Tomography for Multiconjugate Adaptive Optics Systems using Laser Guide Stars

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Outline:

- Why tomography?
- MCAO system concepts
- An MCAO reconstruction algorithm that works for laser guide star beams
- Simulation examples
- Conclusions

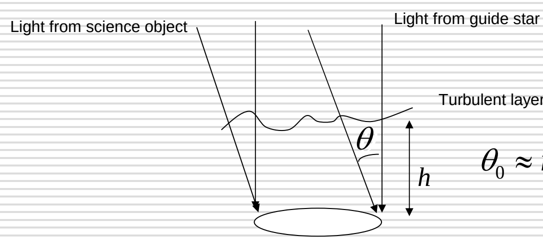
Why tomography on ELTs*?

- AO correction on a wide field
  multiplex the science
- Reduce laser guide star cone effect
  more severe on larger telescopes

*ELT = Extremely Large Telescopes (i.e. D > 10 meters)

Anisoplanatism error

- If the guide star is not the science object...

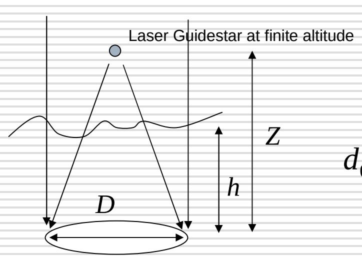


$$\theta_0 \approx r_0 / h$$

- Typical: r_0 = 20 cm, θ_0 = 4 arcsec at λ = 0.5 μ
- Mean height of turbulence: $h_0 = r_0/\theta_0$ = 10 km

Residual wavefront error: $\sigma_A^2 = (\theta/\theta_0)^{5/3}$

Laser guidestar specific error: Cone effect

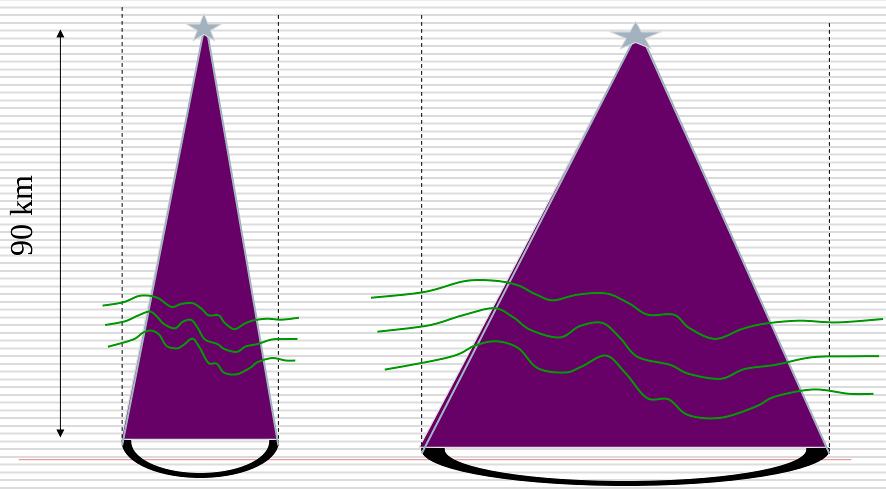


$$d_0 \approx 2 \frac{Z}{h} r_0$$

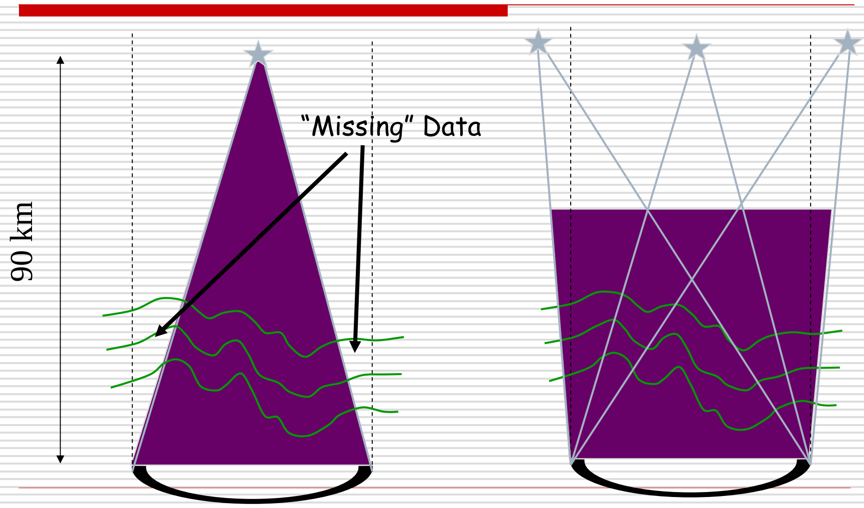
- Example: h=8 km, r_0=20cm at λ = 0.5 μ
- d_0=4.5m

Residual wavefront error: $\sigma_{cone}^2 = (D/d_0)^{5/3}$

Cone effect gets worse as telescope size increases



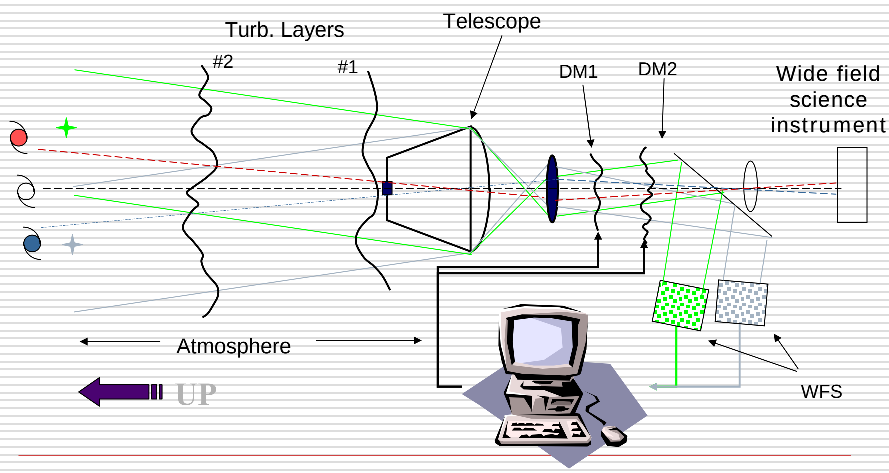
Mitigating cone effect with multiple laser guidestars



