CfAO Summer School 2007:
Astronomical Instrumentation
for
Adaptive Optics

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Outline

• What’s special about AO instruments?
• Spatial information in AO corrected data
• Aliasing & the Sampling Theorem
• AO cameras
• Focal reducers: coupling instruments to telescopes
• Spectrographs, coronagraphs & polarimeters
Background & Bibliography

• Elementary

• Intermediate
  – “Electronic Imaging in Astronomy: Detectors & Instrumentation”, I. S. McLean, Wiley

• Advanced
Cameras, Spectrographs & Polarimeters

- Astronomical instruments quantify the intensity of light from distant sources:
  \[ i = i(\theta, \varphi, E, t) \]
  - Where did the photon come from (direction)?
    - “Spatial” information, \( i(\theta, \varphi) \) is recorded by a camera
  - What is the photon energy (wavelength)?
    - Energy information, \( i(E), E = h\nu \), is recorded by spectroscopy
  - When did the photon arrive, \( i(t) \)
  - What is the orientation of \( E \) (polarization)?
    - The wave equation is a vector wave equation!
What’s Special About AO Instruments?

• AO-equipped telescopes deliver increased \textit{spatial information} content
  – AO instruments \textbf{must} capture this
  – Spectroscopic, polarization and time-domain information of AO corrected data is not unique
    • AO instruments record this conventionally
    • AO spectrographs are distinct from seeing limited instruments because they can be diffraction limited
    • AO enables new modalities
      – High contrast
SPATIAL INFORMATION
in
AO IMAGES
Imaging as Filtering

- An astronomical image $i(x)$ is the convolution of the scene $s(x)$ with the PSF $p(x)$

$$i(x) = s(x) * p(x) + n(x)$$

or in the Fourier domain as

$$I(\nu) = S(\nu) \cdot T(\nu) + N$$

The recorded image is the original image multiplied by the Fourier transform of the PSF—the optical transfer function $T(\nu)$

- The image is corrupted by noise, $n$
The Optical Transfer Function

- Typically
  \[ T(v) \to 1 \text{ as } v \to 0; \ T(v) \to 0 \text{ as } v \to v_c \]
  - Low spatial frequencies are preserved
  - High spatial frequencies are suppressed
The Optical Transfer Function

• The sharper the PSF the broader frequency range \( T(\nu) \) extends over
  – The optical transfer function \( T(\nu) \) in

\[
I(\nu) = S(\nu) \cdot T(\nu) + N
\]

... tells us how much information at each spatial frequency reaches the image plane

• The functional form of \( T(\nu) \) tells us how hard our astronomical camera has to work
Telescope OTF

- A basic result of Fourier optics that the OTF($
\nu$) is the autocorrelation function of the pupil at $r = \lambda f \nu = D_{tel} \nu_n$
- OTF for a circular pupil can be computed from simple geometry

\[ T_{tel}(r/D) = \frac{2}{\pi} \left[ \cos^{-1}(r/D) - (r/D)\sqrt{1 - (r/D)^2} \right] \]
System OTF

- Convolution is associative
  \[ f * (g * h) = (f * g) * h \]
- The convolution theorem lets us write
  \[ T_{sys} = T_{atm} \ T_{tel} \ T_{AO} \ T_{pix} \]
  - Instead of performing multiple convolutions we can just multiply OTFs
- Limitations
  - Applies to linear systems
  - PSF must be spatially invariant
Atmospheric OTF

- Fried showed how to calculate the OTF from the structure function, $D_\phi$, for a random phase errors, $\phi(r)$

$$D_\phi (r) = \left\langle |\phi(r_1) - \phi(r_2)|^2 \right\rangle$$

where $r = r_1 - r_2$

$$T_{atm} (r) = \exp \left[ -\frac{1}{2} D_\phi (r) \right]$$

- Multiply out $D_\phi$ into the covariance function $\langle \phi \phi * \rangle$, which forms a Fourier pair with the phase error power spectrum $\langle \Phi \Phi^* \rangle = |\Phi|^2$ so that

$$D_\phi (r) = 4\pi \int_0^\infty |\Phi(\kappa)|^2 [1 - J_0(2\pi \kappa r)] \kappa r \, dr$$
Simplified AO OTF

