Atmospheric Seeing.

Sources.

A. Quirrenbach notes.

Adaptive Optics In Astronomy. chapter 2

F. Roddier et-al, CUP, ISBN 0-521-55375-x

Laser Beam Scintillation with Applications.

L.C. Andrews et-al, SPIE, ISBN 0-8194-4103-1

The atmosphere is always turbulent.

Reynolds Number $R = VL/\nu$

$\nu \approx 1.5\text{e-}5$

So Reynolds number typically exceeds a million.

Turbulence is an energy cascade, from large spatial scales to small scales. On smallest scales the energy is dissipated as heat.

The inner and outer scales.

- The size of outer scale difficult to measure, and is clearly quite variable. Reported values vary between a few m and hundreds of m.
  - A small outer scale would have significant impact on the performance of large telescopes. Little evidence has turned up at existing large telescopes.
- The inner scale is somewhat variable depending on turbulence strength, but is typically in the range of 3 to 10mm.

Velocity structure function

Structure function defined by $D_v(R_{1,}R_2)=\langle |v(R_1)-v(R_2)|^2 \rangle$

In the inertial range $L_o \gg l \gg l_o$ the velocity structure function depends only on the rate of energy generation per unit mass , ε and the kinematic viscosity ν.

Dimensions ε $Js^{-1}kg^{-1}=m^2s^{-3}$, and ν m^2s^{-1}

$D_v(R_{1,}R_2)=\alpha\cdot f(|R_1-R_2|/\beta)$ $\alpha=\nu^{1/2}\epsilon^{0.5}$ $\beta=\nu^{3/4}\epsilon^{-1/4}$

In the inertial range, structure fn cannot depend on ν, thus mandating

This is usually written

$$D_v(R_{1,}R_2)=\alpha\cdot k\cdot(|R_1-R_2|/\beta)^{2/3}$$

$$D_v(R_{1,}R_2)=C_v^2\cdot|R_1-R_2|^{2/3}$$

Refractive index structure function

Turbulence alone does not produce seeing.

Must mix fluids of different refractive index (low speed).

Temperature structure function, $$D_T(R_{1,} R_2) = C_T^2 \cdot |R_1 - R_2|^{2/3}$$

Has same functional dependence as velocity structure function, since temperature variations are entrained in the turbulence velocity field.

Refractive index follows temperature.

$$N \equiv (n-1) \propto \rho$$

$$D_n(R_{1,} R-2) = D_N(R_{1,} R_2) = C_N^2 \cdot |R_1 - R_2|^{2/3}$$

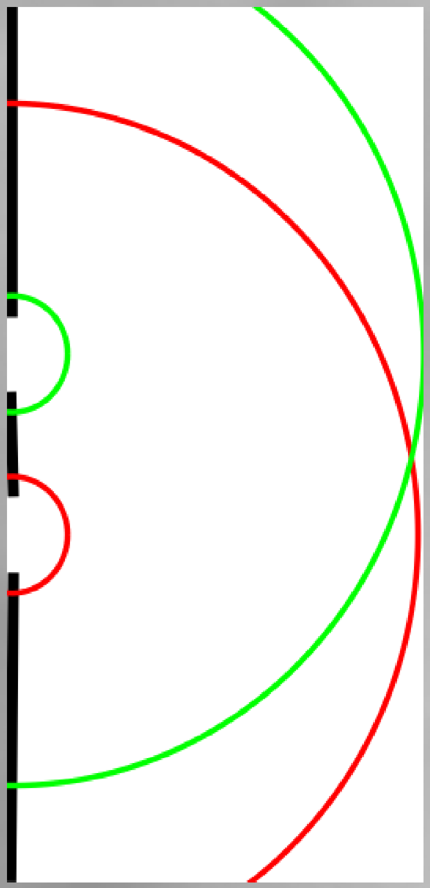
$$C_N = (7.8 \cdot 10^{-5} P[mbar]/T^2[K]) \cdot C_T$$

Wavefronts

The wavefront is a surface of equal time of flight.

The concept of a wavefront can break down when diffraction is important.

Youngs slits.