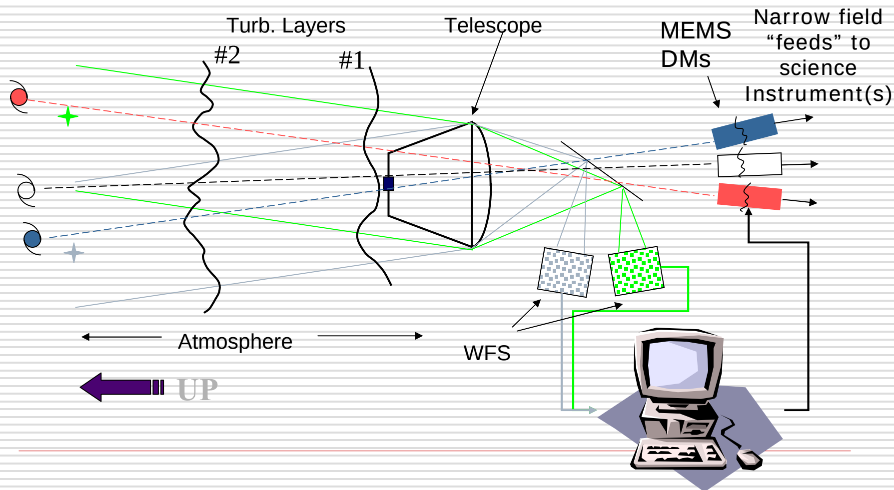
AO system concepts for the near IR bands

- MCAO – Multi-conjugate adaptive optics
  - Diffraction-limited imaging
  - Correction over a “wide” field (~30 arcsec)
  - Multiple DMs, Multiple LGS
- MOAO – Multi-object adaptive optics
  - Diffraction-limited spectroscopy (IFUs)
  - Correction over small fields (~5 arcsec) at many focal plane positions
  - Each unit has a single DM, corrects for the science direction
  - Multiple LGS measure the volume of turbulence

Multiconjugate AO Set-up

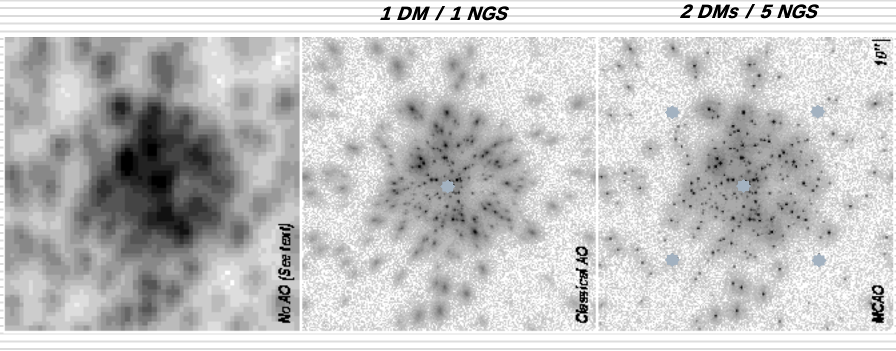


Multi-Object AO Setup



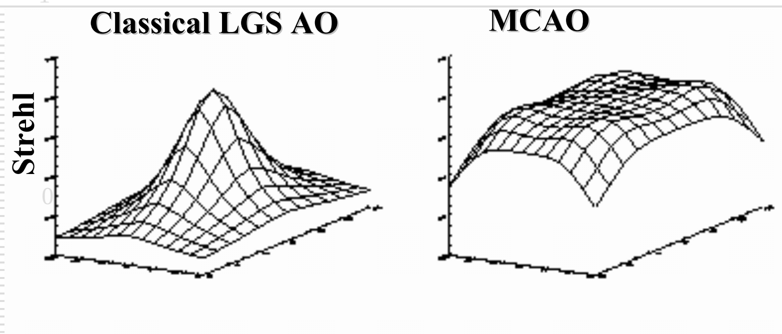
MCAO performance example

No AO	Classical AO	MCAO
	1 DM / 1 NGS	*2 DMs / 5 NGS*



320 stars / K band / 0.7'' seeing *165''* *Stars magnified for clarity*

MCAO Performance



Surface plots of Strehl ratio over a 1.5 arc min FoV. 13x13 actuator system, K band, CP turbulence.

MCAO Pros and Cons

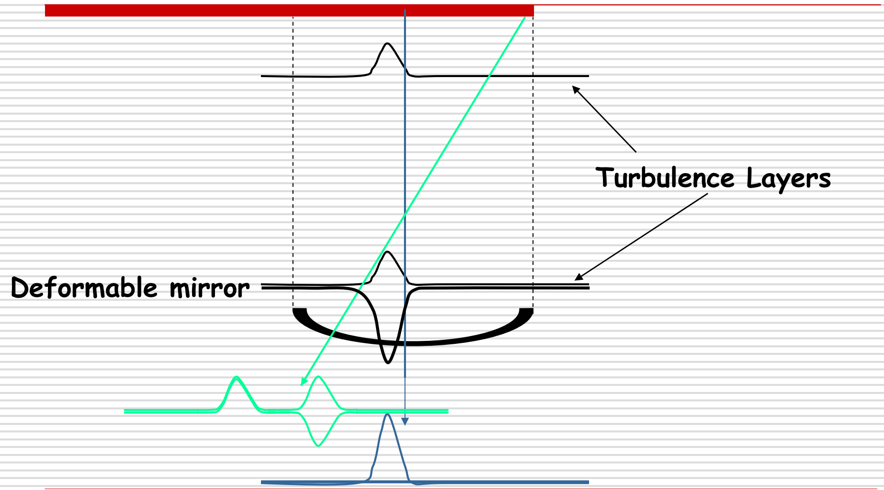
PROS:

