# Method of phase diversity in multi-aperture systems utilizing individual sub-aperture control

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# Outline

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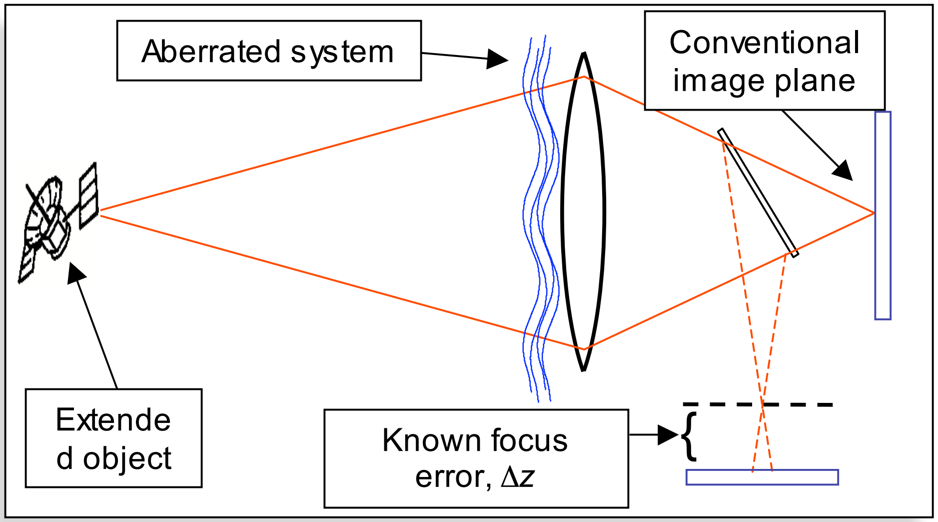
# Motivation



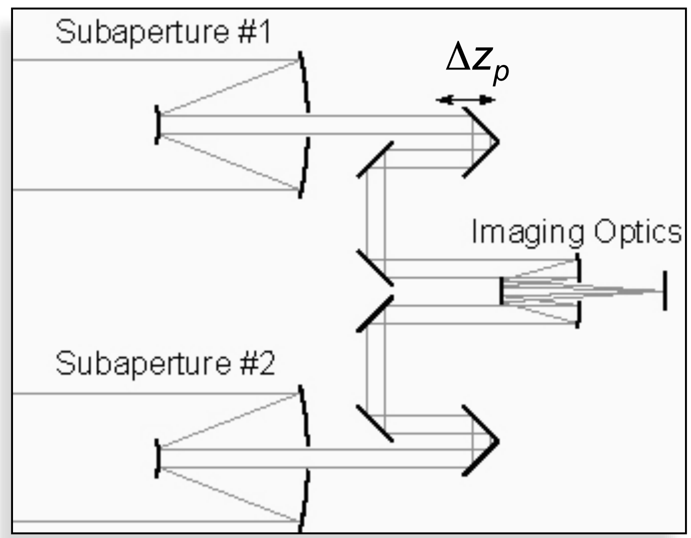

- In a conventional set-up, a global defocus is added in the second image by moving the detector

$$\phi_{foc} = \frac{\pi}{\lambda}\left(\frac{\Delta z}{f^2}\right) r^2$$

- For sub-aperture piston phase diversity, each sub-aperture can be controlled to introduce piston diversity in additional images

$$\phi_{sub,p} = \frac{4\pi}{\lambda}\Delta z_p$$

sub-aperture index

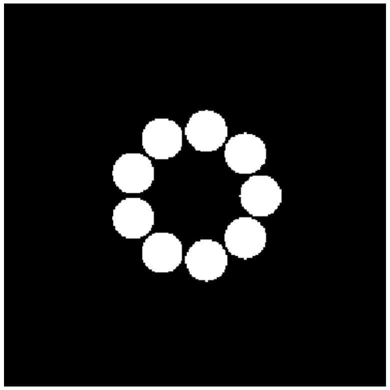


# Motivation

- Sub-aperture piston phase diversity advantages
  - No new hardware required
  - Easily integrated with multi-aperture FTIS operation
  - Piston diversity is applied over the entire image plane allowing for greater variation in scene statistics

