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CFAO- Fall Retreat
PSFs and Deconvolution

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Motivation

- Small residual phase errors exist in images corrected by adaptive optics
- Deconvolution can correct residual blurring
  - Limited by SNR
  - Limited by knowledge of PSF
    - PSF computed from WFS data
    - Limited by accuracy of wave front sensor, knowledge of system

Outline

- Deconvolution by Wiener-Helstrom filter
- Deconvoution with simulated data and perfect PSF/wavefront-estimate
- Deconvolution with simulated data and underestimated wavefront error
  - Effect on constant, *c*, in W-H filter
- Deconvolution with real data
  - Suspicion that wavefront error is underestimated

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Incoherent Imaging Model

Image model: $g(x,y) \approx f(x,y) * s(x,y) + \sigma_n n_1(x,y)$

where f = object brightness, $PSF = s(x,y) = \left| \mathcal{F}\left[P(u,v) e^{i\phi(u,v)} \right] \right|^2$, σ_n = rms noise,

$P(u,v)$ = pupil function, $\phi(\text{u,v})$ = phase errors (aberrations)

$n_1(x,y)$ = unit - variance, independent, zero - mean Gaussian random noise

In Fourier domain:

$$G(f_x, f_y) = F(f_x, f_y) S(f_x, f_y) + \sigma_n N_1(f_x, f_y)$$

$N_1(f_x, f_y)$ = independent, zero - mean complex Gaussian random noise

where $S(f_x,f_y)$ = Optical Transfer Function (OTF), MTF = $|S(f_x,f_y)|$

(Alternatives: SNR on $|F|^2$, Poisson noise model)

Wiener-Helstrom Restoration Filter

Minimize mean-squared error: $E = \left\langle \sum_{f_x, f_y} \left| G_{cor}\left(f_x, f_y\right) - F(f_x, f_y) \right|^2 \right\rangle$

Winer-Helstrom Restoration (linear)

$$G_{cor}\left(f_x, f_y\right) = W\left(f_x, f_y\right) G\left(f_x, f_y\right) \qquad W = \frac{S^*}{|S|^2 + c\,\Phi_N / \Phi_o}$$

where $\Phi_o = \left\langle |F|^2 \right\rangle$ and $\Phi_N = \left\langle |\sigma_n N_1|^2 \right\rangle$