- **Enlarged Field of View**
  - PSF variability problem drastically reduced
- **Cone-effect solved**
- Gain in SNR (less sensitive to noise, predictive algorithms)
- Marginally enlarged Sky Coverage (LGS systems)

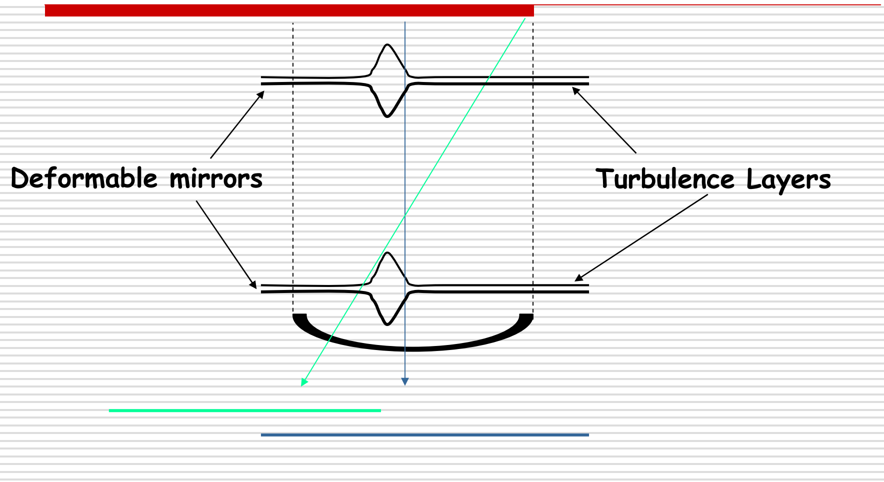
CONS

- **Complexity:** Multiple Guide stars and DMs
- **Other limitations**: Generalized Fitting, anisoplanatism, aliasing

Why multiconjugate?

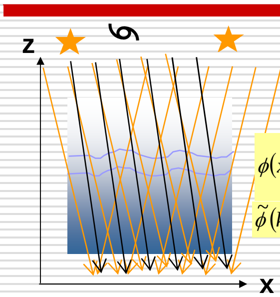


Why multiconjugate?



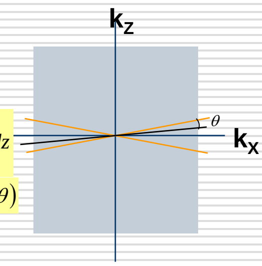
What is tomography?

Fourier slice **theorem (Kak, 1988)**



$$\phi(x) = \int_0^\infty \delta n(x - \theta z, z)\,dz$$

$$\tilde{\phi}(k_x) = \delta\tilde{n}(k_x, k_z = k_x\theta)$$



$$\theta k_x < \Delta k_z$$

$$\Delta k_z \Delta z = 1$$

$$\Downarrow$$

$$k_x < 1/\theta\,\Delta z$$

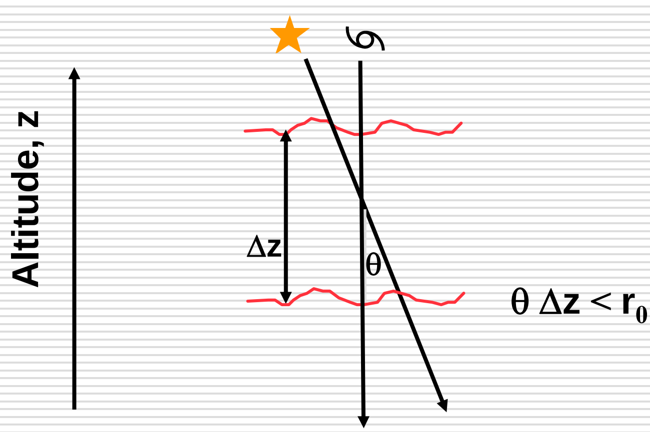
$$\theta\,\Delta z < r_0$$

- Each wavefront sensor measures the integral of index variation along the ray lines
- The line integral along z determines the k_z=0 Fourier spatial frequency component
- Projections at several angles sample the k_x,k_y,k_z volume

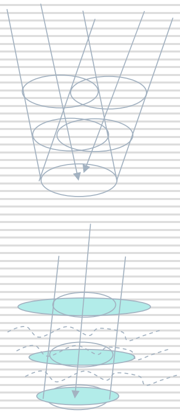
How many guide stars are needed?