The phase structure function.

Calculation of the phase structure function requires integrating the optical phase perturbations through a layer of atmosphere.

The phase structure function is

$$D_\phi(r) \equiv \left\langle \left| \phi(x) - \phi(x+r) \right|^2 \right\rangle$$

If we assume that the wavefront phase has Gaussian statistics, there is an interesting relationship between structure function and wavefront coherence function,

$$B_h(r) \equiv \left\langle \psi(x)\psi^*(x+r) \right\rangle \equiv \left\langle \exp\left(i\left[\phi(x) - \phi(x+r)\right]\right) \right\rangle$$

$$\equiv \exp\left(-\tfrac{1}{2}\left\langle \left| \phi(x) - \phi(x+r) \right|^2 \right\rangle\right) \equiv \exp\left(-\tfrac{1}{2} D_\phi(r)\right)$$

The wavefront coherence function is the seeing contribution to the seeing limited optical transfer function.

Definition of the Fried's Parameter.

What is the phase structure function after propagating a flat wavefront through turbulence.

For sufficiently weak turbulence we can integrate the phase perturbations through the atmosphere giving:

$$D_{\phi}(r) = 6.88\left(\frac{r}{r_o}\right)^{5/3}$$

Where r_o the Fried's parameter is defined by.

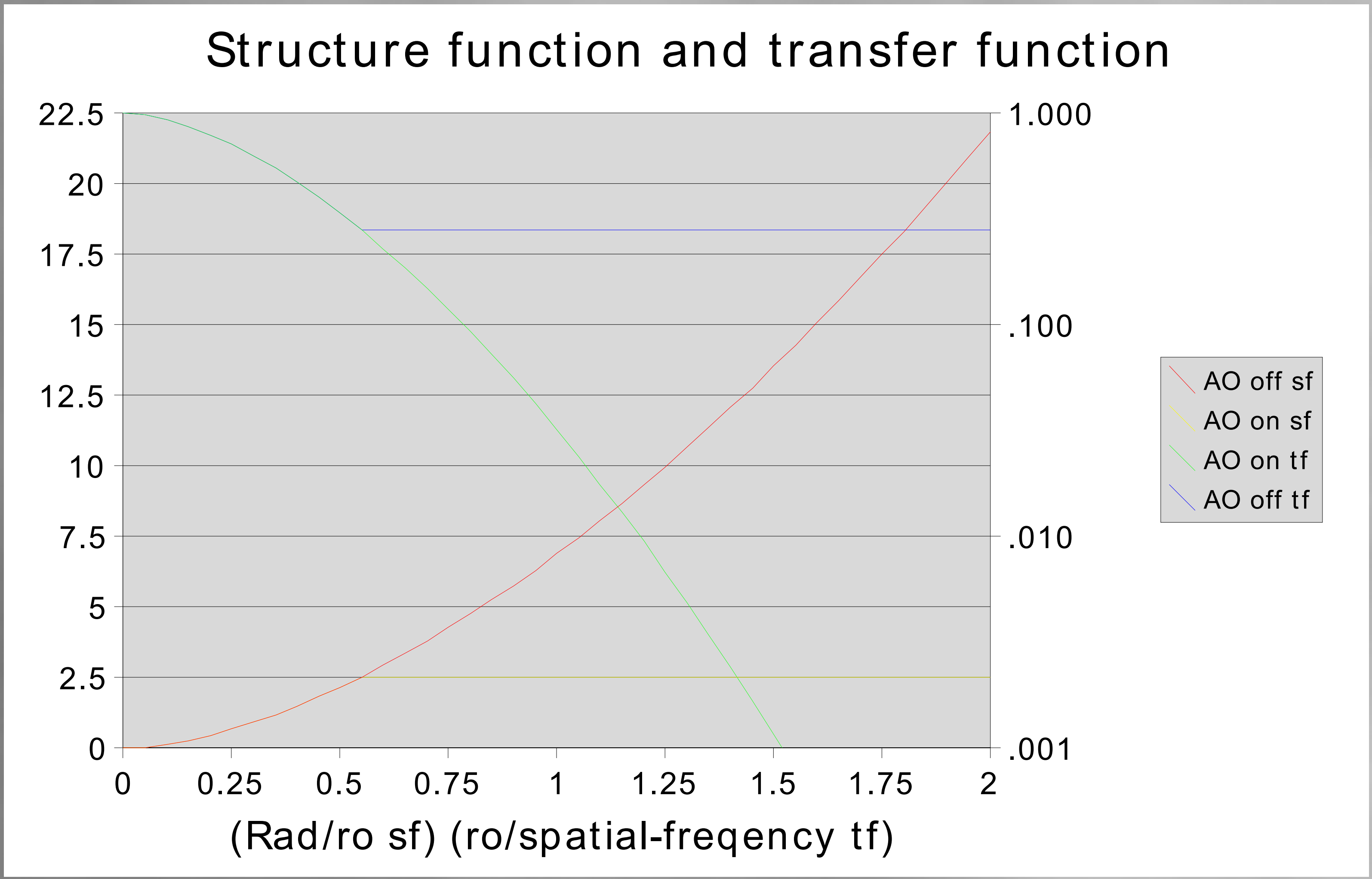
$$r_o = \left[0.423 k^2 \int C_N^2(l)\, dl\right]^{(-3/5)}$$