Wiener filter boosts spatial frequencies where SNR is high

- suppresses spatial frequencies where SNR is low

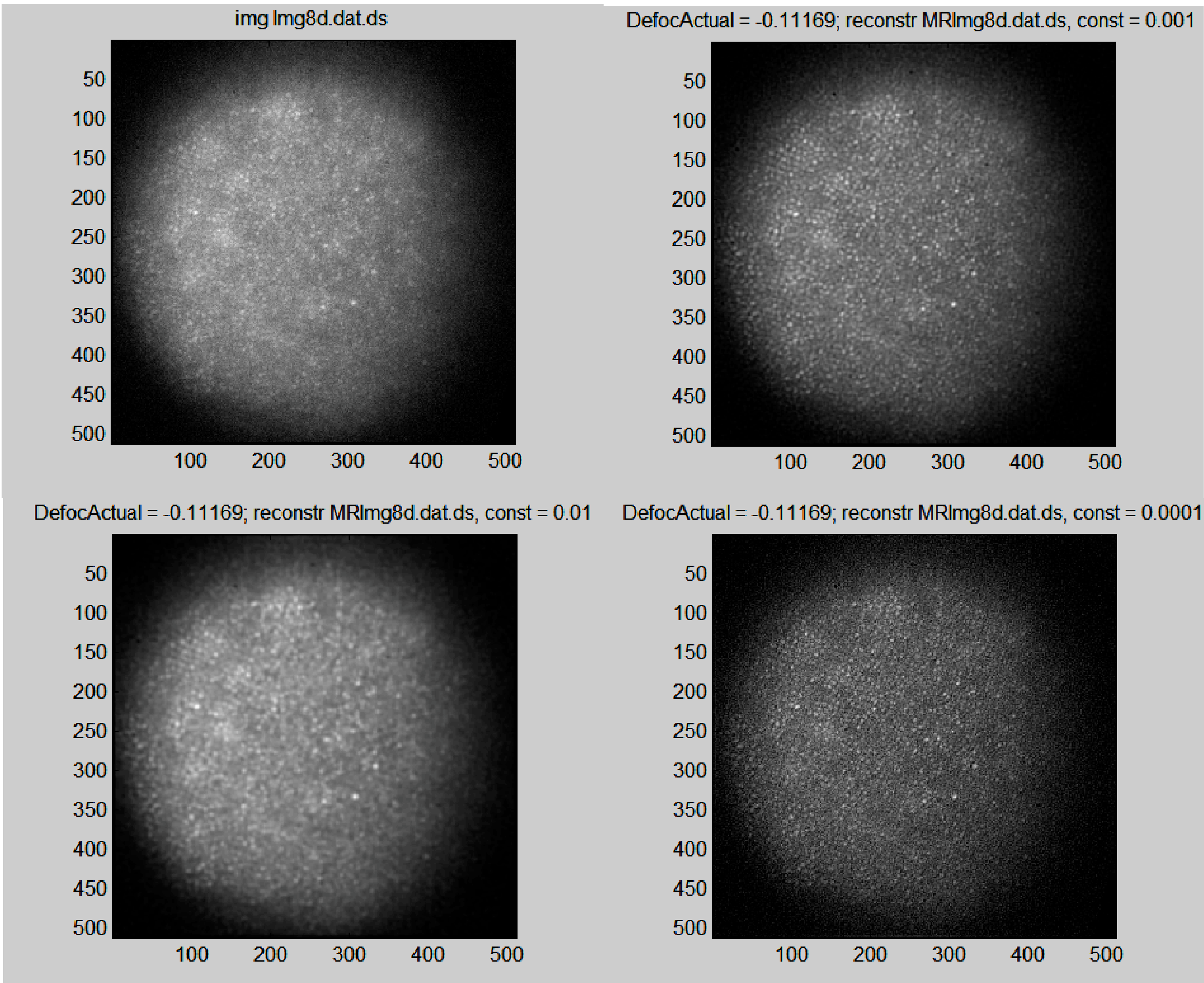
c = constant factor (allows for tradeoff between noise & resolution)

- $c = 1$ ==> Least-squared-error solution
- Greater c gives smoother, lower-resolution image
- **Smaller c gives sharper edges, noisier image (c = 0.005 to 0.5)**

Choice of Φ_o , which depends on spatial freq.

- Some choose it to be a constant in the W-H filter: $c\,\Phi_N / \Phi_o$=const.
- Use freq.-dependent term; Estimate it, e.g., by curve-fitting a model

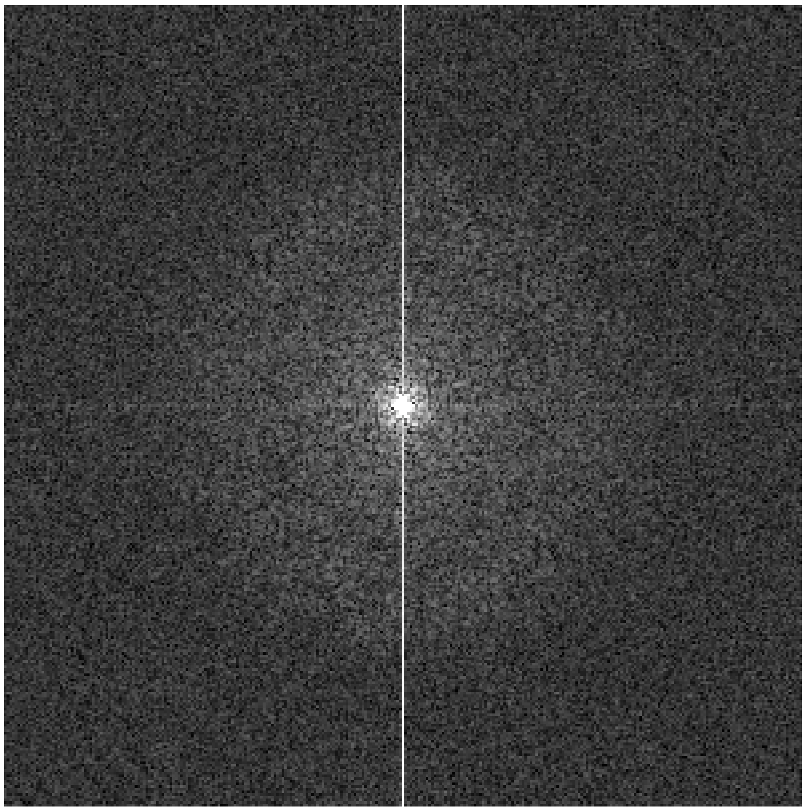
CVS Real Data Comparing different values of *c* for PSF computed from wavefront sensor data