• An AO system filters the power spectrum of atmospheric phase errors within its spatial frequency bandwidth
  – AO boosts the OTF at high frequencies
• The WFS measures the atmospheric wavefront $\phi(r)$ with error $\delta \phi(r)$ and based on this the DM presents $\phi_{DM}(r)$
  – The corrected wavefront is $\phi_{AO}(r)$

$$\phi_{AO}(r) = \phi(r) - \phi_{DM}(r)$$

• The phase presented by the DM is $\phi_{DM}(r)$ is found by convolving the measured wavefront with the influence function, $h(r)$

$$\phi_{DM}(r) = \int [\phi(r') + \delta \phi(r')] h(r - r') dr'$$
The power spectrum of the residual phase errors is found from

$$\phi_{AO}(r) = \phi(r) - \phi_{DM}(r)$$

by substituting for $\phi_{DM}(r)$ and taking the FT to find $\langle \Phi\Phi^* \rangle$

$$|\Phi_{AO}(\kappa)|^2 = [1 - H(\kappa)]^2 |\Phi(\kappa)|^2 + H^2(\kappa)\Delta_\phi^2$$

$H$ is the transfer function for the DM and $\Delta_\phi^2$ is the variance of the wavefront measurement error

- The atmospheric power spectrum is filtered by $(1-H)^2$

If we know $|\Phi_{AO}|^2$ we can find $D_{AO}$ and hence $T_{AO}$
Camera OTF

- The OTF for a perfect camera is the pixel OTF
  - The effect of a perfect single pixel, width $\delta x$, is to convolve the image with a top hat function $\Pi(x/\delta x)$
  - The pixel OTF is a sinc function
    $$T_{\text{pix}}(v) = \text{sinc} (\pi v \delta x)$$
The System OTF for Keck AO

\[ |\Phi(\kappa)|^2 = \kappa^{-1/3} \]

\[ D_s(r) = e^{-3.44(\nu_0 r)^{1/3}} \]

\[ D_s(r) = 4\pi \int_0^\infty |\Phi(\kappa)|^2 [1 - J_1(2\pi \kappa r)] \kappa \, d\kappa \]

\[ D_{\text{pupil}} D_{\text{AO}} \]

\[ |T(\nu_n)|^2 + |N|^2 \]

\[ |T(\nu_n)|^2 \]

\[ |T(\nu_n)|^2 + |N|^2 \]
Good News/Bad News

• AO delivers sharper images
  – Images must be sampled more finely than non-AO images
  – AO instruments call for more samples per object to record the spatial information
• Sampling is done with *pixels* (picture elements)
  – Pixels are expensive
  – If we want fine sampling then we have to trade sampling for field of view
ALIASING & THE SAMPLING THEOREM
• A signal with a spatial wavelength of $\lambda = 1$ (solid) is sampled (dots) at an interval $\lambda_s = 1.1$
  
  – The reconstructed signal (dashed) appears at the wrong wavelength of $\lambda_a = (\lambda^{-1} - \lambda_s^{-1})^{-1} = 11$
Aliasing 2-d

- Target consisting of radial spokes
  - Spatial frequency decreases outward
- Low spatial frequencies are faithfully recorded
- High spatial frequencies near the origin are corrupted
Astronomical Detectors for Visible Light

- Most astronomical instruments use arrays of pixels to sample images
  - Detectors based on the photoelectric effect
  - Si is used for optical; band gap at 1.1 \( \mu m \)
    - Si CCD (refers to how the photodiodes are read)
Astronomical Detectors for Infrared

- Semiconductors materials with smaller bandgaps are used for longer wavelengths
  - Ge (1.6 µm), $\text{In}_x\text{Ga}_{1-x}\text{As}$ (1.68–2.6 µm), $\text{Hg}_{1-x}\text{Cd}_x\text{Te}$: (2.5–10 µm), $\text{InSb}$: 4.6 µm

[Image of Rockwell HgCdTe Arrays]
The Good News: The Sampling Theorem

- So far only we only described the action of one pixel
  - To construct an image we need to sample an array of points
  - The *Sampling Theorem* tells us how close together those points have to be to record the signal with no loss of information
Sampling in Fourier Space

- Sampling in image space is described by multiplication with the bed-of-nails or *Shah function*

\[ III(x) = \sum_{j=-\infty}^{j=\infty} \delta(x - j) \]
• A continuous function $f(x)$ is sampled by multiplication with $III(x/\delta x)$
  – The sampled function is $III(x/\delta x) f(x)$
Sampling in Fourier Space

• In Fourier space the effect of sampling is to convolve the Fourier spectrum with the FT of $\mathcal{I}$

$$FT\{ f(x) \mathcal{I}(x/\delta x) \} = F(k) * FT\{ \mathcal{I}(x/\delta x) \}$$

but

$$FT\{ \mathcal{I}(x) \} = \mathcal{I}(\nu)$$

• Thus

$$FT\{ f(x) \mathcal{I}(x/\delta x) \} = \delta x F(k) * \mathcal{I}(\nu \delta x)$$