- Each line in k-space has a thickness ~ thinness of layers in atmosphere
- Fill desired k_x space over bow-tie of field angles
- Angle between guidestars must be ~less than the ratio of r_0 to layer thickness

Spatial interpretation of the k_x<1/[Δz(θ-θ_{gs})] requirement



The M*AO wavefront reconstruction problem



Measurements from guide stars:

$$s_k(x) = M\left(\phi(x,\theta_k)\right) + \nu_k(x) \qquad k = 1,\ldots n_{gs}$$

s = wfs slopes

ϕ = atmospheric phase integrated along direction θ

M = wfs operator

ν = wfs noise

Problem(s):

- MCAO: Find commands for muli-conjugate DMs that best correct the wavefront in any direction over a given field of view
- MOAO: Find the best estimate of the wavefront in a particular science direction

Spatial domain reconstructors[1]

- Obvious extension of present single-conjugate AO reconstructor methods

Control on each DM → $\psi(\mathbf{x}, z)$; Reconstructor matrix → $\mathbf{G}$; Measurements from each WFS,… → $\mathbf{s}[\,\cdot\,]$; …a function of index of refraction variations along the path → $\delta n(\mathbf{x}, z)$

$$\psi(\mathbf{x}, z) = \mathbf{G}\mathbf{s}\left[\delta n(\mathbf{x}, z)\right]$$

- Minimum-variance solution:

$$\mathbf{G} = \mathbf{\Sigma}_{\delta n}\mathbf{H}^T\left(\mathbf{H}\mathbf{\Sigma}_{\delta n}\mathbf{H}^T + \mathbf{\Sigma}_{\upsilon}\right)^{-1} \quad \text{where} \quad \mathbf{H} = \frac{\partial \mathbf{s}}{\partial \psi}$$

Covariance of refractive index variations → $\mathbf{\Sigma}_{\delta n}$; Covariance of measurement noise → $\mathbf{\Sigma}_{\upsilon}$; $\mathbf{H}$: (“push” matrix)

[1]Ellerbroek, B., *Efficient computation of minimum-variance wave-front reconstructors with sparse matrix techniques*, JOSA-A, 19, 9, Sept. 2002, pp 1803-1816.

An aside:

Minimum variance solutions

- The normal equation:

$$\boldsymbol{\psi} = \mathbf{G}\mathbf{s}$$

$$\min_{\mathbf{G}} J = \left\langle \left| \delta n - \boldsymbol{\psi} \right|^2 \right\rangle$$

$$\frac{\partial J}{\partial G} = 0 \Rightarrow \left\langle (\delta n - \boldsymbol{\psi}) \mathbf{s}^T \right\rangle = 0$$

$$\left\langle \delta n \mathbf{s}^T \right\rangle - \mathbf{G} \left\langle \mathbf{s}\mathbf{s}^T \right\rangle = 0$$

$$\Rightarrow \mathbf{G} = \left\langle \delta n \mathbf{s}^T \right\rangle \left\langle \mathbf{s}\mathbf{s}^T \right\rangle^{-1}$$

(For the sake of simplifying the explanation we're imagining here that atmospheric turbulent layers exist only at DM conjugate locations. We'll treat the more general case later on.)

- All minimum variance solutions have this form

The best linear reconstructor is the cross-correlation of the unknown with the data, times the inverse of the auto-correlation of the data

Minimum variance solution

- Assuming a linear model for WFS response to index variations:

$$\mathbf{s} = \mathbf{H}\delta n + \boldsymbol{\upsilon}$$

H represents the line integral along z through the turbulence, followed by the x,y gradient operations done by the Hartmann (slope) wavefront sensor.

- We use the normal equation to determine the reconstructor matrix

$$\mathbf{G} = \left\langle \delta n \mathbf{s}^T \right\rangle \left\langle \mathbf{s}\mathbf{s}^T \right\rangle = \boldsymbol{\Sigma}_{\delta n} \mathbf{H}^T \left(\mathbf{H} \boldsymbol{\Sigma}_{\delta n} \mathbf{H}^T + \boldsymbol{\Sigma}_{\upsilon} \right)^{-1}$$

Relationship of minimum variance to least-squares

$$\mathbf{G} = \langle \delta n \mathbf{s}^T \rangle \langle \mathbf{s}\mathbf{s}^T \rangle$$

$$= \boldsymbol{\Sigma}_{\delta n} \mathbf{H}^T \left(\mathbf{H} \boldsymbol{\Sigma}_{\delta n} \mathbf{H}^T + \boldsymbol{\Sigma}_{\upsilon} \right)^{-1}$$

$$= \left(\mathbf{H} \boldsymbol{\Sigma}_{\upsilon}^{-1} \mathbf{H}^T + \boldsymbol{\Sigma}_{\delta n}^{-1} \right)^{-1} \boldsymbol{\Sigma}_{\upsilon}^{-1} \mathbf{H}^T$$

Matrix inversion theorem

In the limit of high signal-to-noise: $\Sigma_\upsilon \to I\sigma_\nu^2$ small

$$= \left(\mathbf{H}\mathbf{H}^T \right)^{-1} \mathbf{H}^T$$

Least-squares solution (best fit to measurement): Moore-Penrose pseudo-inverse

Fourier domain approach

- A function is represented as a sum of sine waves

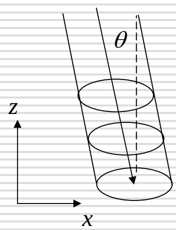
f is spatial frequency from now on. Sorry for the change of notation but I need *k* to enumerate guide stars.

$$\phi(\mathbf{x}) = \iint \tilde{\phi}(\mathbf{f}) e^{i 2\pi \mathbf{f}\cdot\mathbf{x}} \mathrm{d}f_x \mathrm{d}f_y$$

We're taking the Fourier transforms only in the transverse (x,y) plane

- Shifts Δx in the spatial domain -> sine waves add $2\pi f \Delta x$ phase, or multiply Fourier coefficent by $e^{2\pi i f \Delta x}$

Phase at the pupil from a source at field position θ



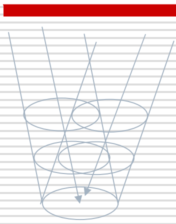
$$\Delta x = \theta z$$

$$\phi(x) = \int_0^\infty \delta n(x - \theta z)\,dz$$

$$\downarrow$$

$$\tilde{\phi}(f) = \int_0^\infty \delta \tilde{n}(f, z)\, e^{i2\pi \mathbf{f}\cdot\boldsymbol{\theta} z}\,dz$$

