

Measuring Anisoplanatism for Astronomical Adaptive Optics Imaging

Danielle Robbins

University of Maryland, Baltimore County

Mentor: Julian Christou, PhD
UCSC, Center for Adaptive Optics





Overview

- Purpose
- Introduction
 - Anisoplanatism
 - Fourier Transform
- Creating the IDL Package
 - Setting up the Package
 - Cross Spectrum, Power Spectrum, Noise, Isoplanatic Parameter
- Conclusion
- Acknowledgements





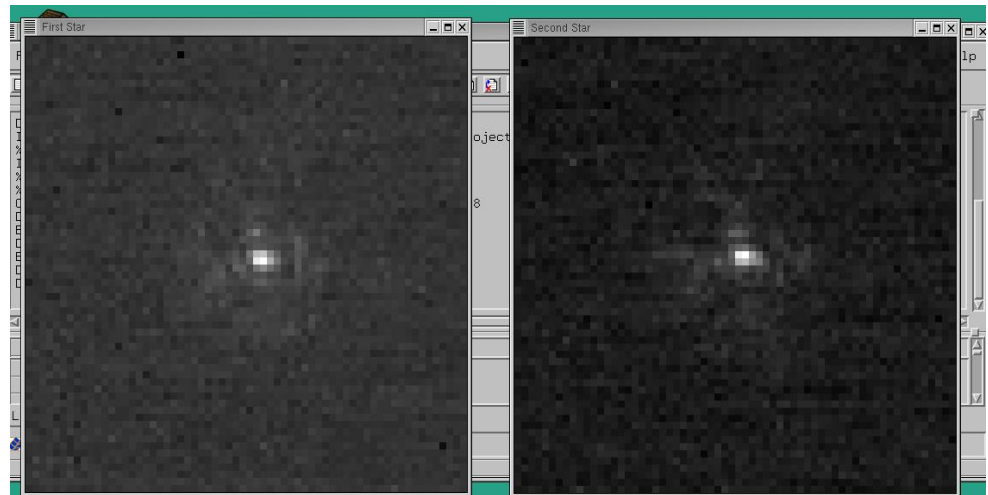
Purpose

- To create an Interactive Data Language package that will:
 - Calculate the Anisoplanatic Parameter
 - To do this:
 - Calculate the average cross-spectrum, and power spectra of Binary Stars
 - Calibrate the power spectra by measuring and subtracting the noise bias.
 - Normalizing the cross-spectrum to the bias corrected power spectrum



What is Anisoplanatism?

- The spatial variation of the Point Spread Function
 - The image produced by an optical system when looking at a point
- Will calculate differences in the PSFs using cross-spectrum analysis, by measuring two stellar images simultaneously



Binary Stars that are 12 arc seconds apart at a spatial scale of 57 mas/pixel



Fourier Transform

- Cross-spectrum and Power Spectra are calculated using the Fourier transforms.

- Fourier Transform of signals

- Takes a signal and separates into component frequencies

Fourier Transform Equation:

$$I(u, v) = \iint_{\infty} i(x, y) e^{-i2\pi(xu + yv)} dy dx$$

- Fourier Transform of images

Takes a spatial signal, i.e. an image $i(x, y)$, and separates into spatial frequencies



Setting Up the IDL Package

- Step One: Input two sets of FITS files
 - FITS: Flexible Image Transport System, standard astronomical imaging data file
 - Each data set is comprised of multiple images for each star
 - Read in first pair of images and compute power spectrum of each and the cross-spectrum between them

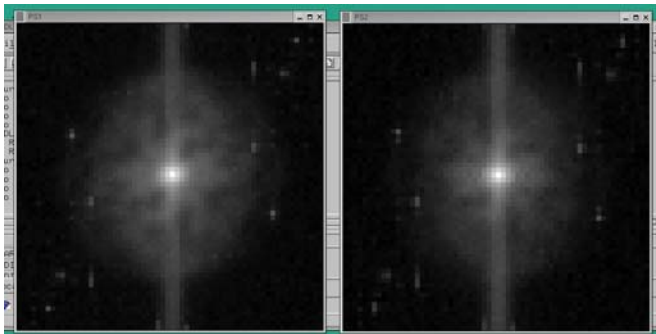


Setting Up the IDL Package

- Step Two: Take the average cross-spectrum, and power spectrum for each set of data, and write these to new FITS files.