The RMS wavefront error within a circle of the Frieds parameter diameter is approximately 1 radian (1.03).

The FWHM of a seeing limited image is approx λ / r_o.

Strehl ratio.

Specular and diffuse components.



scaling

- Fried's parameter $r_o \propto \lambda^{6/5}$
- Image FWHM $\propto \lambda^{-1/5}$
- Greenwood and Isoplanatic angle scale like Fried's.

Rytov variance.

For plane wave defined by $\sigma_I^2 = 1.23\, C_N^2\, k^{7/6} L^{11/6}$

Rytov variance above 1 indicate strong seeing.

Rytov variance below 0.1 are considered weak seeing.

For moderate to strong seeing the concept of a wavefront is no longer useful (multipath).

For weak turbulence the Rytov variance is the variance of the light intensity in the telescope aperture. For non-uniform turbulence:

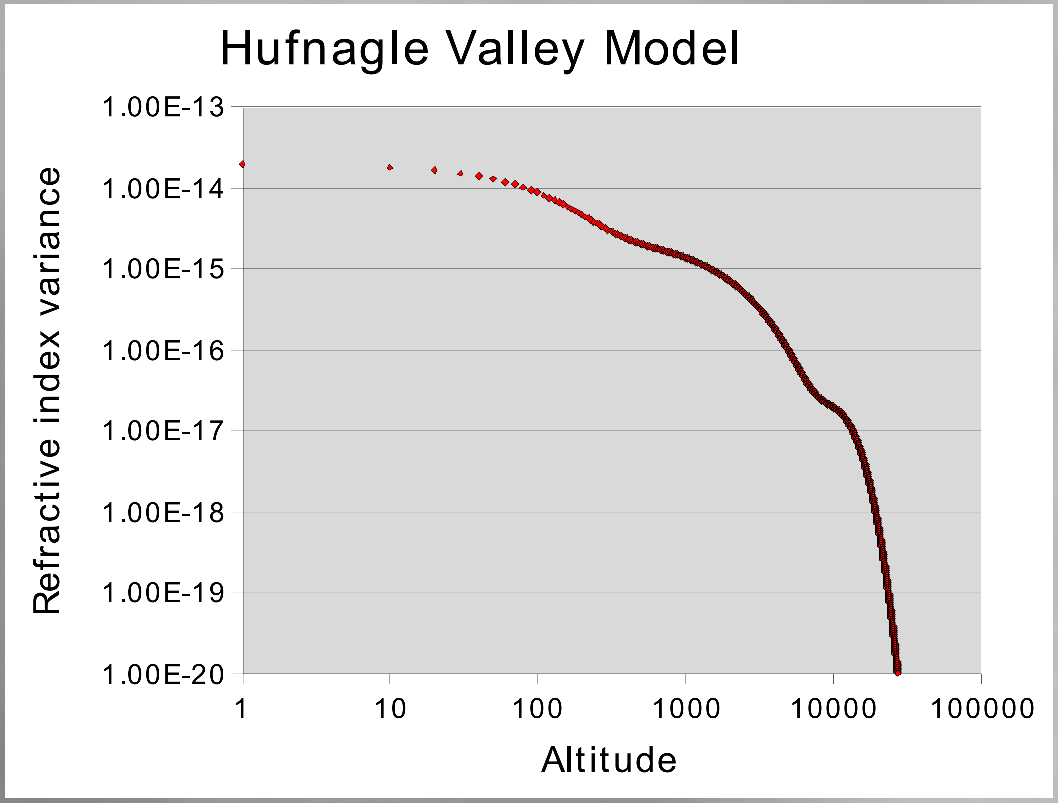
$$\sigma_I^2 = 2.26\, k^{7/6} \int C_N^2(l)\, l^{5/6}\, dl$$

