Image Processing and Deconvolution

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Outline

• Image Formation & Fourier Optics

• Image Quality & PSFs

• Using the PSF for deconvolution
  Linear – optimal filtering
  Non-Linear
    Conjugate Gradient Minimization
    - steepest descent search à la least squares
  Lucy Richardson (LR) – Maximum Likelihood
  Regularization schemes
  Other PSF calibration techniques
The Fourier Transform

\[ \text{FT}\{f(r)\} = F(w) = \int_{-\infty}^{\infty} dr \ f(r) \exp\{ - 2\pi irw \} \]

Properties:

Convolution:

\[ f(r) * h(r) = \int_{-\infty}^{\infty} da \ f(a)h(r - a) \]

Convolution Theorem:

\[ \text{FT}\{f(r) * h(r)\} = F(w)H(w) \]
The Fourier Transform

\[
\text{FT}\{f(r)\} = F(w) = \int_{-\infty}^{\infty} dr \ f(r) \exp\{-2\pi irw\}
\]

Properties:

Power Spectrum:
\[
\text{PS}\{f(r)\} = F(w)F^*(w) = |F(w)|^2
\]

Autocorrelation:
\[
\text{AC}\{f(r)\} = \int_{-\infty}^{\infty} da \ f(a)f(a-r)
\]

Autocorrelation Theorem:
\[
\text{FT}[\text{AC}\{f(r)\}] = |F(w)|^2
\]
Two delta functions produce a set of “fringes”, the spatial frequency of which is inversely proportional to the separation and which are oriented along the separation vector. The “visibility” of the fringes corresponds to the intensity differences.
Fourier Relationships

- Resolution of an aperture of size $D$ is $\frac{\lambda}{D}$ radians.

- Diffraction limit of an aperture of size $D$ is $\frac{D}{\alpha \lambda}$ cycles/radian.

  - resolution depends on wavelength and aperture

- Large spatial structures correspond to low-spatial frequencies.

- Small spatial structures correspond to high-spatial frequencies.
Point Spread Functions

- The response of an optical system to an unresolved target.
- Fourier optics says the the Point Spread Function (PSF) is the modulus squared of the Fourier transform of the complex wave front at the pupil.

\[ h(\vec{r}) = |u(\vec{r})|^2 = |\text{FT}[P(\vec{\omega})]|^2 \]

where

\[ P(\vec{\omega}) = |P(\vec{\omega})| \exp[i\phi(\vec{\omega})] \]

Intensity

Complex wave-front at pupil

Pupil mask
The Transfer Function

The Optical Transfer Function (OTF) is the spatial frequency response of the optical system and is the Fourier Transform of the PSF.

The Modulation Transfer Function (MTF) is the modulus of the OTF.

From the autocorrelation theorem the OTF is the autocorrelation of the complex wavefront at the pupil.
Adaptive Optics Performance

- How well does an AO System perform?
- Adaptive Optics Point Spread Functions

Astronomy

VS

- How do we quantify the difference in these PSFs?
Adaptive Optics Performance - MTF

Changes in Spatial Frequency distribution, the MTF

MTF = |OTF|

[Diagram showing changes in spatial frequency distribution]
Adaptive Optics Performance?

• How well does an AO System perform?
• Tools for Measurement:
  
  - Strehl Ratio:
    The ratio of the peak value of the measure PSF to that of the perfect PSF for the optical system where both are normalised to the same volume.
    \[
    S = \frac{\tilde{h}(r)_{\text{peak}}}{h(r)_{\text{peak}}}
    \]
  
  - Maréchal approximation:
    relates the Strehl ratio to the rms wave front error at the pupil.
    \[
    S = \exp\left\{-\sigma^2 \varphi^2\right\}\exp\left\{-\sigma^2 \chi^2\right\}
    \]
Adaptive Optics Performance - Strehl

- Maréchal approximation:
  relates the Strehl ratio to the rms wave front error at the pupil.
Adaptive Optics Performance - Sharpness

- How well does an AO System perform?
- Tools for Measurement:

  - Image Sharpness:
    \[ S_1 \text{ or Beam Variance Metric (BVM) – Size of PSF} \]
    \[
    S_1 = \frac{\sum \hat{h}_i^2}{\left(\sum \hat{h}_i\right)^2}
    \]

  \[ S_3 \text{ or Normalized peak value – related to Strehl Ratio} \]
    \[
    S_3 = \frac{\hat{h}_{\text{peak}}}{\sum \hat{h}_i}
    \]
Adaptive Optics Performance - Sharpness