- Sub-aperture piston phase diversity disadvantages
  - Objects cannot change faster than the time it takes to collect all the diversity images
    - FTIS operation has same limitation

# System Model



- Pupil
  - 9 apertures, circularly arrayed, no central obscurations
  - *f*/30 system
- Monochromatic Light, $\lambda$ = 591.25 nm
  - $2\pi$ piston errors are undetectable
- Gaussian Noise
  - Average 20,000 photons / pixel
  - $\sigma$ = 100
  - SNR = 200
- Scene
  - 256 x 256 urban scene
  - Shown at far right blurred by OTF and ~0.25$\lambda$ RMS unknown aberration

# System Model

- Complex pupil function given by:

$$H_k(u) = \sum_{p=1}^{P} H_{k,p}(u) = \sum_{p=1}^{P} A_p(u) \exp\left\{ i \frac{2\pi}{\lambda} \left[ \theta_{div,k,p}(u) + \sum_{j=1}^{J} \alpha_{p,j} \phi_j(u) \right] \right\}$$

$H_{k,p}(u)$ → $p^{th}$ Sub-aperture complex pupil function for $k^{th}$ image

$A_p(u)$ → $p^{th}$ sub-aperture amplitude transmission function

$\theta_{div,k,p}(u)$ → $p^{th}$ sub-aperture phase diversity for $k^{th}$ image

$\alpha_{p,j}$ → Unknown phase coefficient

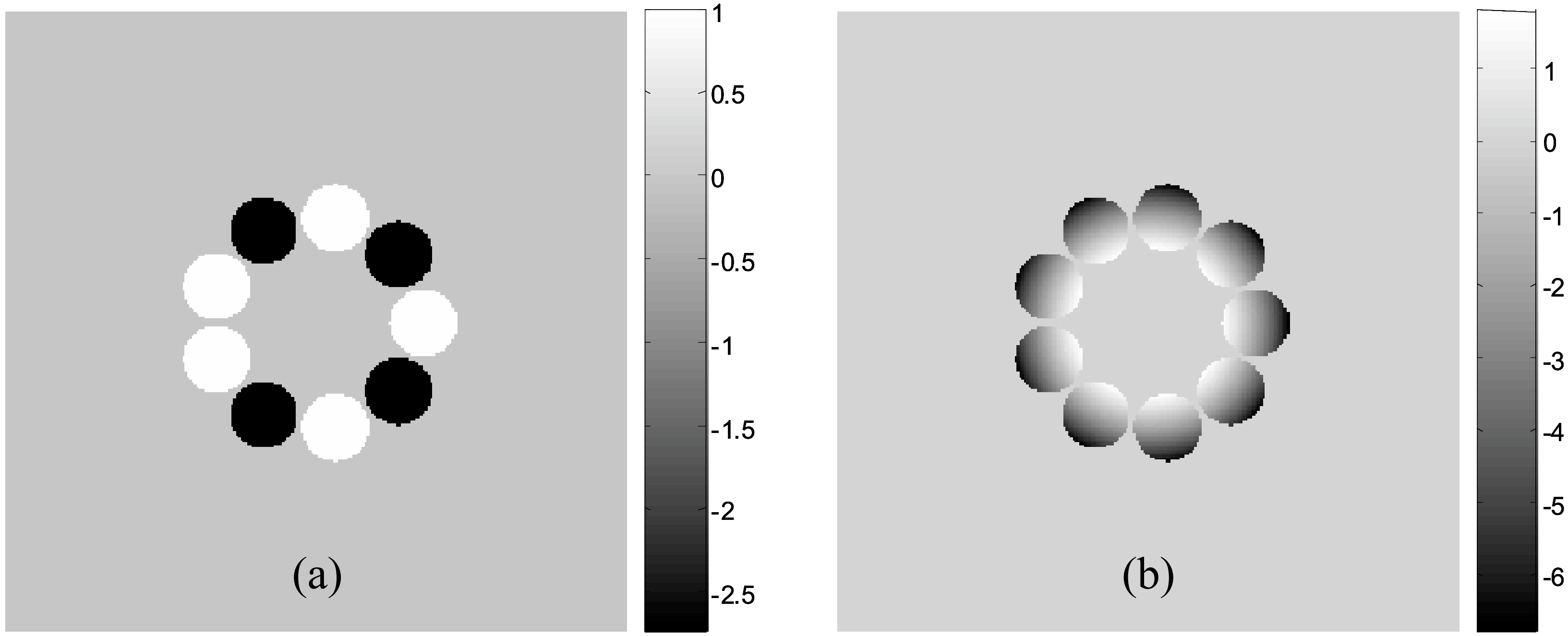
$\phi_j(u)$ → $j^{th}$ phase basis function