A minimum variance solution in the Fourier domain[2]



Measurements from guide stars:

$$\tilde{s}_k(f) = M(f)\tilde{\phi}(f, \theta_k) + \tilde{v}_k(f) \qquad k = 1, \ldots n_{gs}$$

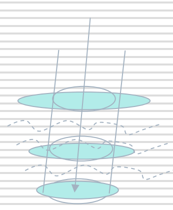
s = wfs slopes

ϕ = atmospheric phase integrated along direction θ

M = wfs operator

v = wfs noise

We'll assume M=1, for simplicity for now, but with a justification later



Problem: MOAO: Find a linear combination of guide star data that best predicts the wavefront in a given science direction, θ

$$\tilde{\psi}(f, \theta) = \sum_{k=1}^{N} \tilde{g}_k(f, \theta)\tilde{s}_k(f) = \tilde{\mathbf{g}}^T \tilde{\mathbf{s}}$$

We'll deal with the MCAO case later

[2]Tokovinin, A., Viard, E., "*Limiting precision tomographic phase estimation*," **JOSA-A**, 18, 4, Apr. 2001, pp873-882.

Fourier domain reconstructor

- The normal equation in the Fourier domain is:

$$\left\langle \left(\tilde{\phi} - \tilde{\psi}\right)\tilde{s}_k \right\rangle = 0$$

- so the minimum variance solution is

$$\tilde{g}_k(f) = S_{\phi\phi_{k'}}(f)\left[S_{\phi_k\phi_{k'}}(f) + S_\upsilon(f)\right]^{-1}$$

← One equation per frequency

↖ Matrix is n_{gs} x n_{gs}

where

$$S_\phi(f) = \left\langle \tilde{\phi}(f)\tilde{\phi}_k(f) \right\rangle$$

$$S_\upsilon(f) = \left\langle \tilde{\upsilon}_k(f)\tilde{\upsilon}_{k'}(f) \right\rangle$$

$$S_s(f) = \left\langle \tilde{s}_k(f)\tilde{s}_{k'}(f) \right\rangle = S_\phi(f) + S_\upsilon(f)$$

Minimum variance solution in the Fourier domain

$$\tilde{\mathbf{g}}^T(\mathbf{f}) = \tilde{\mathbf{c}}^{T*}(\mathbf{f})\left[\mathbf{A}^*(\mathbf{f}) + \upsilon'^*(\mathbf{f})\mathbf{I}\right]^{-1}$$

where

$$\tilde{c}_k(\mathbf{f}) = \int C_n^2(h) e^{-2\pi i \mathbf{f}(\boldsymbol{\theta}_k - \boldsymbol{\theta})h}\, dh \;\; / c_0 \qquad = \left\langle \tilde{\phi}\tilde{\phi}_k \right\rangle / W_0 c_0$$

$$\tilde{a}_{kk'}(\mathbf{f}) = \int C_n^2(h) e^{-2\pi i \mathbf{f}(\boldsymbol{\theta}_k - \boldsymbol{\theta}_{k'})h}\, dh \;\; / c_0 \qquad = \left\langle \tilde{s}_k \tilde{s}_{k'} \right\rangle / W_0 c_0$$

$$c_0 = \int C_n^2(h)\, dh \qquad c_0 = 2.36(\lambda/2\pi) r_0^{-5/3}$$

$$\upsilon'(\mathbf{f}) = W_\nu(\mathbf{f}) / (W_0(\mathbf{f}) c_0)$$

Noise-to-'signal' ratio

$$W_0(\mathbf{f}) = 9.69 \times 10^{-3} (2\pi/\lambda)^2 f^{-11/3}$$

Kolmogorov power spectrum of turbulence

An interpretation[3]...

$$\tilde{\psi}(f,\theta)=\int e^{2\pi i\theta hf}C_n^2(h)\sum_{k=1}^{N}e^{-2\pi i\theta_k hf}\tilde{s}_k'(f)dh \;/c_0$$

$$\tilde{\mathbf{s}}'(f)=\left[\mathbf{A}^*(f)+\mathbf{I}\upsilon'^*(f)\right]^{-1}\tilde{\mathbf{s}}(f)$$

"Consistency-filtered" wavefront measurement

$$\psi(x,\theta)=\int C_n^2(h)\sum_{k=1}^{N}s_k'(x-h(\theta-\theta_k))dh \;/c_0$$

$$\psi(x,\theta)=\int \delta n_{est}(x-h\theta,h)dh$$

$$\delta n_{est}(x,h)=\sum_{k=1}^{N}C_n^2(h)s_k'(x+h\theta_k)\;/c_0$$

[3]Gavel, *Tomography for multiconjugate adaptive optics systems using laser guide stars*, SPIE 5490, June, 2004.

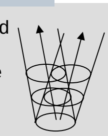
The minimum variance solution = back projection tomography

Measure

$$s_k(x,\theta)=\int \delta n(x-h\theta_k,h)dh+\upsilon_k$$

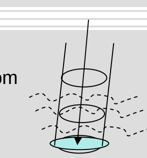
Reconstruct

1 Back-Project filtered measurements through the volume toward the guide stars



$$\delta n_{est}(x,h)=\sum_{k=1}^{N}C_n^2(h)s_k'(x+h\theta_k)\;/c_0$$

2 Propagate light from Science target



$$\psi(x,\theta)=\int \delta n_{est}(x-h\theta,h)dh$$

Why the Hartmann wavefront sensor model can be "ignored" (M=1)

without loss of generality

$$\mathbf{s}_k(x) = \nabla\phi_k(x) + \boldsymbol{\upsilon} \Rightarrow \tilde{\mathbf{s}}_k(f) = -2\pi i \mathbf{f}\tilde{\phi}_k(f) + \tilde{\boldsymbol{\upsilon}}$$