- Image Sharpness vs. Strehl ratio
Adaptive Optics Performance - Sharpness

- Image Sharpness vs. Wavefront Error
The Imaging Equation

Shift invariant imaging equation

\[ g(r) = f(r) \ast h(r) + n(r) \]  
(Image Domain)

\[ g(r) = \int ds \ f(s)h(r - s) = f(r) \ast h(r) \]

\[ G(f) = F(f) \ast H(f) + N(f) \]  
(Fourier Domain)

- \( g(r) \) – Measurement
- \( h(r) \) – Point Spread Function (PSF)
- \( f(r) \) – Target
- \( n(r) \) – Contamination - Noise
Why Deconvolution?

- Better looking image
- Improved identification
  - Reduces overlap of image structure to more easily identify features in the image (*needs high SNR*)
- PSF calibration
  - Removes artifacts in the image due to the point spread function (PSF) of the system, i.e. extended halos, lumpy Airy rings etc.
- Higher resolution
  - In specific cases depending upon algorithms and SNR
- Better Quantitative Analysis?
- Applicable to both Astronomy and Vision Science as well as other (e.g. microbiological) applications.
What is Deconvolution?

- Invert the shift invariant imaging equation

\[ \text{solve for } f(r) \text{ given both } g(r) \text{ and } h(r) \] – INVERSE PROBLEM
Deconvolution

Given the measurement $g(r)$ and the PSF $h(r)$ the object $f(r)$ is computed.

\[
\text{FT}\{f(\vec{r})\} = F(\vec{f}) = |F(\vec{f})| \exp[i\phi(\vec{f})] = \frac{G(\vec{f})}{H(\vec{f})}
\]

and inverse Fourier transform to obtain $f(r)$.

**Problem:**

The PSF and the measurement are both band-limited due to the finite size of the aperture.

The object/target is not.
Deconvolution

The quotient of the Fourier Transforms simply means dividing the moduli and subtracting the phases, i.e.

\[
F(\vec{f}) = \frac{G(\vec{f})}{H(\vec{f})} = \frac{|G(\vec{f})\exp[i\phi_G(\vec{f})]|}{|H(\vec{f})\exp[i\phi_H(\vec{f})]|} = \\
\frac{|G(\vec{f})|}{|H(\vec{f})|}\exp[i\phi_G(\vec{f})-\phi_H(\vec{f})]
\]
Image & Fourier Components

Left: Fourier amplitudes (ratio)
Right: Fourier phases (difference) for the object.

note circle = band-limit

13-Aug-04
Deconvolution via Linear Inversion

Inverse Filtering: Applies a filter to the Fourier quotient before inverting:

\[ \hat{f}(\vec{r}) = F^{-1} \left\{ \frac{G(\vec{f})}{H(\vec{f})} \Phi(\vec{f}) \right\} \]

\[ \Phi(f) = 0 \text{ for } f > f_c \]

Filter options:

Chat Function – The OTF of a Circular Aperture

Wiener Filter – Noise dependent filter

\[ \Phi(\vec{f}) \approx \frac{|H(\vec{f})|^2}{|H(\vec{f})|^2 + |N(\vec{f})|^2} \]
Deconvolution via Linear Inversion

Signal-to-noise Mask

\[ \Phi(\vec{f}) \approx \frac{|H(\vec{f})|^2}{|H(\vec{f})|^2 + |N(\vec{f})|^2} \]

Approximate as

\[ \Phi(\vec{f}) \approx \frac{|G(\vec{f})|^2}{|G(\vec{f})|^2 + |N(\vec{f})|^2} \]
Deconvolution via Wiener Filter

measurement                       PSF                          reconstruction

Note the negativity in the reconstruction – not physical
The problem of reconstructing the original target falls into a class of Mathematics known as **Inverse Problems** which has its own Journal. References in diverse publications such as *SPIE* Proceedings & *IEEE* Journals.