$P$ → Number of sub-apertures

$u$ → 2-D array of pixel indices

$J$ → Number of phase basis functions

# Diversity Implementation




(a) Sub-aperture piston diversity: 4 sub-apertures pistoned relative to others

(b) Global focus diversity: image plane shifted to introduce focus error

# Theory – Error Metric

- Nonlinear optimization problem:
  - Minimize the error function (generalized form of Gaussian Maximum Likelihood function):

$$E(f,\alpha)=\sum_{k=1}^{K}\sum_{u}\left|D_k(u)-\tilde{F}(u)\tilde{S}_k(u)\right|^2$$

$D_k$ → Fourier Transform of Detected Images

$F$ → Fourier Transform of Object

$S_k$ → Unnormalized Optical Transfer Functions (OTFs)

~ → Denotes estimate of the quantity

$K$ → Number of Diversity Images

Optimizing over these:

$f$ → Object parameters (i.e. pixel values)

$\alpha$ → Phase parameters (i.e. Zernike coefficients)

# The Gonsalves Method

- Wish to reduce the number of parameters over which we're optimizing
  - For an arbitrary set of phase parameters, what set of object parameters minimizes the error metric?

$$\frac{\partial E(f,\alpha)}{\partial \tilde{F}(u')} = \frac{\partial}{\partial \tilde{F}(u')} \sum_{k=1}^{K} \sum_{u} \left| D_k(u) - \tilde{F}(u)\tilde{S}_k(u) \right|^2 = 0$$

$$F_M(u') = \begin{cases} \dfrac{\sum_{k=1}^{K} D_k(u') S_k^*(u')}{\sum_{l=1}^{K} \left| S_l(u') \right|^2} & u' \in \chi_1 \\ F_M^*(-u') & u' \notin \chi_1 \end{cases}$$

# Theory – Error Metric

- Using the Gonsalves method, we can reduce the number of parameters over which we're optimizing

$$E(\alpha) = \sum_u \sum_{k=1}^{K} \left| D_k(u) \right|^2 - \sum_{u \in \chi_1} \frac{\left| \sum_{j=1}^{K} D_j(u) S_j^*(u) \right|^2}{\sum_{l=1}^{K} \left| S_l(u) \right|^2}$$

$\chi_1$ is the set of pixel indices where all of the OTFs are *not* zero valued

  - Now, we're only optimizing over the phase parameters, $\alpha$
  - Recover the object after optimization using an image reconstruction technique such as a Wiener Filter.

# Theory – Analytic Derivatives

- Analytic derivatives improve algorithm speed by avoiding finite difference calculations

$$\frac{\partial E}{\partial \alpha_{j,p}} = -4\frac{2\pi}{\lambda}\sum_{u'}\phi_j(u')\,\mathrm{Im}\left\{\sum_{k=1}^{K} H_{k,p}(u')\sum_{u} Z_k(u) H_k^*(u'-u)\right\}$$

$$Z_k(u) = \begin{cases} \dfrac{-\sum_{m=1}^{K}\left|S_m\right|^2\left(\sum_{n=1}^{K} D_n S_n^*\right)D_k^* + \left|\sum_{n=1}^{K} D_n S_n^*\right|^2 S_k^*}{\left(\sum_{m=1}^{K}\left|S_m\right|^2\right)^2} & u \in \chi_1 \\ 0 & u \notin \chi_1 \end{cases}$$