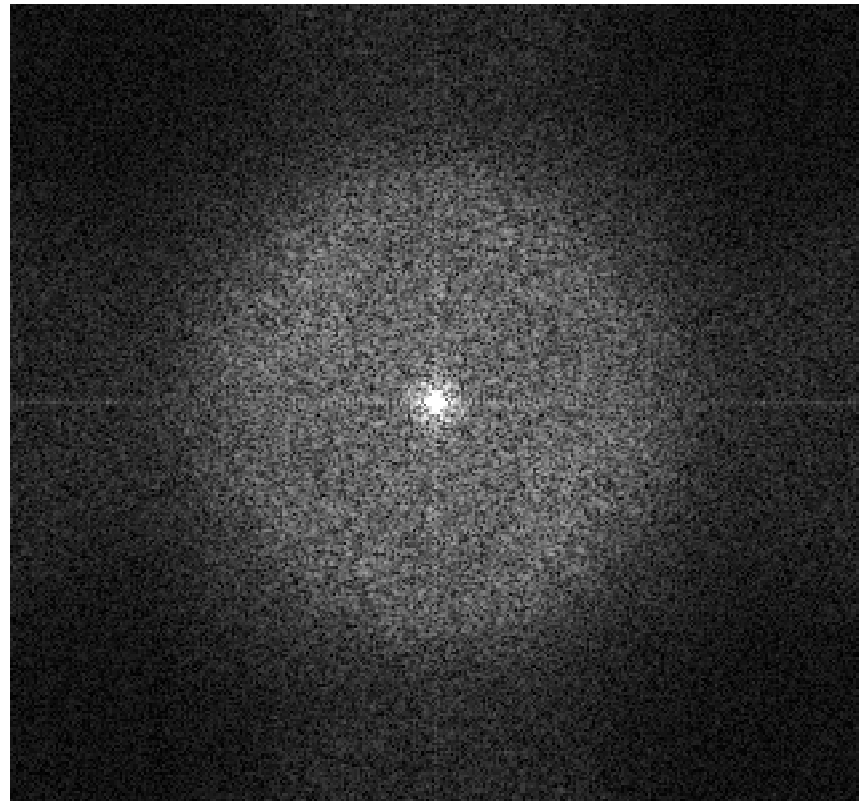


Imagery is improved!

