Designing Adaptive Optics Systems

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Designing Adaptive Optics
Systems: A User’s Guide

- What do we want?
- Performance metrics
- Light
- The atmosphere
- AO hardware
- Example systems
- Software, calibration, & operation
What do we want?

- Operating regime
  - Wavelengths of correction
  - Field of view
  - Sky coverage fraction
- Performance
  - Resolution
  - Encircled energy
  - Contrast
  - Strehl ratio
- On sky efficiency
- Reliability
Resolution

The *Rayleigh criterion*: in a diffraction-limited optical system, two point sources are separately distinguishable at a separation $\sim \lambda / D$

In AO systems with a Strehl $>\sim 0.15$, the FWHM of the corrected image is $\sim \lambda / D$

F. Rodier introduced the concept of “Strehl-resolution” = width you have to enclose to get the same energy as in the FHWM of the ideal PSF
Image contrast

- Ratio of PSF core to PSF halo brightness
- Integration time required to detect a faint object next to a bright object

Keck Adaptive Optics

The Galactic Center at 2.2 microns (without adaptive optics)

Total exposure for mosaic for similar SNR ~ 20 minutes

Average resolution: (brightest stars): 0.4"

Resolved sources

Keck AO Image of the Galactic Center
Ghez, Wizinowich, Acton & Lai
Energy in a spectrograph slit

- Wider slit necessary to gather enough signal
- Signal more concentrated, less background
- Higher spectral resolution for a given dispersion spectrograph
Energy in a spectrograph slit

The SNR-optimal slit-width transitions to $\lambda/d$ when the Strehl gets $> 0.1$
Strehl ratio

\[ S = \frac{\max_\theta [PSF(\theta)]}{PSF(0,0)|_{\phi=0}} \]

Images taken with the IRCAL camera at Lick Observatory, James Graham, UC Berkeley
Strehl is related to wavefront variance via Marechal’s approximation

\[ \langle S \rangle = \frac{\langle PSF(0,0) \rangle}{PSF(0,0)\big|_{\phi=0}} \approx \exp\{-\sigma_{\phi}^2\} \]

- Valid approximation for small \( \sigma_{\phi} \)
- Extended region of validity for AO-corrected wavefronts
Light

- Light originates as photons each emitted by the oscillation of a single atom.
- Light travels as a wave via all possible paths (paths of ‘least time’).
- Light, when detected, is realized as single-photon events distributed according to the intensity of the wave.
Diffraction

Photon momentum

\[ p = \frac{h}{\lambda} \]
\[ \Delta x = \infty \]
\[ \Delta p = 0 \]

Uncertainty principle

\[ \Delta x \Delta p \approx h \]

Law of diffraction

\[ \theta \approx \frac{\Delta p}{p} = \frac{\lambda}{D} \]
Diffraction-limited image formation

$\sim f^{\lambda/D}$

The Marechal criterion

$$\sigma_\phi = \left\langle \left[ \phi(x) - \left\langle \phi(x) \right\rangle_A \right]^2 \right\rangle_A^{1/2} < \pi$$
Fresnel number

\[ n = \frac{D}{\sqrt{\lambda L}} \]

- Rayleigh Range
  Transition from geometric to diffractive ray propagation \((a \leftrightarrow D)\)

- Fresnel zone
  Smallest geometrically resolved phase perturbation after propagating distance \(L\)

- Scintillation
  Variation in intensity at distance \(L\) due to a phase perturbation

\[ L = \frac{D^2}{\lambda} \]

\[ a = \sqrt{\lambda L} \]
M is transverse magnification ($10^{-2}$)

\[ D' = MD \quad L' = M^2 L \quad \theta' = \frac{D'}{L'} = M^{-1} \theta \]

Preserves the Lagrange invariant:

\[ \Xi = \theta'D' = \theta D \]

Also preserves Fresnel numbers:

\[ n = \frac{a}{\sqrt{\lambda L}} = \frac{a'}{\sqrt{\lambda L'}} \]
Atmospheric aberrations
Images of a bright star, Arcturus

Lick Observatory, 1 m telescope

\[ \theta \sim \frac{\lambda}{r_0} \sim 1 \text{ arc sec} \]

\[ \theta \sim \frac{\lambda}{D} \]

Speckles (each at diffraction limit of telescope)
Aberrations arise from turbulent mixing in atmospheric layers.