Sampling replicates the Fourier spectrum at a spatial frequency intervals equal to the sampling frequency
Sampling in Fourier Space

Continuous function

Sampling function

Over sampled

Critically sampled

Under sampled

Band-limited Fourier spectrum

Replicated spectrum

Aliased spectrum
The Sampling Theorem

• To record a band-limited signal with spectrum

\[ F(k) \neq 0; |\nu| \leq \nu_c \]
\[ F(k) = 0; |\nu| > \nu_c \]

The sampling frequency must be

\[ 1/\delta x \geq 2\nu_c \]

– A sampled, band-limited signal can be reconstructed perfectly at any intermediate point by interpolation
The Sampling Theorem for AO Images

• For a diffraction-limited system the telescope pupil defines the bandlimit (slide 11)

In spatial units ($m^{-1}$)

\[
\nu_c = \frac{D_{tel}/\lambda f}{1/\lambda F}
\]

\[
= 1/\lambda F
\]

In angular units ($\text{radian}^{-1}$)

\[
\nu_c = \frac{D_{tel}/\lambda}{\lambda}
\]
Keck AO Observations with NIRC2

• NIRC2/LGS AO image of IC 10
  – Narrow camera (10 mas pixels)
  – $I/H/K'$
Observed Power Spectra for Keck AO

- Observed power spectrum $\propto |OTF|^2$
  Diffraction limited information is evident
- Correction is better at the longer wavelength
  - Only detector & sky noise at $\nu > 0.8 \nu_n$
  - 25 mas pixels would have been fine at $K'$ and 19 mas pixels $H$
  - Could have greater FOV & if the noise is detector noise better SNR with coarser pixels
• IC 10 color magnitude diagram from AO
AO CAMERAS
AO Cameras

• An AO camera images the focal plane at a spatial frequency that satisfies the sampling theorem
• Insures that all light collected by the telescope is relayed to the detector
• Provides a collimated beam for filters or other optical elements
  – Conventional cameras, may correct residual telescope aberrations & thereby increase the field of view, e.g., prime focus corrector
• Controls background & stray light
Design Problems

• Physical pixel size may not be ideal
  – Optical CCD's and infrared detector arrays have 10-40 μm pixels
  – Detector noise depends on pixel size
    • Small pixels have smaller dark current & read noise

• Arrays have limited size
  – Typically 1024 × 1024 for IR & 4096 × 4096 for CCDs
  – Over-sampling is expensive
    • Trade spatial sampling & field of view
Diffraction Limited Sampling

- Cut off frequency of a diffraction limited system is $1/\lambda F$
  - Nyquist sampling frequency is twice this: $2/\lambda F$
  - Spatial sampling is $\delta x = \lambda F/2$
  - For typical optical systems $F \approx 15$
    - Require $\delta x = 15 \mu m$ wide pixels at $\lambda = 2 \mu m$
  - A camera that is well sampled at $2 \mu m$ will be severely under-sampled at $1 \mu m$
    - Need variable magnification with wavelength
Plate Scale

- Diffraction limit of a 10-m telescope at 1 µm
  - $\lambda/D_{\text{tel}} = 20$ mas
- Nyquist sampling interval
  - $\lambda/2D_{\text{tel}} = 10$ mas
- The “plate scale” is the angular scale at the focal plane

$$S = \frac{206,265}{f} \text{ [arc sec m}^{-1}]$$

- $f = FD_{\text{tel}}$ is the system focal length [m], $D_{\text{tel}}$ is the telescope diameter [m] and $F$ is the system focal ratio
Plate Scale

• For example $F/15$ focus of Keck
  \[ S = 1.375 \left( \frac{F}{15} \right) \left( \frac{D_{\text{tel}}}{10 \text{ m}} \right) \text{ arc sec mm}^{-1} \]
  or for $\delta x$ wide pixels
  \[ s = 13.75 \left( \frac{F}{15} \right) \left( \frac{D_{\text{tel}}}{10 \text{ m}} \right) \left( \frac{\delta x}{10 \mu \text{m}} \right) \text{ mas pixel}^{-1} \]
• The NIRC2 camera has three plate scales
  – 10 mas
  – 20 mas
  – 40 mas
• Which one would you use at K band (2.2 $\mu \text{m}$)?
TELESOPES & FOCAL REDUCERS:
Coupling Instruments To Telescopes
Cassegrain two-mirror telescope with primary mirror focal length $f_1$ has an effective focal length of $mf_1$.
Equivalent 1-Lens Telescope