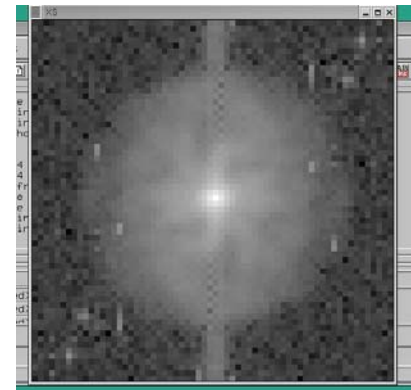
Power Spectrum:

$$\langle I_1(f) I_1(f)^* \rangle = \langle |I_1(f)|^2 \rangle$$



Cross-spectrum:

$$\langle I_1(f) I_2(f)^* \rangle$$



Setting Up IDL Package

- Step Three: Calibrate power spectra for the biased noise term, using the radial average
 - Calculate noise from the power spectrum because it is auto-correlated

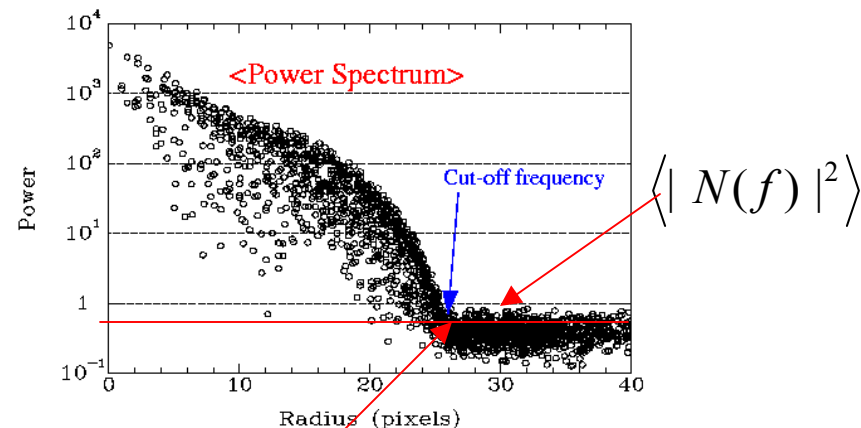
$$\text{Power Spectrum} = \langle |I_1(f)|^2 \rangle$$

$$i(r) = \hat{i}(r) + n(r)$$

$$I(f) = \hat{I}(f) + N(f)$$

$$\langle I(f)I^*(f) \rangle = \langle |I(f)|^2 \rangle = \langle |\hat{I}(f) + N(f)|^2 \rangle$$

$$\langle |I(f)|^2 \rangle = \langle |\hat{I}(f)|^2 \rangle + \langle |N(f)|^2 \rangle$$



$$f_c = \frac{1}{\alpha} = \frac{D}{\lambda}$$

where α is the diffraction-limit

α = Diffraction Limit (Optical Resolution)
 D = Diameter of the Optical System
 f_c = Cut - Off Frequency
 λ = Wavelength



Setting Up the IDL Package

- Step Four: Calculate anisoplanatism, the Isoplanatic Parameter, using the ratio of the average cross-spectrum to the average calibrated power spectrum
 - Isoplanatic Parameter measures the spatial differences between PSF
 - It is closely related to cut-off frequency and diffraction limit

$$\gamma(f) = \frac{\langle I_1(f)I_2(f)^* \rangle}{\langle I_1(f)I_1(f)^* \rangle} = \frac{\langle I_1(f)I_2(f)^* \rangle}{\langle |\hat{I}_1(f)|^2 \rangle}$$

Isoplanatic Situation:

$$\gamma(f) = 1 \text{ for } f < f_c$$

Anisoplanatic Situation:

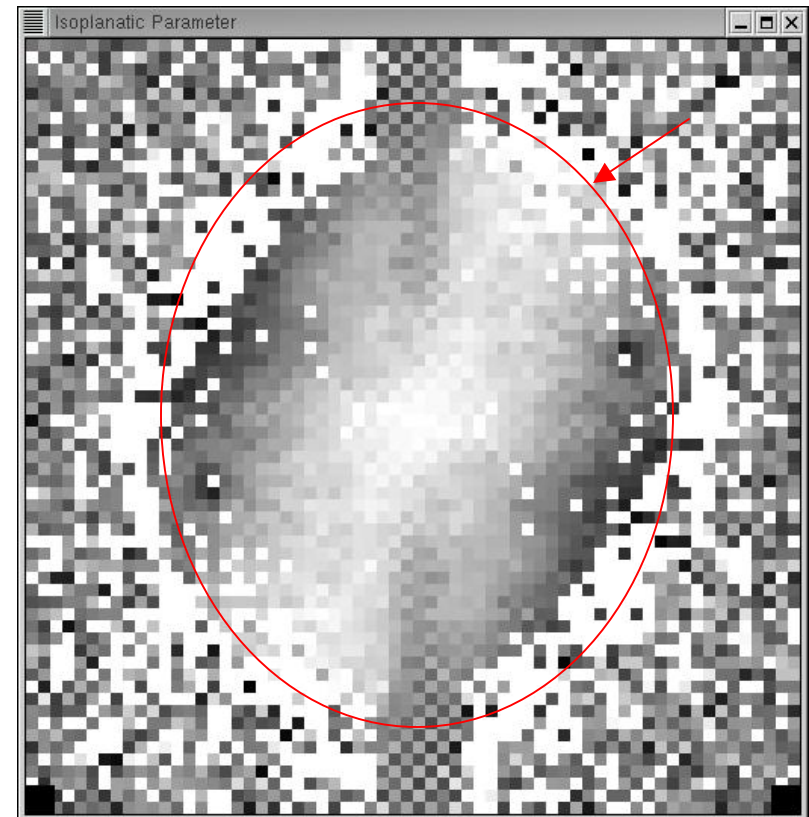
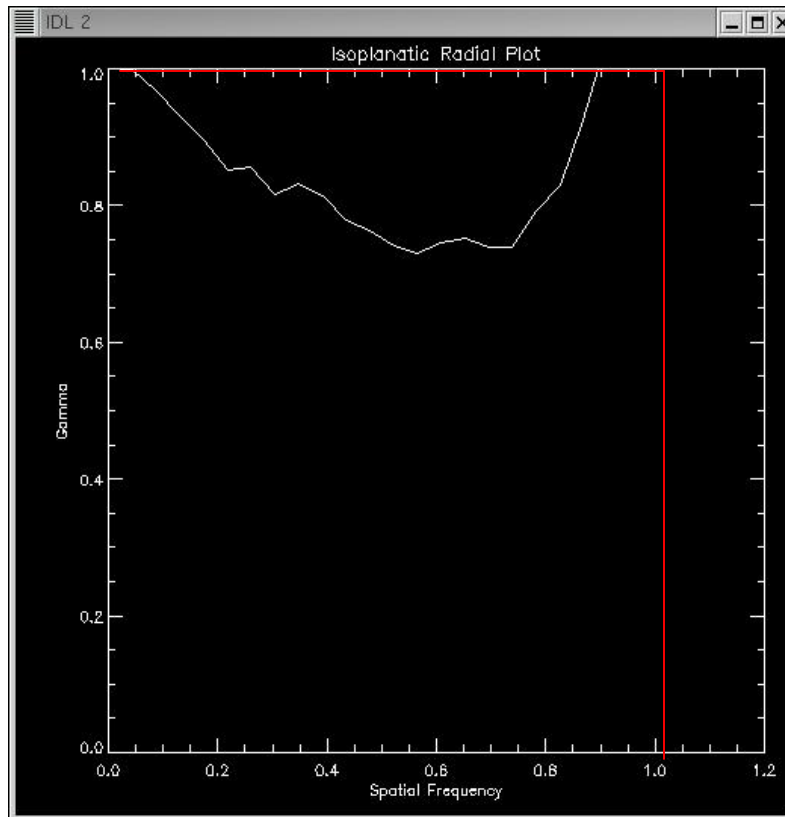
$$0 < \gamma(f) < 1 \text{ for } f < f_c$$

$$\gamma(f) = 0 \text{ for } f > f_c$$

γ = Isoplanatic Parameter
 f_c = Cut - Off Frequency



Anisoplanatic Parameter $\gamma(f)$



- Once the Isoplanatic Parameter is calculated for the set of binary stars that looked very similar in the beginning, the image above shows their differences





Conclusion and Future Steps

- An IDL Package was written and tested with one binary star data set
- Test results match previous analysis



Acknowledgements

- Dr. Julian Christou
- Center for Adaptive Optics NSF Grant
 - This project is supported by a Research Experiences for Undergraduates (REU) supplement to the National Science Foundation Science and Technology Center for Adaptive Optics , managed by the university of California at Santa Cruz under a cooperative agreement No. AST-9876783.



Deconvolution

- Calculating the anisoplanatic parameter improves deconvolution methods for object estimates:

Let an image be represented by :

$$i(r) = o(r) * p(r)$$

where $o(r)$ is the object and $p(r)$ is the PSF to find the Fourier Spectrum

$$\tilde{O}(f) = \frac{\langle I(f)P_1^*(f) \rangle}{\langle P_1(f)P_1^*(f) \rangle} = \frac{\langle P_2(f)P_1^*(f) \rangle}{\langle |P_1(f)|^2 \rangle} = O(f)\gamma(f)$$

this is the Fourier spectrum filtered by the isoplanatic parameter.