  - With only a few FFTs, we can compute all of the derivatives with respect to the phase parameters

# Performance Criteria

- Strehl Ratio
  - Determine residual phase error: $\phi_{res} = \phi_{actual} - \phi_{estimate}$
  - Calculate incoherent PSF: $s_{res}(x\ ,\ y)$
  - Cross-correlate residual PSF with ideal PSF to remove shift due to arbitrary global tip and tilt in the estimated phase
  - Calculate Strehl ratio:

$$Strehl = \frac{s_{res}\left(0,0\right)}{s_{ideal}\left(0,0\right)}$$

- Convergence Percentage
  - Trials converged when Strehl ratio > 90%

- Number of Function Evaluations
  - Number of times the error metric and analytic gradients were evaluated for converged trials

# Digital Simulations

- 25 Realizations of unknown pupil error:
  - ~0.25λ RMS of piston, tip and tilt error
- 25 Realizations of initial guess:
  - ~0.001λ RMS of piston, tip and tilt error

➔ 625 trials for a given amount of RMS diversity

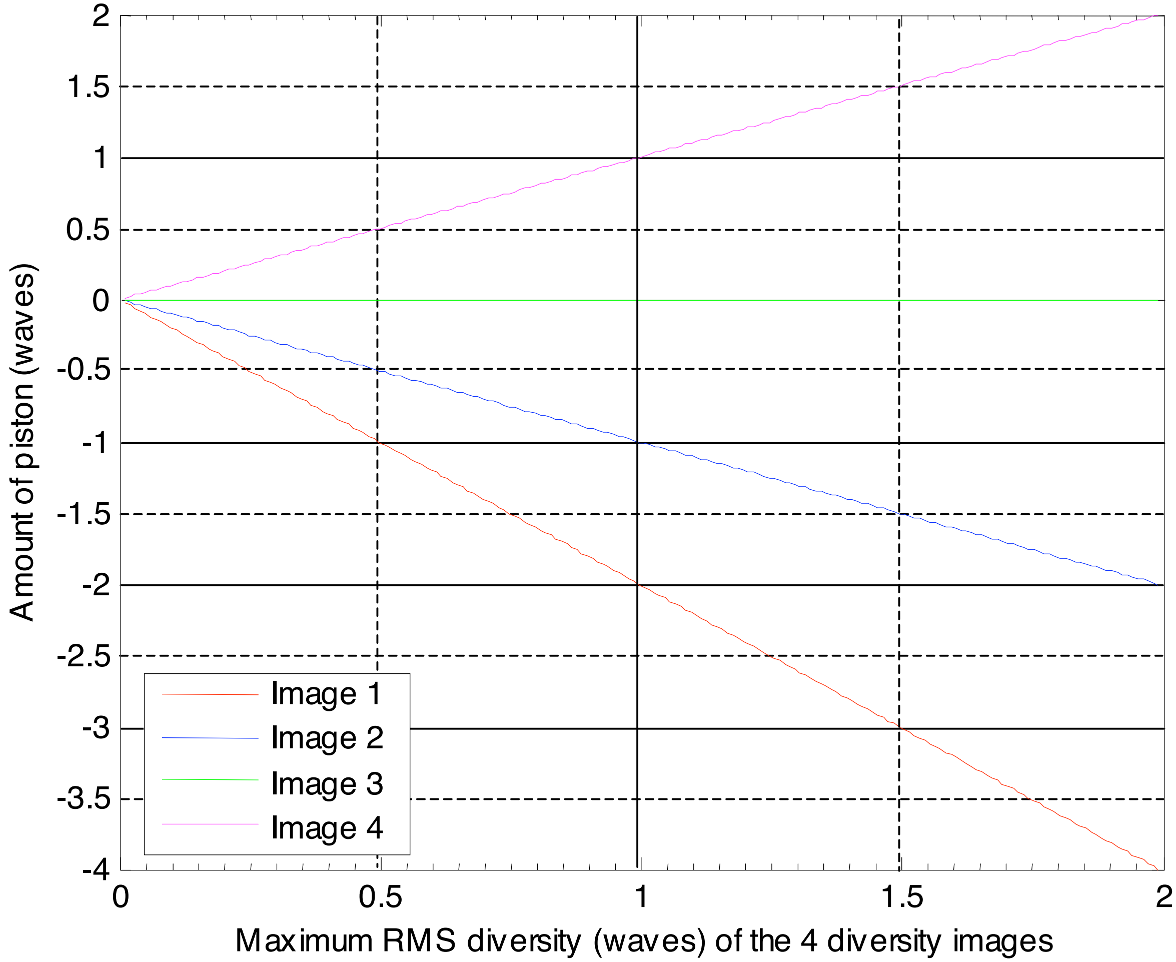
$$\phi_{div}(x,y) \Rightarrow \phi_{RMS} = \left[\overline{\phi_{div}^2} - \overline{\phi_{div}}^2\right]^{1/2}$$