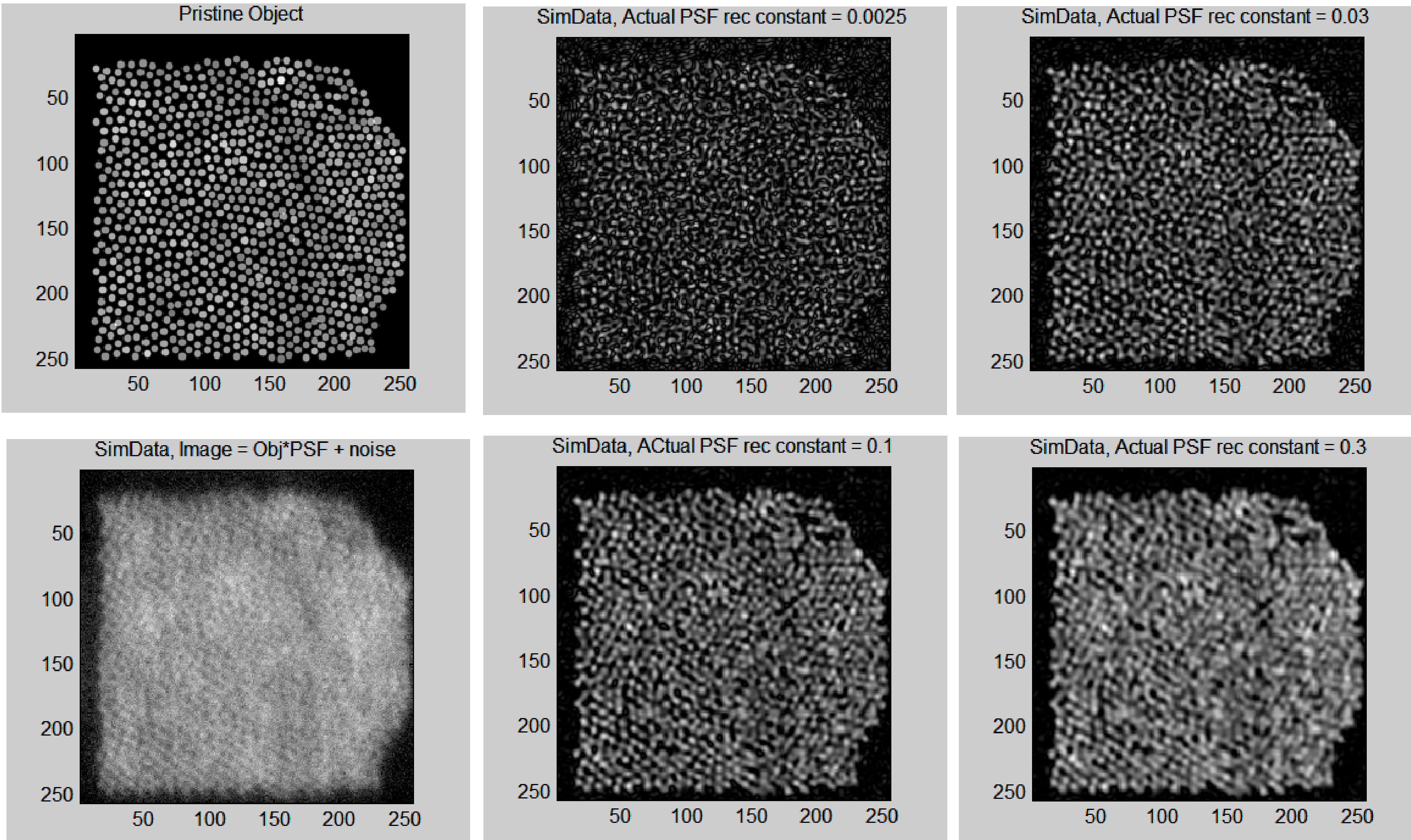
Best *c* is lower than expected

Fourier Transforms

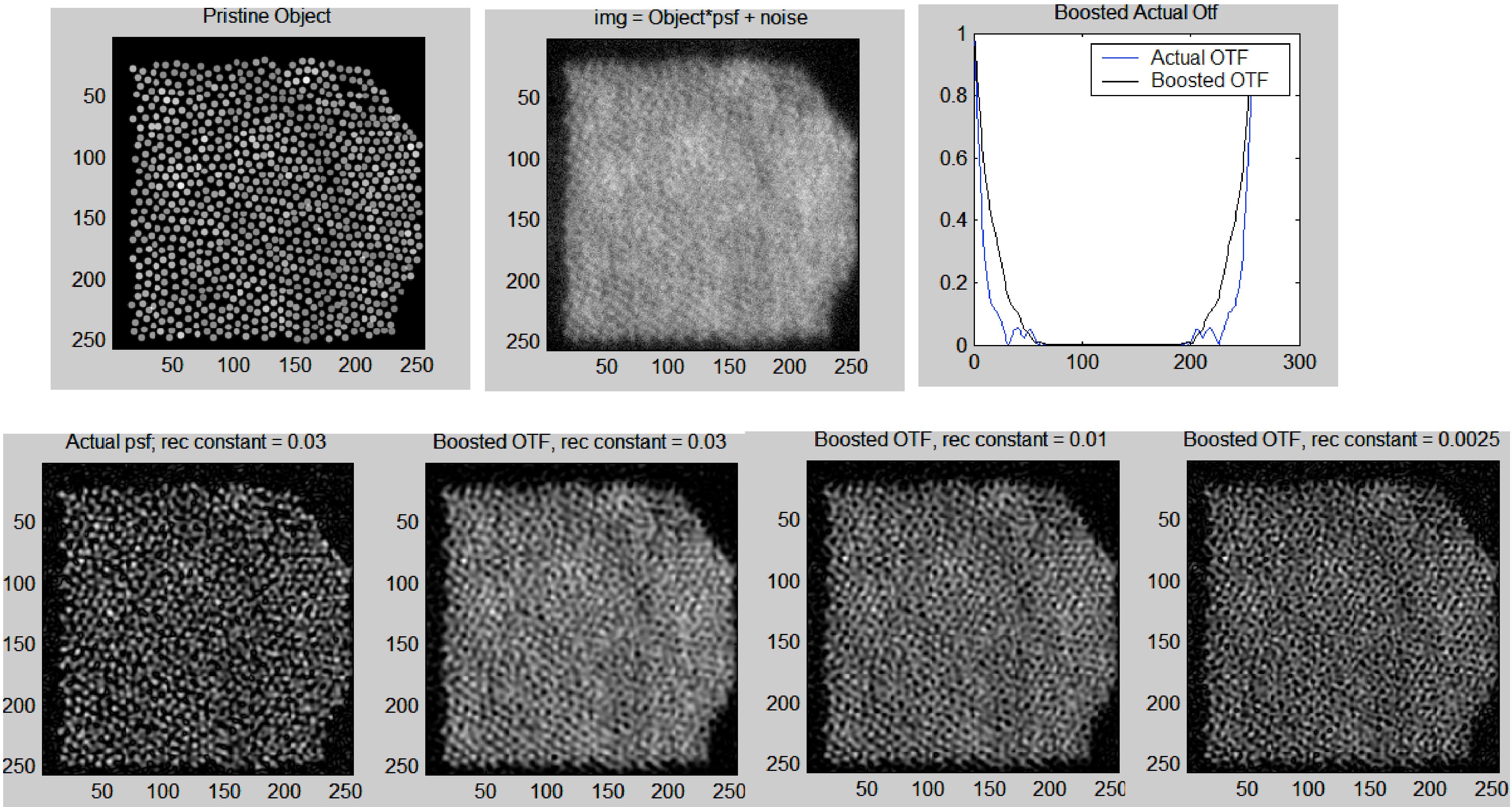

Before Restoration


After Restoration

Noisy Simulated Data, True PSF, Varying *c*



Simulated Dat: deconvolution with boosted OTF (underestimating aberrations) ==> lower value for *c*



Q: Are the residual aberrations underestimated for the real data?