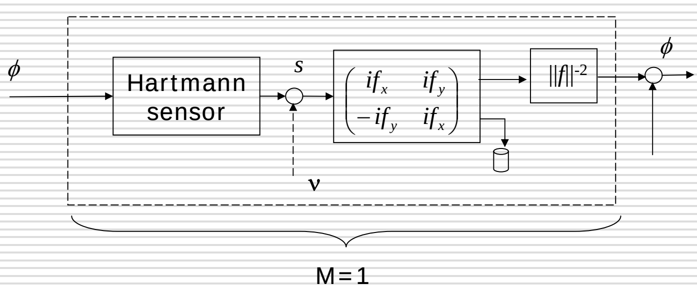
Perform an invertible transformation on the WFS data

$$\begin{pmatrix} \tilde{s}_0(f) \\ \tilde{s}_1(f) \end{pmatrix} = \begin{pmatrix} if_x & if_y \\ -if_y & if_x \end{pmatrix} \begin{pmatrix} \tilde{s}_x(f) \\ \tilde{s}_y(f) \end{pmatrix} = 2\pi \begin{pmatrix} f_x^2 + f_y^2 \\ 0 \end{pmatrix} \phi + \begin{pmatrix} if_x\nu_x + if_y\nu_y \\ -if_y\nu_x + if_x\nu_y \end{pmatrix}$$

- Since the 2nd equation doesn't depend on the data, $M = \|f\|^2$ is an appropriate scalar measurement model (the 1st equation above).
- Without loss of generality, we'll assume the WFS has a post-filter $1/2\pi\|f\|^{-2}$. This allows us to set M=1 (up to some frequency cut-off associated with the WFS spatial sampling). The noise spectrum is modified accordingly $\langle \nu(f)^2 \rangle \sim 1/f^2$

i.e. We can assume the wavefront sensors measure phase directly

The "direct phase" Hartmann sensor



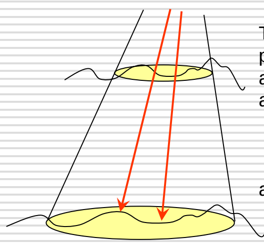
Important: you must consider the boundary conditions when using the Fourier transform on aperture-masked slope data. See:

[4]Poyneer, L., Gavel, D., and Brase, J., *Fast wave-front reconstruction in large adaptive optics systems with use of the Fourier transform*, **JOSA-A**, 19, 10, October, 2002, pp2100-2111.

We can extend the back projection method to **spherical waves**, resulting in a tomography solution for **guide stars at finite altitude, Z**

Spatial domain

Forward propagation



Turbulence at position x at altitude h appears at position

$$x\left(\frac{Z-h}{Z}\right)-\theta h$$

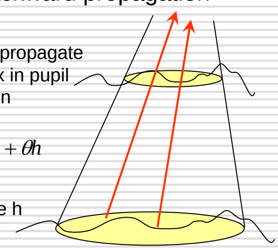
at the pupil

Backward propagation

So back-propagate position x in pupil to position

$$x\left(\frac{Z}{Z-h}\right)+\theta h$$

at altitude h



Frequency domain

Frequencies f at altitude h scale down to frequencies

$$f\left(\frac{Z-h}{Z}\right)$$

at the pupil

Frequencies f at the pupil scale up to frequencies

$$f\left(\frac{Z}{Z-h}\right)$$

at altitude h

Where Z = altitude of guide star

Guide Stars at Finite Altitude: The Math[3]

Minimum error variance spherical wave tomography formulae

Back-project

$$\delta n_{est}(\mathbf{x},h)=\sum_{k=1}^{N}C_n^2(h)s_k'\left(\mathbf{x}\frac{Z}{Z-h}+h\boldsymbol{\theta}_k\right)\ /c_0$$

Forward propagate

$$\psi(x,\theta)=\int\delta n_{est}(x-h\theta,h)dh$$

Consistency filters

$$\tilde{\mathbf{s}}'(f)=\left[\mathbf{A}^{s*}(f)+\mathbf{I}\upsilon'^{*}(f)\right]^{-1}\tilde{\mathbf{s}}(f)$$

$$c_k^s(\mathbf{f},h)=\left(\mathrm{Z}/(\mathrm{Z}-h)\right)^{-11/3}C_n^2(h)\exp(-2\pi i\mathbf{f}\cdot\boldsymbol{\theta}_k h)\ /c_0$$

$$a_{kk'}^s(\mathbf{f})=\int_0^Z\left(\mathrm{Z}/(\mathrm{Z}-h')\right)^{-5/3}C_n^2(h')\exp\left(2\pi i\mathbf{f}\cdot(\boldsymbol{\theta}_{k'}-\boldsymbol{\theta}_k)h'\right)\mathrm{d}h'\ /c_0$$

$$v_{kk'}'(\mathbf{f})=v_k^2(\mathbf{f})\delta_{kk'}/\left(9.69\times10^{-3}f^{-11/3}c_0\right)$$

Covariance & PSF calculation

$$S_\varepsilon(f)=c_0W_0(f)\left[1-\int_0^Z\int_0^Z\sum_k\tilde{g}_k(\mathbf{f},h)\times\left(\frac{Z}{Z-h'}\right)^2\left(\frac{Z-h}{Z-h'}\right)^{-11/3}C_n^2(h')e^{-2\pi i\mathbf{f}\cdot(\boldsymbol{\theta}_k-\boldsymbol{\theta})\frac{Z-h}{Z-h'}h'-2\pi i\mathbf{f}\cdot\boldsymbol{\theta}h}\mathrm{d}h\mathrm{d}h'\ /c_0+\int_0^z\sum_k\left|\tilde{g}_k(\mathbf{f},h)\right|^2v_k'\left(\mathbf{f}\frac{Z-h}{Z}\right)\mathrm{d}h\right]$$

$$D_\varepsilon(x)=FT^{-1}\{S_\varepsilon(f)\}$$

$$MTF(x)=MTF_{Telescope}(x)\times\exp\left[-\frac{1}{2}D_\varepsilon(x)\right]$$

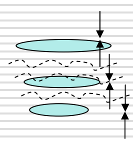
$$PSF=\left|FT\{\tau(x)\}\right|^2$$

[3]Gavel, *Tomography for multiconjugate adaptive optics systems using laser guide stars*, SPIE 5490, June, 2004.