- Stratosphere
- Tropopause
- Boundary layer
- Wind flow over dome
- Heat sources within dome

- 10-12 km
- ~ 1 km
"Typical" $C_n^2$ profile of atmospheric turbulence

$$\delta N = -77.6 \times (P/T^2) \delta T$$

$$C_n^2(z) = \left\langle \delta n^2(x, y, z) \right\rangle_{x,y}$$
Light propagation through the turbulent atmosphere

- Index variations

\[ C_n^2(z) = \left\langle \delta n^2(x, y, z) \right\rangle_{x,y} \]

- Total optical path variation*

\[ \phi(x) = \left( \frac{2\pi}{\lambda} \right) \int_0^\infty \delta n(x, y, z) dz \]

*Under geometric conditions (n >> 1)

- Wavefront aberration statistics: Structure Function \( D_\phi \)

\[ D_\phi(r) = \left\langle [\phi(x) - \phi(x + r)]^2 \right\rangle = 6.88 (r/r_0)^{5/3} \]

Kolmogorov spatial statistics (a ‘fractal’ process)
Image formation using Fourier optics

- Field at the pupil plane
  \[ E(u) = A(u)e^{i\phi(u)} \]

- Field at the focal plane
  \[ F(\theta) = \frac{1}{\lambda} \int E(u)e^{i2\pi u \cdot \theta/\lambda} du \]

- Average PSF at the focal plane
  \[ \langle PSF(\theta) \rangle = \left\langle |F(\theta)|^2 \right\rangle \]
  \[ \frac{1}{\lambda^2} \int \int A(u)A(u + r) \left\langle e^{i(\phi(u) - \phi(u + r))} \right\rangle e^{-i2\pi r \cdot \theta/\lambda} dudr \]
Image formation (continued)

\[ \langle A(u)A(u + r) \rangle_A = B_0(r) \]  

MTF of Pupil

\[ \left\langle e^{i(\phi(u) - \phi(u+r))} \right\rangle = e^{\frac{1}{2} \langle (\phi(u) - \phi(u+r))^2 \rangle} = e^{-\frac{1}{2}D_\phi(r)} \]  

MTF of Atmosphere

\[ \langle PSF(\theta) \rangle = \frac{A_0}{\lambda^2} \int B_0(r)B_A(r)e^{-i2\pi\cdot\theta/\lambda} \, dr \]  

PSF = \mathcal{F}(MTF)

- **Strehl**: \( \frac{PSF(0)}{PSF(0)} \big|_{\phi = 0} \)
Goal of adaptive optics

Maximize Strehl $\Leftrightarrow$ Minimize mean square wavefront error

(i.e. $S = B_A(0) \sim \exp\{-1/2 D_\phi\}$, so make $D_\phi$ small)
Astronomical adaptive optics system architecture

- Deformable mirror is made optically conjugate to telescope primary mirror via the pupil image relay.
Statistical characteristics of atmospheric wavefronts

Transverse correlation distance

\[ r_0 = \left[ 0.423 \left( \frac{2\pi}{\lambda} \right)^2 \int_0^\infty C_n^2(z)dz \right]^{-3/5} \]

- Depends on \( \lambda \)
- Typical: \( r_0 = 20 \text{ cm} \) at \( \lambda = 0.5 \mu \)

Correlation angle

\[ \theta_0 = \left[ 2.905 \left( \frac{2\pi}{\lambda} \right)^2 \int_0^\infty C_n^2(z)z^{5/3}dz \right]^{-3/5} \]

