
Adjustable gain and leak knobs are provided for tweaking the control during operation.
The integral feedback control law follows the desired input closely and is insensitive to small mismodeling errors, such as miscalibration, nonlinearities, and distortion.

Further Studies

Merely because the integral control law seems to work well in practice does not mean that it can not be improved.

We can improve the control law by predicting the behavior of the atmospheric aberrations.   Often the atmosphere has aberrations that are distributed probabilistically with Komolgorov structure function. We are working on ways to characterize the atmospheric aberrations during telescope operation, and then use this information to improve the control law.

References

We have described feedback control only for AO systems. For an introduction to control of general systems, some good texts are:

G. C. Goodwin, S. F. Graebe, and M. E. Salgado, “Control System Design”, Prentice Hall, 2001

G. F. Franklin, J. D. Powell, and A. Emami-Naeini, “Feedback Control of Dynamic Systems”, 4th ed., Prentice Hall 2002.

For further information on control systems research in AO, see the CfAO website publications and their references.