Two-Mirror Telescope

Equivalent 1-Lens Telescope with effective focal length $f$
Focal Reducers

• A focal reducer is an optical system that changes the final $F$-ratio
Focal Reducers

• A *field lens* at the telescope focus images the exit pupil of the telescope onto the aperture stop of the focal reducer
  – Ensures that no light is lost at the collimator

• A *collimator* renders light parallel

• A *camera* images the light onto a focal plane detector
  – Gratings or filters can be put in the space between the collimator and the camera
Geometry of Focal Reducers

- Telescope with primary mirror focal length $f_1$
  - Image is formed a distance $f_1 \delta$ from the exit pupil
- Angle on the sky is $\theta$; the angle at the exit pupil is $\psi$
  - Diameter of the focal reducer elements depend on the FOV
  - The diameter of the field lens is $2f\theta$, where $f$ is the effective telescope focal length and $\theta$ is the angular radius of the FOV

![Diagram](image_url)
Geometry of Focal Reducers

- If the aperture stop of the focal reducer is at the collimator lens and $\alpha$ is the angle at which the chief ray from the edge of the field enters the collimator then

$$\theta f = \psi f_1 \delta = \alpha f_{\text{col}}$$

- $\theta$ – angle on the sky
- $f$ – telescope focal length
- $f_{\text{col}}$ – collimator focal length
- $f_1$ – primary mirror focal length
Geometry of Focal Reducers

- The diameter of the collimator lens is found from the Lagrange invariant \( \theta D_{tel} = \alpha D_{col} \)

\[
\frac{\alpha}{\theta} = \frac{f}{f_{col}} = \frac{D_{tel}}{D_{col}}
\]
Magnification

- The focal length of the telescope/focal reducer combination is the effective telescope focal length $f$ times the magnification of the focal reducer
  - The magnification of the focal reducer is the ratio of the camera to collimator focal lengths
  - The system focal length is the diameter of the telescope times the $F$-ratio of the camera
Focal Reducers: Example

• Keck/NIRC2, narrow camera has a magnification $m = 3.66$
  
  $D_{tel} = 10 \text{ m}, F_{narrow} = 3.66 \times 15 = 55, \delta x = 27 \mu \text{m pixels}$

$$s = \frac{206,265}{(FD_{tel})} = 375 \text{ mas/mm}$$

$$\delta \theta = s \delta x = 0.01 \text{ arc sec/pixel}$$

• The Nyquist frequency at 1 $\mu$m for 10-m telescope

$$2D_{tel}/\lambda = 100 \text{ arc sec}^{-1}$$
Focal Reducers: Example

- NIRC2 has three different magnifications to for: 10, 20, & 40 mas pixels
  - Achieved by substituting the camera with one of different focal length
NIRC2
NIRC2: Inside the Black Box

Camera module
Focal Reducer for the Infrared

- Collimator doubles as a field lens
  - Forms an image of the telescope exit pupil at the aperture stop
- An aperture stop is placed between the collimator and camera lenses
  - Distance between the collimator and camera equals \( f_{\text{col}} + f_{\text{cam}} \)
Focal Reducer for the Infrared

• A system with entrance & exit pupils at infinity is *telecentric*
  
  – All the chief rays—those that go through the center of the aperture stop—emerge parallel to the optical axis
  
  – Since a real image of the telescope exit pupil is found at the focal plane of the collimator, an aperture stop at this location precisely defines the acceptance angle of the focal reducer

• This stop is vital to rejecting unwanted thermal radiation from the telescope and sky
SPECTROGRAPHS
More Good News

• The majority of astronomical instruments are focal reducers
  – Cameras
  – Spectrographs
  – Even AO systems!

• Focal reducers often host useful optical components
  – Filters, polarizers, prisms, diffraction gratings, &c.
Spectrographs

DEIMOS
• Why are astronomical spectrographs so big?
Size Matters

• Astronomical spectrographs are huge because most are *not diffraction limited*, but seeing-limited
  – Most astronomical spectrographs are dispersive—dispersive spectrographs encode wavelength as position
  – Image size determines spectral resolution
    • To prevent light loss at the slit spectrograph size scales linearly with primary mirror diameter at constant spectral resolution
  – Not true for interferometric spectrometers, e.g., Fourier transform spectrometers
• AO spectrographs can work at the diffraction limit and consequently are more compact
Dispersive Spectrometers