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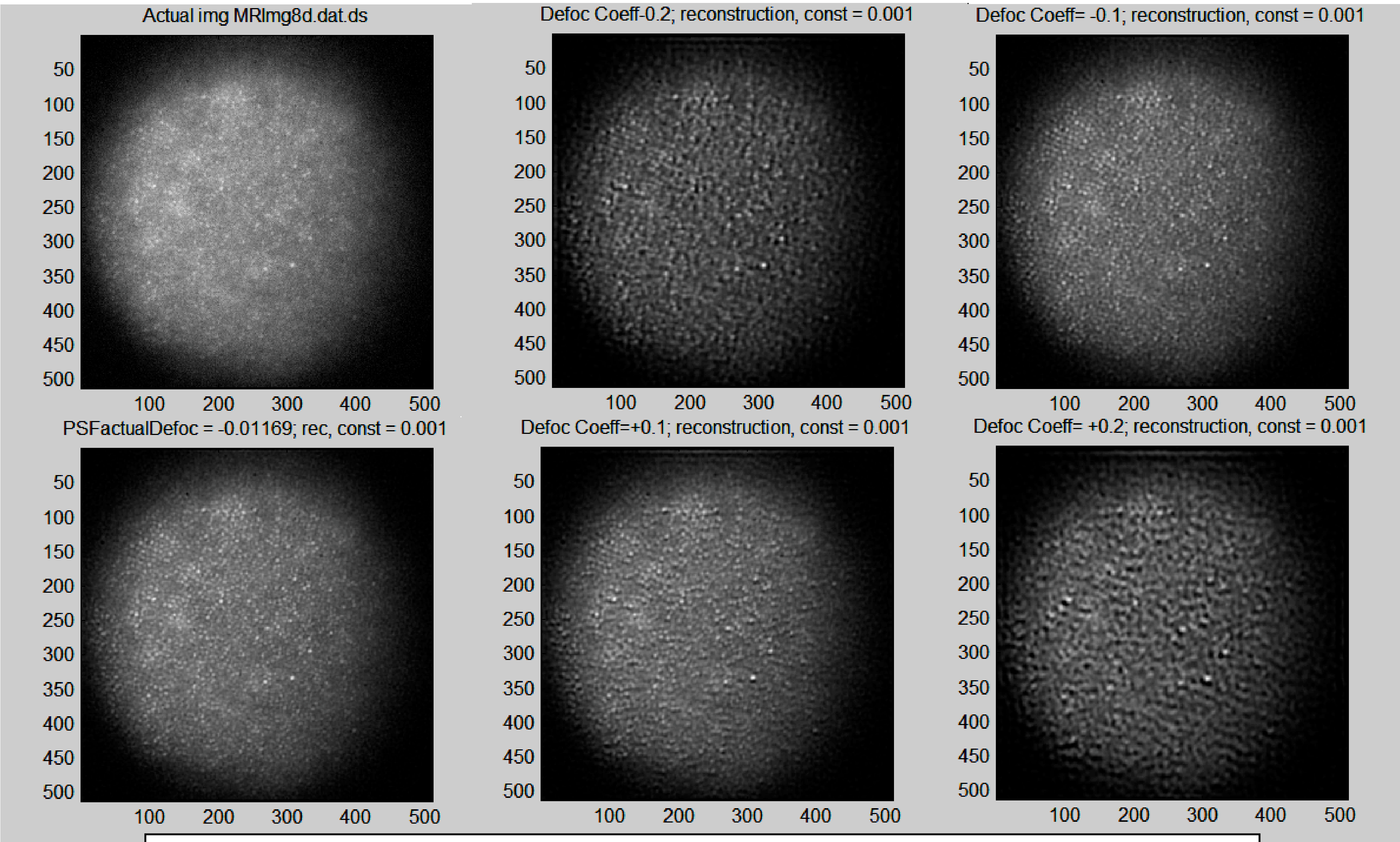
Real CVS Data
Comparing different defocus coefficients



If aberrations are underestimated, it is not the focus term in this case

Conclusions

- Post-adaptive optics image restoration using measured residual error does further improve image quality

- Suspicion that residual phase errors are underestimated somewhat
  - Based on optimal value of Wiener constant *c*
  - Consistent with simulated data results
  - Consistent with theory
    - Smaller *c* causes additional boosting of FT
  - Focus error (most likely culprit) estimated well for one example
  - Other sources of error possible
    - Calibration of Shack-Hartmann WFS
    - Wavefront undersampling by WFS
    - Difference in OPD for two wavelengths
    - Non-common-path errors
    - etc.