- 4 Diversity images per trial with diversity amounts:

$$\left\{ -\phi_{div} \quad -\frac{\phi_{div}}{2} \quad 0 \quad \frac{\phi_{div}}{2} \right\}$$

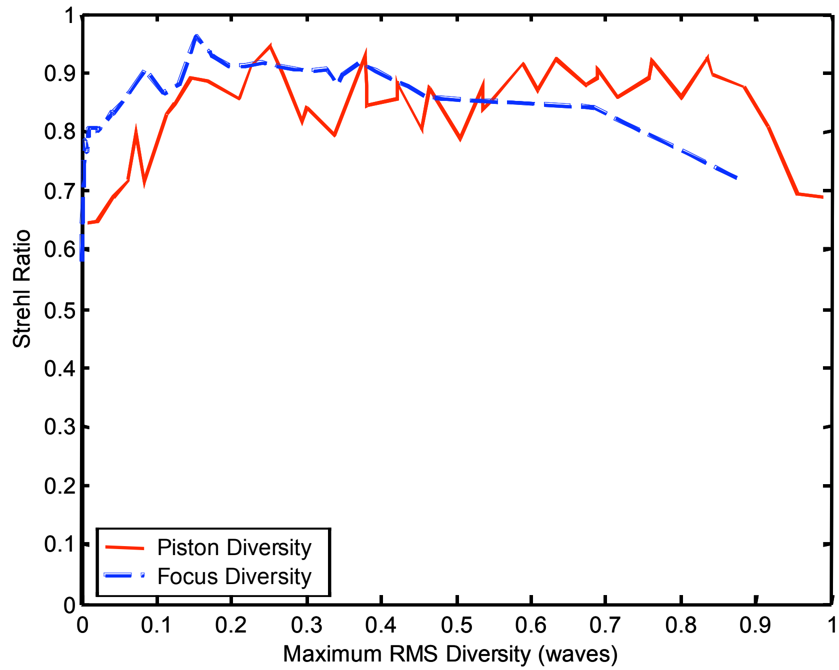
# Periodic Behavior

- Since we're using monochromatic light, we expect piston diversity to have a periodic nature