- Dispersive spectrometers are a class of instruments that encode wavelength as position on a focal plane detector.
- Dispersion can be caused by refraction or diffraction.
- Key element is
  - Prism ($dn/d\lambda \neq 0$)
  - Grating
- Gratings are favored
  - Flexible
    - Transmission or reflection
    - Groove spacing
    - Plane or powered surface
  - Efficient
    - Grating can be blazed
  - Lightweight
Spectrometers as Imagers

• A dispersive spectrometer is fundamentally a device which makes an image of a source
  – The position of the image encodes wavelength
• Typically the spectrometer makes an image of an aperture or slit
  – In fiber fed spectrometers, the spectrometer makes an image of the light exiting an optical fiber
• The location and size of the image is determined jointly by the laws of geometric optics and the grating equation
Fiber source $d = 100 \mu m$
Filter wheel
Collimator $f = 179 \text{ mm}$
Grating $B = 64.5^\circ$
$1 = 12.5 \mu m$
Camera $f = 200 \text{ mm}$
CCD 13 $\mu m$ pixels
$2 = 10^\circ$

Toy Spectrometer
$m\lambda = \sigma (\sin \alpha - \sin \beta)$
Condition for Constructive Interference

\[ m\lambda = \sigma (\sin \alpha \pm \sin \beta) \]

**Sign Convention**

Reflection grating: \( \alpha \) & \( \beta \) have the same sign if they are on the same side of the grating normal. Transmission grating: \( \alpha \) & \( \beta \) have the same sign if the diffracted ray crosses the normal.
Orders & Grating Blaze

\[ \lambda = 633\text{nm}; \frac{1}{\sigma} = 600\ \text{mm}^{-1} \]
Positional Encoding

- The angle $\beta$ is given by the grating equation
  - Image position set by the camera focal length

$$m \lambda = \sigma (\sin \alpha + \sin \beta)$$

$$\beta = \arcsin\left(\frac{m \lambda}{\sigma} - \sin \alpha\right)$$

$$p = f_{\text{cam}} \tan(\beta)$$
Mapping Wavelength to Angle

- Holding $\alpha$ and $m$ constant, $\beta$ varies with $\lambda$

$$
\beta = \arcsin \left( \frac{m \lambda}{\sigma} - \sin \alpha \right)
$$
Mapping Wavelength to Position

• Holding $\alpha$ and $m$ constant, $p$ varies with $\lambda$

\[ p = f_{CAM} \tan(\beta - \beta_0) / \Delta p \]

$f_{CAM} = 200$ mm

$\Delta p = 13 \, \mu m$
Dispersion

• Dispersion gives the angular spread of diffraction, $\delta \beta$, for a source with wavelength spread, $\delta \lambda$
  – Using the grating equation, hold the angle of incidence, $\alpha$, and the order, $m$, constant

\[
m\lambda = \sigma (\sin \alpha + \sin \beta)\]
\[
m \delta \lambda = \sigma \cos \beta \delta \beta\]
\[
\left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha,m} = \frac{m}{\sigma} \frac{1}{\cos \beta}\]
Dispersion

• Over a limited range of wavelength dispersion is $\approx$ constant
  – Roughly linear relation between wavelength & position

\[
\left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha,m} = \frac{m}{\sigma \cos \beta}
\]
Dispersion

• With higher dispersion it is possible to distinguish closely spaced wavelengths
• High dispersion corresponds to
  – High order (large $m$)
  – Narrow grooves/high groove density
  – Large $1/\cos(\beta)$, i.e., oblique $\beta$ close to $\pi/2$

\[
\left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha,m} = \frac{m}{\sigma} \frac{1}{\cos \beta}
\]
Spectral Resolution

\[ \Delta \alpha = \Delta s / f_{col}, \quad \Delta \beta = \left( \frac{\partial \beta}{\partial \alpha} \right)_\lambda \Delta \alpha, \quad \Delta p = f_{cam} \Delta \beta \]
Spectral Resolution: Diffraction Limit

• Even if the input is a point source, the image has a finite size on the CCD array, $\Delta p$, due to diffraction
  
  – The angular size of camera images, $\delta \beta = \lambda / D_{\text{cam}}$, limits the spectral resolution

\[
\delta \lambda = \frac{\partial \lambda}{\partial \beta} \delta \beta
\]

\[
= \frac{\sigma \cos \beta}{m} \delta \beta
\]

\[
\delta \beta = \frac{\lambda}{D_{\text{cam}}}
\]

\[
R = \frac{\lambda}{\delta \lambda} = \frac{(\sin \alpha + \sin \beta)}{\cos \beta} \frac{D_{\text{cam}}}{\lambda} \approx 2 \frac{D_{\text{cam}}}{\lambda} \tan \theta_B
\]
Spectral Resolution: the Diffraction Limit