Multidisciplinary Field with many applications:

- **Applied Mathematics**
  - Matrix Inversion (*SIAM*)

- **Image and Signal Processing**

- **OSA Topical Meetings** on *Signal Recovery & Synthesis, Image Reconstruction, etc.* ...
Deconvolution - Iterative non-linear techniques

• Iterative Least Squares

• Statistical Techniques
  • Maximum Entropy
  • Maximum likelihood - Richardson-Lucy
  • Maximum a posteriori (MAP)

• Multiscale Techniques
  • Wavelet Transforms
  • Pixon - Bayesian image reconstruction
A Simple Iterative Deconvolution Algorithm

- Error Metric Minimization – object estimate & PSF convolve to measurement
  \[ E = \sum_{g \in \mathbb{R}} \left[ g_i - \left( \tilde{f}_i * h_i \right) \right]^2 \]

- Strict positivity constraint - reparameterize the variable
  \[ \tilde{f}_i = \phi_i^2 \]

- Conjugate Gradient Search (least squares fitting) requires the first-order derivatives w.r.t. the variable, e.g. \( \frac{\delta E}{\delta \phi_i} \)

- Equivalent to maximum-likelihood (the most probable solution) for Gaussian statistics

- Permits “super-resolution”
Maximum-likelihood
Lucy-Richardson Algorithm

Discrete Convolution

\[ g_i = \sum_j h_{ij} f_j \]

where \( \sum_j h_{ij} = 1 \) for all \( j \)

From Bayes theorem \( P(g_i|f_j) = h_{ij} \) and the object distribution can be expressed iteratively as

\[
 f_j = f_j \sum_i \left( \frac{h_{ij} g_i}{\sum_k h_{jk} f_k} \right)
\]

so that the LR kernel approaches unity as the iterations progress

Richardson-Lucy Application
Simulated Multiple Star

measurement             PSF                      reconstruction

Note – super-resolved result and identification of a 4th component

Super-resolution means recovery of spatial frequency information beyond the cut-off frequency of the measurement system.

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Richardson-Lucy Application
Simulated Galaxy

SNR = 2500  SNR = 250  SNR = 25

Truth

2000 iterations  200 iterations  26 iterations

Diffraction limited

All images on a logarithmic scale

LR works best for high SNR

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Richardson-Lucy Application
Noise Amplification

- Maximum-likelihood techniques suffer from noise amplification
- Problem is knowing when to stop
- SNR = 250

Measurement
26 iterations
200 iterations
500 iterations
1000 iterations
2000 iterations
5000 iterations
diffraction limited

All images on a logarithmic scale
Richardson-Lucy Application
Noise Amplification

• For small iterations RL produces spatial frequency components not strongly filtered by the OTF, i.e. the low spatial frequencies.

• Spatial frequencies which are strongly filtered by the OTF will take many iterations to reconstruct (the algorithm is relatively unresponsive), i.e. the high spatial frequencies.

• In the presence of noise, the implication is that after many iterations the differences are small and are likely to be due to noise amplification.

• This is a problem with any of these types of algorithms which use maximum-likelihood approaches including error metric minimization schemes.
Richardson-Lucy Application
Regularization Schemes

Sophisticated and silly!

- Why not smooth the result? – a low-pass filtering!
  SNR = 250 – 5000 iterations

- What is the reliability of the high SNR region?
- Is it oversmoothed or undersmoothed?
Object Regularization

- In an incoherent imaging system, the object is also real and positive.

- The object is not band-limited and can be reconstructed on a pixel-by-pixel basis – leads to super-resolution (recovery of power beyond spatial frequency cut-off).

- Limit resolution (and pixel-by-pixel variation) by applying a smoothing operator in the reconstruction.
  \[ f_v = (m * \gamma)_v \]

- Parametric information about the object structure can be used (Model Fitting):
  - Multiple point source
    \[ f(r) = \sum_j^N [A_j \delta(r - r_j)] \]
  - Planetary type-object (elliptical uniform disk)
    \[ F(\vec{f}) = \mathcal{F}[f(\vec{r})] = A\pi\alpha\beta^{2J_1(\pi\vec{f})} \frac{\pi}{\vec{f}} \]
Object Regularization

Planetary/hard-edged objects (avoids ringing)

Use of the finite-difference gradients $\Delta f(r)$ to generate an extra error term which preserves hard edges in $f(r)$.