For MCAO one can optimally project the volume estimates onto a finite number of DMs to minimize generalized anisoplanatism[5]

$$\tilde{d}_m(f) = \int \tilde{g}_m^{DM}(f,h)\,\delta\tilde{n}_{est}(f,h)\,dh \qquad m = 1,\ldots,n_{DM}$$

DM commands — Combining gains — Index estimates



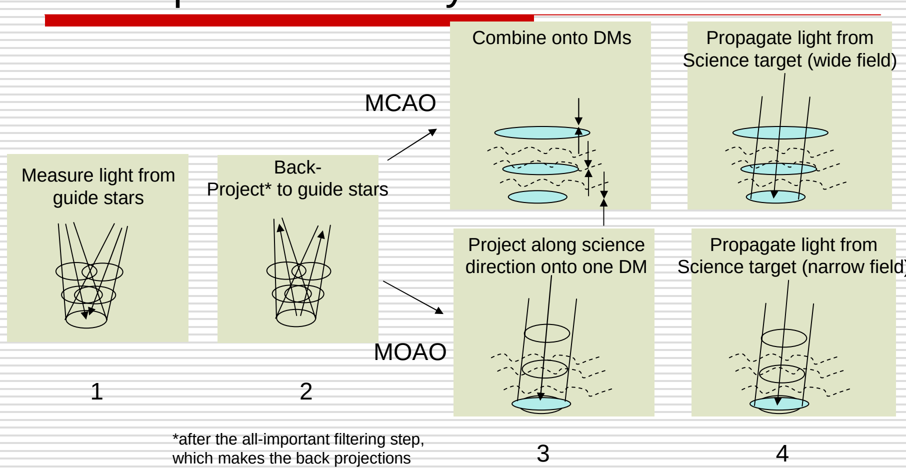
$$\tilde{\mathbf{g}}^{DM}(f,h) = \mathbf{A}^{DM\,-1}(f)\,\tilde{\mathbf{b}}^{DM}(f,h)$$

$$\tilde{b}_{m'}^{DM} = J_0\left[2\pi f \Theta (H_m - h)\right]$$

$$\tilde{a}_{mm'}^{DM} = J_0\left[2\pi f \Theta (H_m - H_{m'})\right]$$

[5]Tokovinin, A., Le Louarn, M., Sarazin, M., "*Isoplanatism in a multiconjugate adaptive optics system*," **JOSA-A**, 17, 10, Oct. 2000, pp1819-1827.

Tomographic Wavefront Reconstruction
a quick summary



*after the all-important filtering step, which makes the back projections consistent with all the data

Tomography solution domains: a comparison

Spatial Domain

- Obvious extension of present AO reconstructor methods
- Big matrix calculations; requires large number of computations O(n^2)

$$\boldsymbol{\psi} = \mathbf{G}M\left[\phi(\mathbf{x})\right]$$

$$\mathbf{G} = \boldsymbol{\Sigma}_{\delta n}\mathbf{H}^T\left(\mathbf{H}\boldsymbol{\Sigma}_{\delta n}\mathbf{H}^T + \boldsymbol{\Sigma}_{\upsilon}\right)^{-1}$$

- Requires reformulating for every parameter change
- Not obviously "tomography"

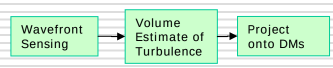
Fourier Domain

- The natural "Karhunen-Loeve" space (uncorrelated modes)
- Smaller calculational problem O(nlogn)

$$\tilde{\psi}(f) = \tilde{\mathbf{g}}^T(f)\tilde{\mathbf{s}}(f)$$

$$\tilde{\mathbf{g}}(f) = S_\phi(f)\left(S_\phi(f) + S_\upsilon(f)\right)^{-1}$$

- It is tomography!
- Insight –breaks into parts



- Finite aperture causes difficulties

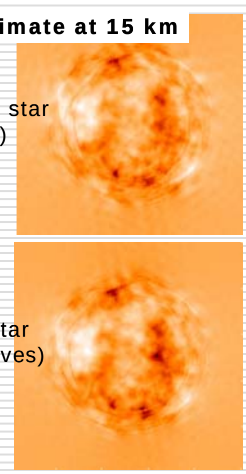
Problem from finite apertured data: Artifacts due to the sharp boundary

- Let's mimic the infinite aperture algorithm, i.e. back-prop filtered measurements, but only from a finite aperture
- Artifacts from the hard edges of the aperture degrade the reconstruction
- Affects both NGS and LGS, but LGS more severely
- An "extension" scheme fixes the problem. Extension is the minimum-variance estimate of the unknown data given the known data