• Telescope diameter does not enter into the formula!
  – For our toy spectrograph
    • $D_{cam} = 75 \text{ mm}$
    • $\tan \theta_B = 2$
    • $\lambda = 0.632 \mu\text{m}$

\[
R_{DL} \approx 470,000
\]
Slit-Limited Spectral Resolution

\[ \Delta \alpha = \Delta s / f_{col}, \quad \Delta \beta = \left( \frac{\partial \beta}{\partial \alpha} \right)_{\lambda} \Delta \alpha, \quad \Delta p = f_{\text{cam}} \Delta \beta \]
Slit-Limited Spectral Resolution

• Generally, the source, is not a point
  – If the extent is greater than the diffraction blur then the spectrometer resolution “slit limited”

\[ \delta \alpha = \frac{\delta s}{f_{col}} \]

\[ \delta \beta = \left( \frac{\partial \beta}{\partial \alpha} \right)_{\lambda,m} \]

\[ \delta \alpha = \frac{\cos \alpha}{\cos \beta} \]

\[ \delta \alpha = \frac{\cos \alpha}{\cos \beta} \frac{\delta s}{f_{col}} \]

\[ \delta \beta = \left( \frac{\partial \beta}{\partial \lambda} \right)_{\alpha,m} \]

\[ \delta \lambda = \frac{m}{\sigma \cos \beta} \delta \lambda \]

\[ \frac{m}{\sigma \cos \beta} \delta \lambda = \frac{\cos \alpha}{\cos \beta} \frac{\delta s}{f_{col}} \]

Hence, \( R_{SL} = \frac{\lambda}{\delta \lambda} = \frac{\sin \alpha + \sin \beta}{\cos \alpha} \frac{f_{col}}{\delta s} \)

Which is bigger \( R_{DL} \) or \( R_{SL} \)?
Slit-Limited Spectral Resolution

- For the toy spectrograph the image size is set by the diameter of the fiber feed (100 μm)
  - The resultant resolution is $R_{SL} \approx 11,500$ or 1/40 of the diffraction limit

- For a spectrograph with slit width $\delta \phi >> \lambda/D_{tel}$ and collimated beam diameter $d_{col}$

$$R_{SL} = 2 \frac{d_{col} \tan \theta_b}{D_{tel} \delta \phi}$$

$$\approx 83,000 \left( \frac{d_{col}}{1 m} \right) \left( \frac{D_{tel}}{10 m} \right)^{-1} \left( \frac{\delta \phi}{1 \text{arc sec}} \right)^{-1} \left( \tan \theta_b / 2 \right)$$

- The grating is $d_{col} \tan(\theta_b) = 2$ m long!
HIRES on Keck

- The optics in high resolution, slit-limited spectrographs are huge
  - For HIRES $R\delta \phi = 39,000$
  - $\tan(\theta_b) = 2.6$
  - 14-inch collimated beam and 36-inch long grating mosaic
POLARIMETRY
Polarimetry Science

• Characterize the scattering properties of circumstellar material
• Polarization-based speckle suppression
Polarimetry & Scattering

• Consider a plane wave propagating in the $z$-direction, incident on a particle located at the origin
  – The direction of the incident and scattered light defines the *scattering plane*
• The amplitude of the incident electric field, $\mathbf{E}_i$, can be written as a sum of $E_{i//}$ & $E_{i\perp}$, parallel and perpendicular to the scattering plane
• The scattered field, $\mathbf{E}_s$, is decomposed into $E_{s//}$ & $E_{s\perp}$
Polarimetry & Scattering

• The *complex amplitude scattering matrix* describes the relationship between $E_i$ and $E_s$

$$
\begin{pmatrix}
E_{s//}
\\
E_{s\perp}
\end{pmatrix} = \frac{e^{ik(r-z)}}{-ikr}
\begin{pmatrix}
S_2 & S_3 \\
S_4 & S_1
\end{pmatrix}
\begin{pmatrix}
E_{i//}
\\
E_{i\perp}
\end{pmatrix}
$$

e.g., for Rayleigh scattering

$$
\begin{pmatrix}
S_2 & S_3 \\
S_4 & S_1
\end{pmatrix} = -ik^3 r
\begin{pmatrix}
\cos \theta & 0 \\
0 & 1
\end{pmatrix}
$$
Polarimetry: The Observables