$\alpha$ & $\beta$ are adjustable parameters.

$$E_{FD} = \alpha \sum_r \left[ \frac{\Delta f(r)}{\beta} - \ln \left( 1 + \frac{\Delta f(r)}{\beta} \right) \right]$$
Object Regularization

Local Gradient across the object defines the object texture (Generalized Gauss-Markov Random Field Model), i.e. $|f_i - f_j|^p$ where $p$ is the shape parameter.
Object Regularization

GGMRF example

truth

raw

over

under
The Imaging Process

Model the preprocessing in the imaging process

- Light from target to the detector
  - Through optical path – PSF $h(r)$
  - Detector
    - Gain – (flat field) $a(r)$
    - Dark current – (darks) $d(r)$
    - Background – (sky?) $b(r)$
    - Hot and dead pixels – included in $a(r)$
    - Noise

- Most algorithms work with “corrected” data
- Forward model the estimate to compare with the measurement

\[
g(r) = \left[\{f(r) \ast h(r)\} + b(r)\right]a(r) + d(r)
\]
Where do the PSFs come from?

Deconvolution requires a PSF.

- **Astronomy**
  - Point sources are everywhere on the sky
  - AO has small fields and the chances of a point source near target is very low.
  - PSF varies strongly across field (anisoplanatism)
  - PSF reference star
    - Different time – different brightness – different correction
  - Average PSF estimate from DM/WFS
  - Non-common path error between WFS and scoring camera requires calibration.

- **Vision Science:**
  - No point source structure easily seen in retinal images
  - DM “frozen” during retinal exposure.
  - Zernike coefficients on the DM are saved
  - Generate PSF from Zernike coefficients
    - Careful to obtain correct image scale ($\mu$rad/pixel)
    - Calibrate non-common path errors between WFS and scoring camera.
Variations on a Theme

• Poor or no PSF estimate – Myopic/Blind Deconvolution

• Deconvolution from wavefront sensing (DWFS)
  Use a simultaneously obtained wavefront to deconvolve the focal-plane data frame-by-frame. PSF generated from wavefront.

• Phase Diversity
  Two channel imaging typically in & out of focus. Permits restoration of target and PSF simultaneously. No PSF measurement needed.
Blind Deconvolution

Solves for both object & PSF

\[ g(r) = f(r) \ast h(r) + n(r) \]

Measurement
unknown object irradiance
unknown or poorly known PSF
contamination

Single measurement:
Under – determined - 1 measurement, 2 unknowns
Never really “blind”
Blind Deconvolution – Physical Constraints

• How to minimize the search space for a solution?

• Uses Physical Constraints.
  – \( f(r) \) & \( h(r) \) are positive, real & have finite support.
  – \( h(r) \) is band-limited – symmetry breaking prevents the simple solution of \( h(r) = \delta(r) \)

• a priori information - further symmetry breaking (\( a * b = b * a \))
  – Prior knowledge (Physical Constraints)
  – PSF knowledge: band-limit, known pupil, wavefront model
  – Object & PSF parameterization – in astronomy, e.g. multiple point sources
  – Noise statistics
  – Multiple Frames: (MFBD)
    • Same object, different PSFs.
    • \( N \) measurements, \( N+1 \) unknowns.
Multiple Frame Constraints

Multiple Observations of a common object

\[ g_1(r) = f(r) * h_1(r) \]
\[ g_2(r) = f(r) * h_2(r) \]
\[ \vdots \]
\[ g_n(r) = f(r) * h_n(r) \]

- Reduces the ratio of unknown to measurements from 2:1 to \( n+1:n \)
- The greater the diversity of \( h(r) \), the easier the separation of the PSF and object.
An MFBD Algorithm

• Uses a Conjugate Gradient Error Metric Minimization scheme
  - Least squares fit.

• Error Metric – minimizing the residuals (convolution error):

\[ E = \sum_{ik} |g_{ik} - \tilde{g}_{ik}|^2 = \sum_{ik} |g_{ik} - (f_i \ast \tilde{h}_{ik})|^2 = \sum_{ik} |r_{ik}|^2 \]

• Alternative error metric – minimizing the residual autocorrelation:

\[ E = \sum_{ik} |r_{ik} \otimes r_{ik}|^2 \]

  Autocorrelation of residuals
  Reduces correlation in the residuals
  (minimizes “print through”)
  So not sum over the 0 location.
**An MFBD Algorithm**

- **Object non-negativity**
  Reparameterize the object as the square of another variable or penalize the object against negativity.

\[
E = \sum_{u \in f_i < 0} \left| \tilde{f} \right|^2
\]

- **PSF Constraints** (when pupil is not known)
  - **Non-negativity**
    Reparameterize or penalize
    \[
    E_{PSF} = \sum_{u \in \tilde{h}_{i,k} < 0} \left| \tilde{h}_{i,k} \right|^2
    \]
  - **Band-limit**
    \[
    E_{bl} = \sum_{k,u > u_c} \left| \widetilde{H}_{k,u} \right|^2
    \]
An MFBD Algorithm