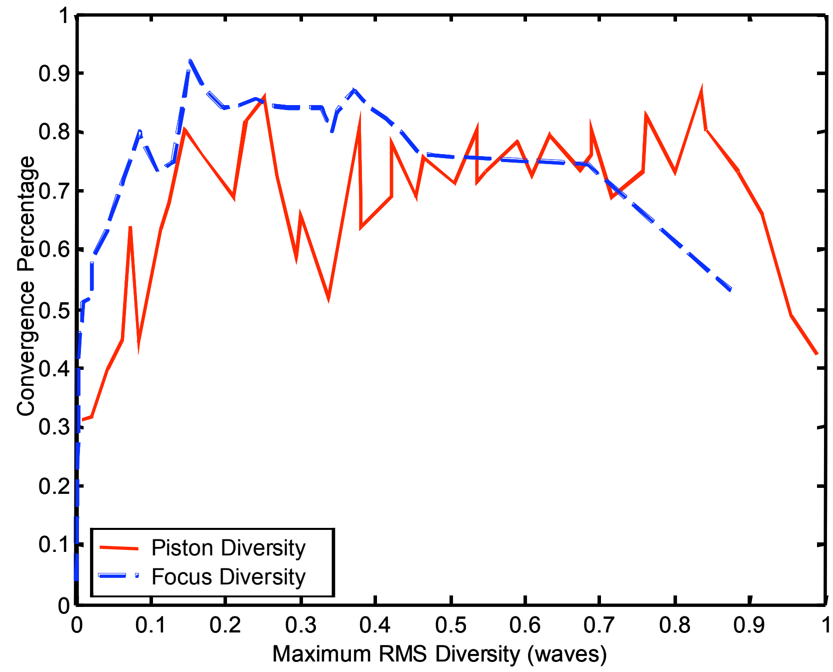
# Results – Strehl Ratio

- Each point is an average of all 625 trials
- Sub-aperture piston diversity exhibits periodic nature due to monochromatic light
- Focus diversity decreases as larger amounts of diversity spread out PSF and depress OTF



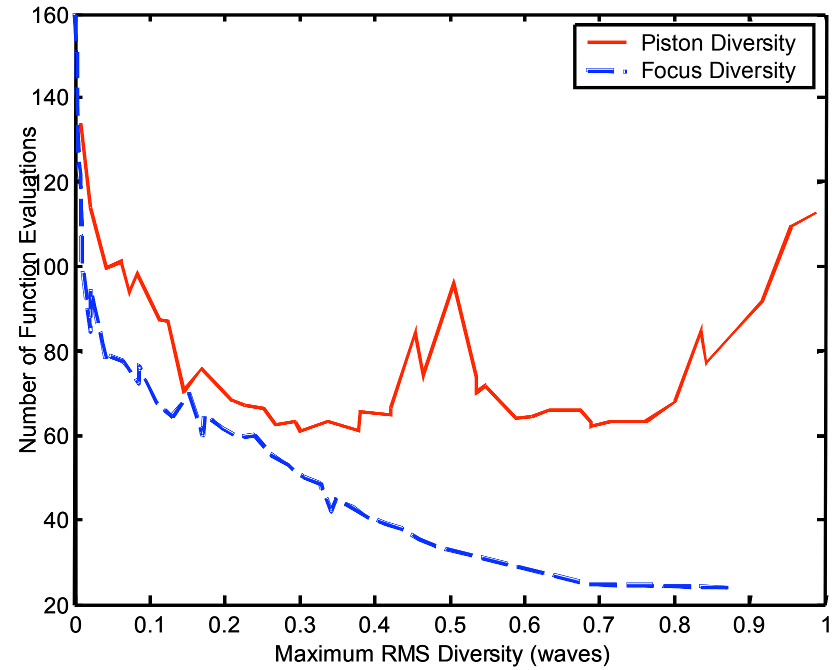
# Results – Convergence %

- Convergence → Strehl ratio > 90%
- Non-zero convergence percentage implies that multiple trials, each with a different initial guess, will likely result in a solution
- Periodic nature of monochromatic sub-aperture piston phase diversity is apparent
  - 1λ : all four images have same amount of diversity, therefore convergence is unlikely