Seeing tends to occur in discrete layers

- Instrument.
- Telescope/dome.
- Boundary layer.
- Jet stream layers.
- Other atmospheric flows.
- Atmospheric layers often occur in pairs (Foy?).

Hufnagle valley model.

- Hufnagle valley model represents statistical average.
  - Widely used but not believed to be accurate except over a statistical ensemble of atmospheres.



Amelioration of seeing.

- Dome and instrument seeing through careful thermal management.
- Forced air flow.
- Site selection.

Stationarity of Frieds Parameter.

In computer simulations the Strehl ratio converges to a few percent within a second of simulated observing time.

In actual observations convergence is much slower.

Real time measurements show that the effective Fried's parameter varies rapidly on times scales of a second or shorter.

Variations of a factor of 2 are typical with larger variations being common.

Makes characterization of AO performance difficult.

Greenwood frequency.

Taylor Hypothesis, or frozen phase screen.

- Assume that the turbulence evolves much more slowly than the time it takes to advect past the aperture. Reasonable assumption for sub-sonic flows.

Greenwood frequency.

- Time over which the wavefront phase decorrelates by 1 radian RMS. For a single layer simply r_o/v

Isoplanatic angle.

Cause of isoplanatic angle.

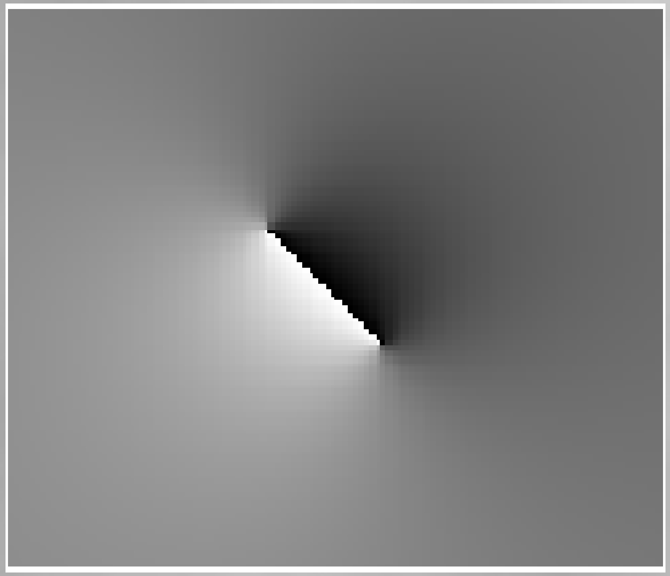
- As beams propagating in different directions through a series of turbulence layers, each beam will sample slightly different turbulence.
- Isoplanatic angle is hugely variable, running from a few arc-seconds up to an arc minute at MKO (in the IR).
- In extreme turbulence situations the isoplanatic angle can approach the diffraction limit of the telescope.

Severe turbulence.

Wavefront model breaks down.

When interference effects become strong enough to cause zeros, positive and negative poles (screw dislocations occur). Location and number of poles is wavelength dependent.

Phase due to pole pair



Ref:

Fried, Vaughn, Branch cuts in the phase function Applied Optics 15, 1992