- PSF Constraints (Using the Pupil)
  
  Parameterize the PSF as the power spectrum of the complex wavefront at the pupil, i.e.

  \[ \tilde{a}_{ik} = \sum_{v} W_v \exp \left\{ j \left[ \frac{2\pi iv}{N} \right] - \phi_{vk} \right\} \]

where

\[ \tilde{a}_{ik} = \sum_{v} W_v \exp \left\{ j \left[ \frac{2\pi iv}{N} \right] - \phi_{vk} \right\} \]
PSF Constraints

- **PSF Constraints (Using the Pupil)**
  - Modally - express the phases as either a set of Zernike modes of order $M$
    \[
    \varphi_{vk} = \sum_{m=1}^{M} q_m Z_{vk}
    \]
  - or zonally as
    \[
    \begin{pmatrix}
    (\frac{v}{\sigma})^2 \\
    (\frac{2}\sigma)
    \end{pmatrix}
    \]
    which enforces spatial correlation of the phases.

- Phases can also be constrained by statistical knowledge of the AO system performance.

- Wavefront amplitudes can be set to unity or can be solved for as an unknown especially in the presence of scintillation.
PSF Constraints

- **Myopic Deconvolution (using known PSF information)**
  - For MFBD penalize PSFs for departure from a “typical” PSF or model (good for multi-frame measurements)

\[
E_{SAA} = \sum_{ik} |h_{ik}^{SAA} - \tilde{h}_{ik}^{SAA}|^2
\]

- Penalize PSF on power spectral density (PSD)

\[
E_{PSD} = \sum_i \left[ \left( \frac{\langle \tilde{H} \rangle_i - \tilde{H}_i}{\text{PSD}_H} \right)^2 \right]
\]

where the PSD is based upon the atmospheric conditions and AO correction.
PSF Constraints

- **Myopic Deconvolution**
  using the reconstructed PSF

\[
E_{\text{PSD}} = \sum_i \left( \frac{\langle H \rangle_i - \tilde{H}_i}{\text{PSD}_H(f)} \right)^2
\]

residual phase structure function

- where

\[
\langle H \rangle = \exp\left[ -\frac{1}{2} D_\phi(\lambda f) \right] T(f)
\]

Perfect optics transfer function

- and

\[
\text{PSD}_H(f) = \sigma^2_{\text{turb}} = \frac{\tau_s T_i}{T_i} \left[ \text{STF}(f) - \left| \langle H \rangle \right|^2 \right]
\]

Speckle transfer function

- where

\[
\tau_s = 0.36 \frac{r_0}{V}
\]

Integration time
Deconvolution from Wavefront sensing

Multiframe deconvolution with a “known” PSF.

The estimate of the Fourier components of the target for a series of short-exposure observations is (also called speckle holography):

\[ F'(f) = \frac{\langle G(f) H^*(f) \rangle}{\langle |H'(f)|^2 \rangle} = F(f) \frac{\langle H(f) H'^*(f) \rangle}{\langle |H'(f)|^2 \rangle} \]

where \(|H'(f)|^2 = H'(f) H'(f)^*\) and \(H'(f)\) is the PSF estimate obtained from the measured wavefront, i.e. the autocorrelation of the complex wavefront at the pupil.

\(F'(f) = F(f)\) when \(H'(f) = H(f)\)

Noise sensitive transfer function. Requires good SNR modeling.
Phase Diversity

Measurement of the object in two different channels. No separate PSF measurement.

\[ g_1(r) = f(r) * h_1(r) \quad \text{and} \quad g_2(r) = f(r) * h_2(r) \]

Two measurements – 3 unknowns – \( f(r), h_1(r), h_2(r) \) but \( h_1(r) \) & \( h_2(r) \) are related by a known diversity, e.g. defocus. Hence 2 unknowns \( f(r), h_1(r) \)

\[ H_1(f) = |H_1(f)| \exp[i\vartheta(f)] \quad \& \quad H_2(f) = |H_1(f)| \exp[i\vartheta(f) + \alpha(f)] \]
Phase Diversity

Phase Diversity restores both the target and the complex wavefront phases at the pupil.