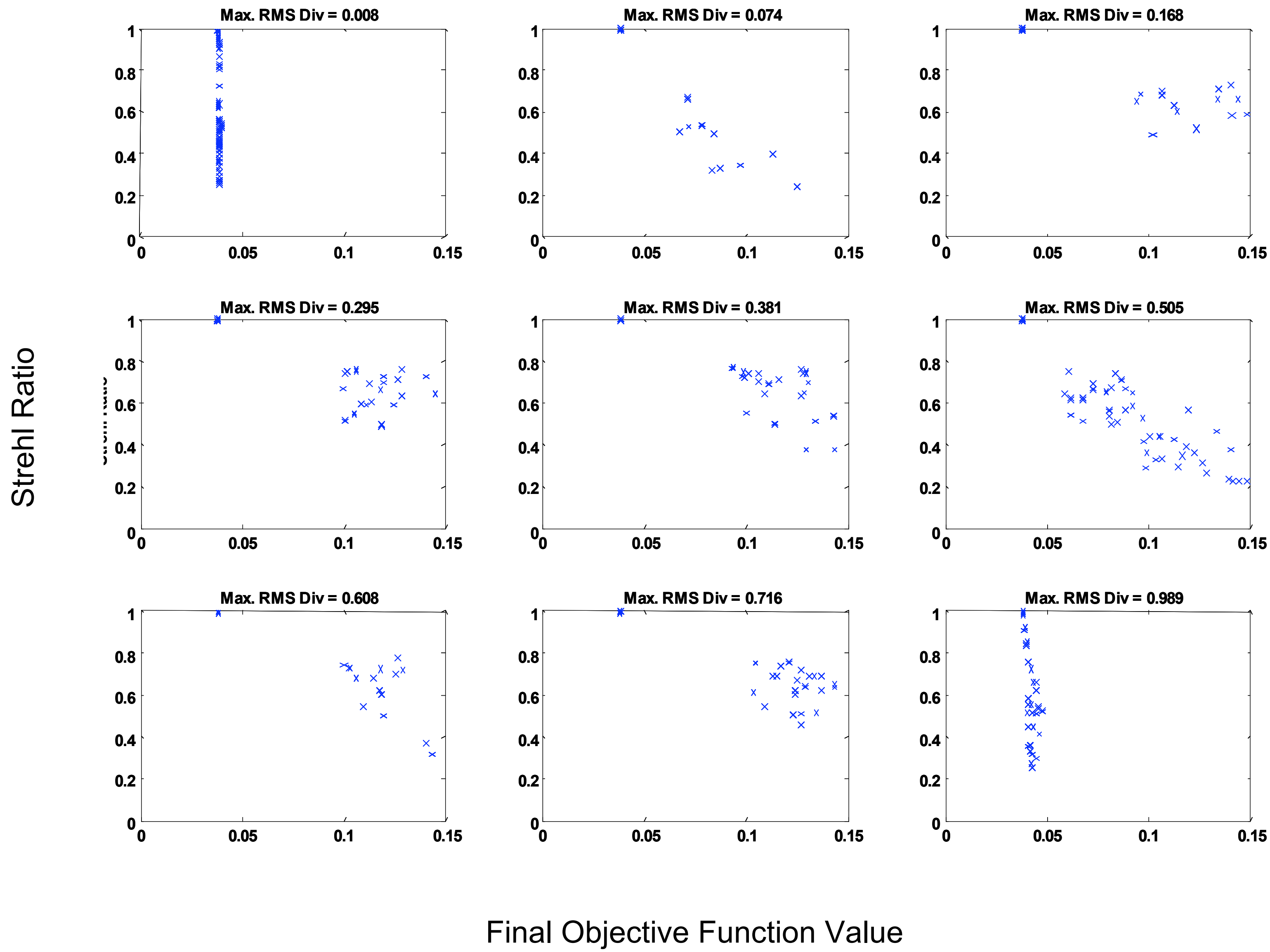


# Results – Function Evaluations

- Focus diversity outperforms sub-aperture piston phase diversity
- Periodic nature of monochromatic sub-aperture piston phase diversity is apparent
  - 1λ : all four images have same amount of diversity, therefore convergence is unlikely
  - 0.5λ : pairs of diversity images share same amount of diversity, effectively reducing the diversity to only two images
    - Algorithm works, but takes longer to converge



# Results – Convergence Criteria



# Conclusions

- Sub-aperture piston phase diversity is a viable method of determining phase errors of a multi or segmented aperture system
  - Performs almost as well as global focus diversity with regards to Strehl ratio and convergence percentage
  - Global focus diversity outperforms sub-aperture piston phase diversity with regards to number of error metric and gradient evaluations

**Acknowledgments**

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