- Typical: \( \theta_0 = 4 \text{ arcsec} \) at \( \lambda = 0.5 \mu \)
- Mean height of turbulence: \( h_0 = r_0/\theta \)
- \( h_0 = 8.2 \text{ km} \)

Correlation time

\[ D_\phi(\tau) = \left[ \langle \left[ \phi(t) - \phi(t + \tau) \right]^2 \rangle \right] = \left( \frac{\tau}{\tau_0} \right)^{5/3} \]

\[ \tau_0 = \left[ 2.91 \left( \frac{2\pi}{\lambda} \right)^2 \int_0^\infty C_n^2(z)v(z)^{3/5}dz \right]^{-3/5} \]

- Typical: \( \tau_0 = 3 \text{ ms} \) at \( \lambda = 0.5 \mu \)
- Mean wind velocity: \( \nu_0 = 0.314 \frac{r_0}{\tau_0} \)
- \( \nu_0 \sim 20 \text{ m/s} \)
Hardware

Wavefront control

Wavefront sensing
Wavefront control with deformable mirrors

- Divide pupil into regions of size \( r_0 \), do “best fit” to wavefront
- Several types of deformable mirror (DM), each has its own characteristic “fitting efficiency”
- Other requirements: dynamic range (the farthest excursion the mirror surface can take), frequency response, influence function of actuators, surface quality (smoothness), hysteresis, power dissipation
Generic construction of piezo type DM actuators

- Actuators are glued to back of glass mirror
- When you apply a voltage (PZT) or a magnetic field (PMN) to the actuator, it expands or contracts in length, thereby pushing or pulling on the mirror
Front view of Xinetics DM (Keck)

349 degrees of freedom; 250 in use at any one time

(paper coasters)
Rear view of Xinetics 349 actuator
DM used in Keck Telescope AO system
Shack-Hartmann wavefront sensor concept - measure subaperture tilts
Measurement error from Shack-Hartmann sensing

- Measurement error depends on size of spot as seen in a subaperture, $\theta_b$, wavelength $\lambda$, subap size $d$, and signal-to-noise ratio SNR:

$$ \sigma_{SNR} = \eta d \frac{\theta_b}{SNR} \approx \eta d \frac{\lambda/r_0}{SNR} $$

(Hardy equation 5.16)
Anisoplanatism error

- If the guide star is not the science object...

\[ \theta_0 \approx \frac{r_0}{h} \]

Isoplanatic angle:

\[ \sigma_A^2 = \left( \frac{\theta}{\theta_0} \right)^{5/3} \]
Adaptive optics residual errors

- **Fitting error**
  \[ \sigma_F^2 = \mu (d/r_0)^{5/3} \]

- **Bandwidth error**
  \[ \sigma_{BW}^2 = \kappa \left( \tau_{CL}/\tau_0 \right)^{5/3} \]

- **Anisoplanatism error**
  \[ \sigma_A^2 = (\theta/\theta_0)^{5/3} \]

- **Wavefront measurement error**
  \[ \sigma_{SNR} = \eta d \frac{\theta_b}{SNR} \approx \eta d \frac{\lambda/r_0}{SNR} \]
Summary of adaptive optics system requirements

- Enough actuators to fit the wavefront
  - Actuator spacing $d \sim r_0$
- Fast enough update rate to keep up with the atmosphere
  - Temporal bandwidth $\tau_{CL} \sim \tau_0$
- Guidestar nearby science target
  - Isoplanatic patch: $\theta < \theta_0$
- Enough light from the guidestar to measure the wavefront accurately
Error Budgets

Total Error

Fitting error

Bandwidth

WF Meas

Anisoplanatism

Bandwidth/Star time Optimization

SNR calculation

Star density models

Guidestar: $m_v=12$

$\theta$

$\theta_0$

$n_e$

$d$

$r_0$

$\tau_0$

$fc$
Example systems

Lick 3-meter
Keck 10-meter
TMT (30-meter)
Lick Laser Guidestar
Adaptive Optics System
Lick AO wavefront control geometry