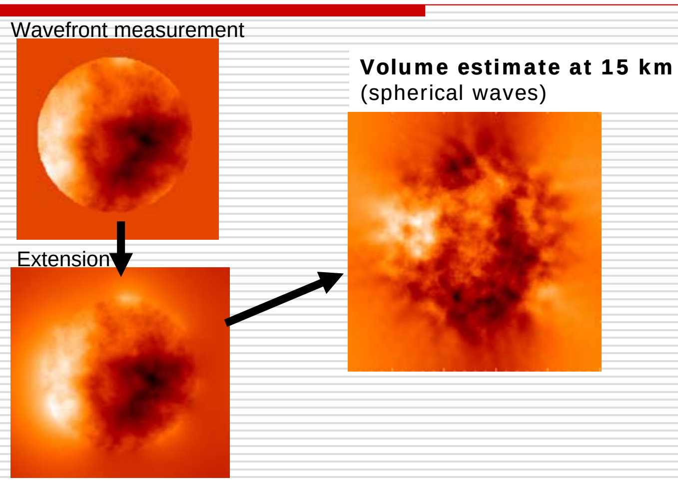
Volume estimate at 15 km

Natural guide star (plane waves)

Laser guide star (spherical waves)



Minimum Variance *Extension* of wavefront measurements to outside the aperture

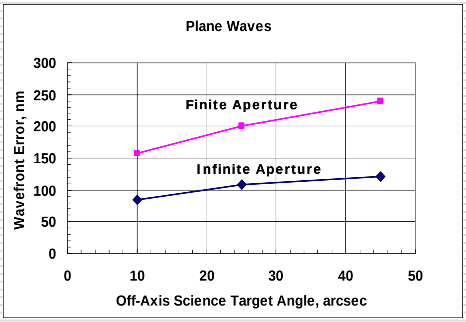


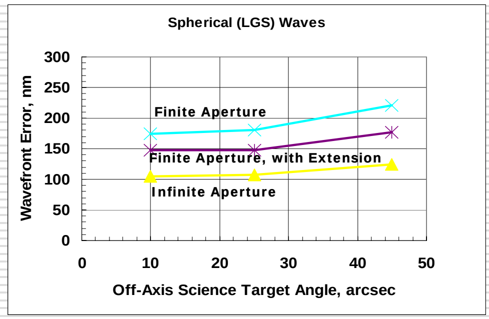
Simulation results

- Telescope Diameter D = 30 meters
- r_0 = 20 cm, CP C_n^2 profile (7 layer)
- z_{LGS} = 90 km
- Tomography only (no anisoplanatism or other error budget terms)

- Compare NGS (plane wave) and LGS (spherical wave)
- Compare Fourier and Spatial Domain solutions

Tomographic Error --- Generalized Cone Effect

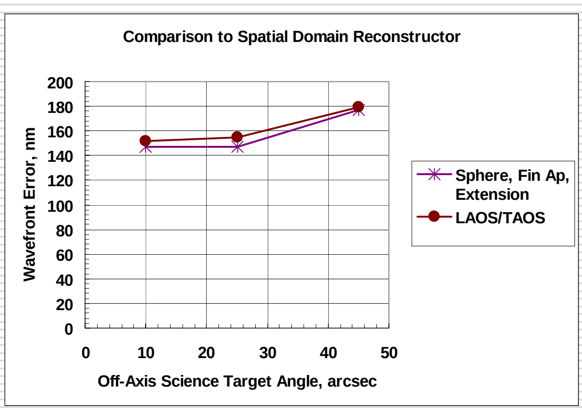




Two types of “cone” effects:

- Expanding of spherical waves
- Finite aperture - by far the biggest effect, partially offset by extension

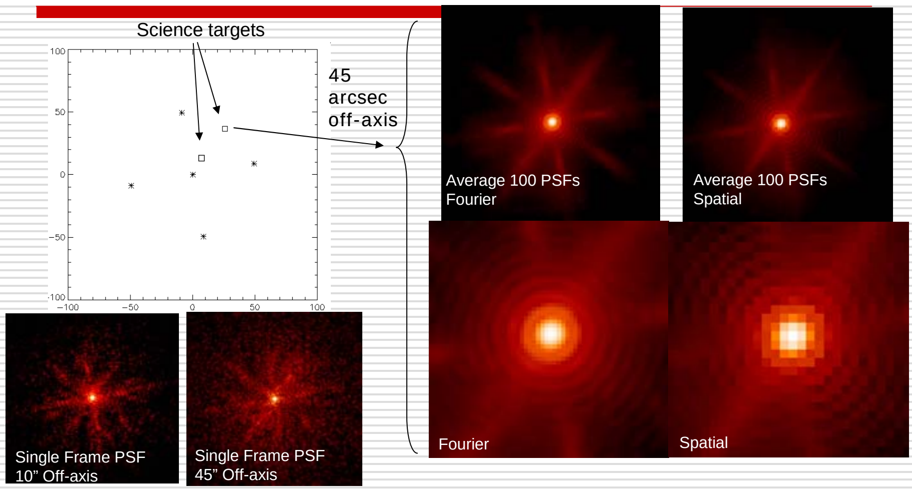
Results compared to a spatial domain reconstructor*



*Linear Adaptive Optics Simulator, Tools for Adaptive Optics Simulation – Brent Ellerbroek, NOAO

Point Spread Function

Finite Aperture, 90km LGS, with Extension, λ = 2 microns



Conclusions

- Multiple guide-star AO provides
  - a wide field of correction
  - a means of defeating laser guide star cone-effect error
- Two M*AO system architectures:
  - MCAO – one DM per turbulent layer
  - MOAO – one DM per science target
- Minimum variance M*AO reconstructors estimate the index variations in the atmospheric volume above the telescope
- One minimimum variance reconstructor (Fourier domain method) is intuitively interpreted as back-projection tomography

Conclusions

- Fourier domain methods are
  - computationally fast
  - provide problem insight (the solution breaks into a series of independent operations, allowing easier parametric design and analysis)
  - can be applied to NGS (plane waves) and LGS (spherical waves) beacons
  - and there is very close simulation agreement between spatial-domain and Fourier domain minimum-variance solutions

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