• If we can measure $\mathbf{S}$ we can establish the nature of the scattering particle

• The *phase function* measures the angular variation of total intensity with scattering angle

\[ I(\theta) = |E_{s//}|^2 + |E_{s\perp}|^2 \propto \frac{1}{2} \left( |S_2|^2 + |S_1|^2 \right) \]

• Stokes $I$ measures total intensity
Polarization

- If we measure the difference

\[ |E_{s//}|^2 - |E_{s\perp}|^2 \propto \frac{1}{2}(|S_2|^2 - |S_1|^2) \]

- This is Stokes Q
- For isotropic particles $S$ is diagonal
  - If we have single scattering $I$ & $Q$ are all we need
- More generally we want $U$ and possibly $V$
  - $U$ is needed to measure arbitrary linear polarization PA
  - $V$ measures circular polarization
Phase Function & Polarization by Spheres

\[ m = 1.33-0.01i; \ x = 2\pi \ a/\lambda \]
Polarization is a Key Diagnostic

• For edge-on disks polarimetry is essential
  – Information about the phase function is lost because of averaging along the line of sight
  – This information is recovered if $Q$ and $U$ are available because of the different angular dependence of the matrix elements of the complex amplitude scattering function

AU Mic

Graham, Kalas & Matthews 2006
Measuring $\langle \cos \theta \rangle$ and $p_{\text{max}}$

- Simultaneous fit of surface brightness and polarization measures $\langle \cos \theta \rangle$ and $p_{\text{max}}$.
- For AU Mic the data are consistent only with very porous grains, which means that they trace the primordial aggregation mechanism.
Polarimetry & Speckles

- Dual channel polarimetry suppresses common & non-common path wavefront errors

- Stellar halo
- Common WFE
- Non-common WFE
- Polarized astrophysical signal
Implementing a Polarimeter

- A polarimeter consists of a modulator and an analyzer
  - The modulator changes some polarization property of the incident radiation
  - The analyzer sorts the light according to its polarization state
Measuring Polarized Light

• The simplest polarimeter is a rotating linear polarizer:

\[ I(\theta) = \frac{1}{2} (I + Q \cos 2\theta + U \sin 2\theta) \]

• Choose three angles and solve the resulting simultaneous equations, e.g., 0°, 60°, & 120°

\[ I = \frac{2}{3} (I_0 + I_{60} + I_{120}), \quad Q = \frac{2}{3} (2I_0 - I_{60} - I_{120}), \quad U = \frac{2}{\sqrt{3}} (I_{60} - I_{120}) \]
Polarimetry Components

• Retarder
  – Introduces a phase delay between two perpendicular polarization states (e- and o-rays)
  – Birefringent crystals
    • e.g., rotating 1/4- and 1/2-wave quartz plates

• A rotating 1/2-wave plate rotates the plane of a linear polarized wave by 2\(\phi\) when the fast axis is rotated by \(\phi\)
1/2-wave plate
Polarimetry Components

- Analyzer
  - HR polaroid
  - Wire grids
  - Polarizing beam splitters
    - Wollaston
- Focal plane mask
  - Needed for dual channel operation
  - Half-plane mask/venetian blind
- Calibration system

![Calcite](image-url)
Calibration

• Polarization calibration is tricky because
  – Polarizing elements are not perfect, e.g.,
    • The waveplate retardance is not exactly half a wave
    • The extinction of the analyzer is not 100%
    • Orientations may not be at their nominal values
  – Instrumental (de)polarization
    • Diffraction (spiders)
    • Non-normal reflections
      – Telescope, relay optics, detector & dichroic beams splitters
    • Birefringent optics (refractive elements)
      – Polarization dependent wavefront errors!
Mueller Matrix

• Both calibrations are accomplished by measuring the 4 × 4 elements of the system Mueller matrix

\[
\begin{pmatrix}
I_s \\
Q_s \\
U_s \\
V_s
\end{pmatrix} =
\begin{pmatrix}
M_{11} & M_{12} & M_{13} & M_{14} \\
M_{21} & M_{22} & M_{23} & M_{24} \\
M_{31} & M_{32} & M_{33} & M_{34} \\
M_{41} & M_{42} & M_{43} & M_{44}
\end{pmatrix}
\begin{pmatrix}
I_i \\
Q_i \\
U_i \\
V_i
\end{pmatrix}
\]

– Only seven of the 16 elements are independent, because the four complex matrix elements, \( S_j \), have four absolute values and three phase differences

• Spatial variation—angles change with field angle
• Time dependent—new configuration (\( \lambda \)), image rotators, flexure
Data Reduction