- Measure local wavefront gradients
- Reconstruct wavefront surface given local gradients

Hartmann CCD sensor image
Lick AO System

- **Telescope**: Shane 120” (3 meter primary, 80 cm secondary)
- **Deformable Mirror**: LLNL-built, 61 actuators, hex grid, \(d_a=50\) cm
- **Wavefront Sensor**: Shack-Hartmann, 40 subapertures, \(d=43\) cm
- **Controller**: Max sample rate: 1000 Hz
- **Laser Guide Star**: Sodium layer, dye laser, 12-15 watts
- **Science Camera**: 1-2 micron infrared, HgCdTe Picknic array, 256 x 256, 0.076 arcsec/pixel (Nyquist in K)
If there is no nearby star, make your own “star” using a laser
The laser produces an artificial star in the mesospheric Sodium layer

A small spot is important:

- Laser beam quality ($M^2$)
- Launch telescope aperture $d_p$ matched to $r_0$ of atmosphere
- Translates to wavefront measurement accuracy

$$\sigma_{\text{wavefront measurement}} = \text{(spot size)} \times \frac{1}{\text{SNR}}$$
Lick laser guidestar AO operations staff

Adaptive Optics System Operator
AO Support Scientist, Elinor Gates

Laser System Operator
Laser technician Kostas Chloros
The advantage to using the laser is a huge increase in sky coverage.

LGS coverage: 50%

NGS coverage: 0.1%

Galactic latitude:
- 90°
- 45°
- 30°
The new generation: adaptive optics on 8-10 m telescopes

Summit of Mauna Kea volcano in Hawaii:

And at other places: MMT, VLT, LBT, Gemini South
The Keck Telescope AO System

Light Path — Keck Telescope diagram shows the path of incoming starlight (1), first on its way to the primary mirror; reflected off the primary, toward the secondary mirror (2); bouncing off the secondary, back down toward the tertiary mirror (3); and finally reflected either off the tertiary mirror to an instrument at the Nasmyth focus (4), or to the Cassegrain focus (5) beneath the primary mirror.

Credit: California Association for Research in Astronomy
Keck adaptive optics bench
Keck AO pupil geometry

- 342 actuators
- 298 subapertures
- $d = 50 \text{ cm}$
- (20 across $D=10\text{m}$)
- Laser guide star
TMT

- $D = 30\text{m}$
- $d = 30\text{cm}$
  - 100 acts across
  - $10^4$ actuators total/DM
- Multiple lasers
- Multiple DMs
Laser guidestar specific error: Cone effect

Example: $h=4$ km, $r_0=10$ cm $\Rightarrow d_0=4.5$ m

$$\sigma_{\text{cone}}^2 = \left(\frac{d}{d_0}\right)^{5/3}$$

$$d_0 \approx 2\frac{Z}{h}r_0$$
Cone effect gets worse as telescope size increases.
Mitigating cone effect with multiple laser guidestars

"Missing" Data

90 km
Software, calibration, and operation

- Real time controller
- Operator’s interface
Calibration Procedures

- Initial alignment
- Image sharpening on internal source using the science camera

Nominal

Mode 2 (Astig 0) +20
Mode 3 (Astig 45) +20
Mode 4 (Coma) +20

Mode 2 (Astig 0) -20
Mode 3 (Astig 45) -20
Mode 4 (Coma) -20
Operating Procedure

- Acquisition
- Field steering
- AO coordination with science instrument
  - Nod sequence
  - Loop control
- Laser:
  - Zenith dependent focus
  - Calibration issues
Conclusion

- We’ve given you a taste of the issues involved in designing astronomical adaptive optics systems.
- Systems are getting more challenging as telescope sizes increase.
- AO is becoming a crucial instrument on next generation telescopes.