- Solve for the wavefront phases which represent the unknowns for the PSFs

- The phases can be represented as either
  - zonal (pixel-by-pixel)
  - modal (e.g. Zernike modes) – fewer unknowns

- The object spectrum can be written in terms of the wavefront phases, i.e.

\[
F(f) = \frac{G_1(f)H_1^*(f) + G_2(f)H_2^*(f)}{|G_1(f)|^2 + |G_2(f)|^2}
\]

- Recent work suggests that solving for the complex wavefront, i.e. modeling scintillation improved PD performance for both object and phase recovery.
Deconvolution Applications

Applications to both Astronomical and Vision Science data.

- Demonstrate improved Contrast in images
  - Visually – more information
- Quantitative measurements improved with deconvolution.
Deconvolved using a PSF estimated from the bright core and a separate PSF star.
Adaptive Optics Solar Imaging

Low-Order AO System

- Lack of PSF information.
- Sunspot and granulation features show improved contrast, enhancing detail showing magnetic field structure

[Data from Thomas Rimmele, NSO-SP]
Extended Sources near the Galactic Center

Keck Imaging at 2.2µm

Deconvolution permits easier determination of extended sources of astrophysical interest
Keck Imaging of Io in Sunlight

Basic processed image:
Kron-b band (2.2 microns)

Basic processed image:
L'-band (3.8 microns)
18 Dec. 2001, 7:34 UT

Basic processed image:
Ms-band (4.7 microns)
18 Dec. 2001, 7:36 UT

MISTRAL deconvolved
Kron-b band (2.2 microns)

MISTRAL deconvolved
L'-band (3.8 microns)
18 Dec. 2001, 7:34 UT

MISTRAL deconvolved
Ms-band (4.7 microns)
18 Dec. 2001, 7:36 UT

Calisto S1
(reconstructed)

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Keck Imaging of Io in Eclipse

Two Different Algorithms

Keck observations to identify hot-spots.

K-Band
19 with IDAC
17 with MISTRAL

L-Band
23 with IDAC
12 with MISTRAL
Artificial Satellite Imaging
Deconvolution & Vision Science

- Can deconvolution be used to quantitatively improve the cone classification?

- Measure the absorptances in two different bleach settings (470nm & 650nm).

  \[
  A_{470} = 1 - \frac{R_{470}}{R_{\text{full}}} \\
  A_{650} = 1 - \frac{R_{650}}{R_{\text{full}}}
  \]

- Compare two-color absorptances for each cone – determines the cone class.
Cone Histogram

L cones

M cones

JW: 1 deg nasal
Cone Histogram

A. No noise
B. Photon noise
C. PSF Blur
D. Blur & noise

absorptance after 650 bleach

absorptance after 470 bleach
Proof of Principle: Synthetic Data

10 simulated images for each bleaching condition (30 total)
PSF and object are known
Proof of Principle: Synthetic Data

Best Estimate of Object

Best Estimate of the 10 PSFs

Actual 10 PSFs used in simulation
Proof of Principle: Synthetic Data
Proof of Principle: Synthetic Data

All cones were correctly identified from deconvolved images
Real Data

<table>
<thead>
<tr>
<th>Full Bleach</th>
<th>470 nm Bleach</th>
<th>650 nm Bleach</th>
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<td><img src="image6.png" alt="Image" /></td>
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13-Aug-04  CfAO Summer School 2004  Macaque retina
Real Data

An isolated cone

Contrast Enhancement showing reduction in overlap

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Macaque retina
Scatter plots of L and M cones

3 pixels  4 pixels  6 pixels  8 pixels  10 pixels
Histograms of L and M cones

3 pixels

4 pixels

6 pixels

8 pixels

10 pixels

Number of Cones

Cone Angle
Cone Assignment Error

Number of cones

\[ \theta (\text{deg}) \]

divide this area

by this area
Reduction in Cone Assignment Error
Imaging rods *in vivo* (Choi et al)

- Deconvolution improves the contrast of not only the rods but also the complex cone structure.
Summary

• AO systems do not produce “perfect” diffractions-limited PSFs.

• AO PSFs differ between Vision Science and Astronomical observations
  • Both show structure but different type which affect the images.

• Deconvolution permits improved quantitative and qualitative analysis.
  • Improves contrast
  • Different algorithms behave differently
  • No “Magic Bullet”
  • Incorporate as much physics as possible.

• For astronomy, isoplanatism is assumed, how to incorporate anisoplanatism for wide field imaging?

• Retinal images are 3D structures – 3D Deconvolution