- The system Mueller matrix for our simple polarizer consisting of a rotating wave plate and an ideal polarizer is

\[ M_{SYS}(\theta, \phi) = M_{LP}M_{ROT}(\theta)M_{WP}(\phi)M_{ROT}(\theta) \]

- Using the top row of the system Mueller matrix the observed intensity is

\[ 2I^o(\theta_j, \phi) = I^i + \left( \cos^2 2\theta_j + \sin^2 2\theta_j \cos \phi \right) Q^i + \]

\[ \left[ \cos 2\theta_j \sin 2\theta(1 - \cos \phi) \right] U^i + \]

\[ \sin 2\theta_j \sin \phi V^i \]

where the output intensity, \( I^o(\theta_j, \phi) \), is measured at angles \( j = 1, 2, 3, \ldots N \)
Data Reduction

- The column vector of measurements are related by a measurement matrix, $M'$, to the input Stokes vector

\[
\begin{pmatrix}
I_1^o \\
I_2^o \\
I_3^o \\
\vdots \\
I_N^o
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
1 & \cos^2 \theta_1 + \sin^2 \theta_1 \cos \phi & \cos \theta_1 \sin \theta_1 (1 - \cos \phi) & \sin \theta_1 \sin \phi \\
1 & \cos^2 \theta_2 + \sin^2 \theta_2 \cos \phi & \cos \theta_2 \sin \theta_2 (1 - \cos \phi) & \sin \theta_2 \sin \phi \\
1 & \cos^2 \theta_3 + \sin^2 \theta_3 \cos \phi & \cos \theta_3 \sin \theta_3 (1 - \cos \phi) & \sin \theta_3 \sin \phi \\
\vdots & \vdots & \vdots & \vdots \\
1 & \cos^2 \theta_N + \sin^2 \theta_N \cos \phi & \cos \theta_N \sin \theta_N (1 - \cos \phi) & \sin \theta_N \sin \phi
\end{pmatrix}
\begin{pmatrix}
I^i \\
Q^i \\
U^i \\
V^i
\end{pmatrix}
\]

- Restating the last equation as

\[
I^o = M' S^i
\]

the least squares estimate of $S^i$ corresponds to minimizing the norm

\[
\|e\|_2^2 = \|I^o - M' S^i_{est}\|
\]

which is found by using the pseudo-inverse of the measurement matrix

\[
S^i_{est} = \left( M'^T M' \right)^{-1} M'^T I^o
\]
**Acronyms & Glossary**

- **Aperture stop**: an opening that determines the amount of light which passes through an optical system to the image, usually the boundary of the primary mirror.
- **Cassegrain telescope**: a two mirror telescope with a convex secondary.
- **Chief ray**: any ray from an off-axis object point which passes through the center of the aperture stop of an optical system. The chief ray enters the optical system along a line directed toward the midpoint of the entrance pupil, and leaves the system along a line passing through the center of the exit pupil.
- **Entrance pupil**: the image of the aperture stop. If there is no physical aperture stop, the entrance pupil is an image of the lens or mirror itself.
- **Exit pupil**: is a virtual aperture in an optical system. Only rays that pass through the exit pupil can exit the system. The exit pupil is the image of the aperture stop in the optics that follow it.
- **Fourier transform**: $F(v) = \int f(x) \exp(-2\pi i v x) \, dx$
- **FOV**: field of view.
- **Gregorian telescope**: a two mirror telescope with a concave secondary.
- **Influence function**: shape adopted by the surface of a deformable mirror when a single actuator is exercised.
- **OTF**: the optical transfer function. The spatial frequency response of a system to a single harmonic stimulus. Fourier transform of the PSF.
- **Mueller Matrix**: a $4 \times 4$ real matrix describes how optical elements modify the Stokes vector.
- **PSF**: the spatial response of a system to a point source.
- **Shah function**: The sampling function or bed of nails $\Pi(x) = \sum \delta(x-j)$
- **sinc(x) = sin(x)/x — Bracewell’s definition includes π, i.e., sin(πx)/πx**
- **Stokes Vector**: $(I, Q, U, V)$ a basis set for describing partially polarized incoherent light. $Q$ & $U$ describe linear polarization, and $V$ describes circular polarization.
- **Structure function**: $D_\phi(r) = \langle |\phi(x) - \phi(x+r)|^2 \rangle$
- **Top hat function**: $\Pi(x) = 1, |x|<1/2, \text{otherwise 0}$
- **Telecentric**: a system with entrance & exit pupils at infinity.