AO PSF estimation from WFS data on PUEO (CFHT)

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- **Implementation**: PUEO @ CFHT
- **Limitations of the method**:
  - Magnitude of guide source ≤ 13.5
  - Kolmogorov model for the high order modes
  - Curvature based AO systems only
- **Fundamental limitations**:
  - PSF in the direction of the guide star (anisoplanatism)
  - Long exposure PSF only (speckle noise)
Imaging: long exposure PSF

- Long exposure (= infinite integration time) through the atmosphere:

\[ < PSF(\hat{\alpha}) > \propto \left| FT[\Psi(\hat{\alpha})P(\hat{\alpha})] \right|^2 \]

- We recall the near-field approximation:

\[ \Psi(x) = \exp[i\varphi(x)] \]

- We use the autocorrelation formula to compute the long exposure OTF:

\[ < OTF(f) >= < OTF(\hat{n}/\lambda) >= \frac{1}{S} \int \int \exp[i\varphi(x) - i\varphi(x + \hat{n})]P(x)P(x + \hat{n})dx \]

- Gaussian hypothesis:

\[ < OTF(f) >= < OTF(\hat{n}/\lambda) >= \frac{1}{S} \int \int D_{\varphi}(x,\hat{n})P(x)P(x + \hat{n})dx \]

- Kolmogorov model:

\[ D_{\varphi}(\rho) = \left\langle |\varphi(x) - \varphi(x + \rho)|^2 \right\rangle = 6.88(\rho / r_0)^{5/3} \]

- So we have:

\[ < OTF(f) >= < OTF_{atm}(f) > OTF_{tel}(f) \]

where:

\[ < OTF_{atm}(f) >= \exp\left[ -\frac{1}{2} D_{\varphi}(\lambda f) \right] = \exp\left[ -3.44(\rho / r_0)^{5/3} \right] \]
Separation low order / high order modes

- \( \varphi_{\varepsilon}(r,t) \) AO corrected wave-front phase

- We have: \( <\text{OTF}_{\text{cor}}(f)> = \exp \left[ -\frac{1}{2} D_{\varphi_{\varepsilon}}(\lambda f) \right] \neq \exp \left[ -3.44(\bar{e} f / r_0)^{5/3} \right] \)

- So we need to estimate: \( D_{\varphi_{\varepsilon}}(\lambda f) \)

- We have \( \varphi_{\varepsilon} = \varphi_{\varepsilon\parallel} + \varphi_{\perp} \)
  - \( \varphi_{\varepsilon\parallel} \) sensed and corrected
  - \( \varphi_{\perp} \) unsensed and uncorrected

- We can show that:
  \[
  D_{\varphi_{\varepsilon}} = D_{\varphi_{\varepsilon\parallel}} + D_{\varphi_{\perp}}
  \]
Computation of $D_{\varphi_{\perp}}$

- $D_{\varphi_{\perp}}$ follows the Kolmogorov model and depends only upon $r_0$

$$D_{\varphi_{\perp}}(\mathbf{n}) = \iint [1 - \cos(2\pi\mathbf{n}\mathbf{k})] \text{PSD}_{\varphi_{\perp}}(\mathbf{k}) d\mathbf{k}$$

where:

$$\text{PSD}_{\varphi_{\perp}}(\mathbf{k}) = \text{PSD}_{\varphi_{\text{kolmogorov}}}(\mathbf{k}) \quad \text{if} \quad k > k_M$$

$$\text{PSD}_{\varphi_{\perp}}(\mathbf{k}) = 0 \quad \text{if} \quad k \leq k_M$$
Computation of $D_{\varphi_{\epsilon\parallel}}$

- Modal decomposition:
  $$\varphi_{\parallel}(r, t) = \sum_{i=1}^{N} \epsilon_i(t) M_i(r)$$
  $$D_{\varphi_{\parallel}}(\tilde{n}) = \sum_{i=1}^{N} \sum_{i=1}^{N} \langle \epsilon_i \epsilon_j \rangle U_{ij}(\tilde{n})$$

  $$U_{ij}(\tilde{n}) = \frac{\iint P(x)P(x + \tilde{n})[M_i(x) - M_i(x + \tilde{n})]M_j(x) - M_j(x + \tilde{n})]dx}{\iint P(x)P(x + \tilde{n})dx}$$

- $U_{ij}(\tilde{n})$ can be pre-computed once for all

- We then just need to compute $\langle \epsilon_i \epsilon_j \rangle$ for each acquisition we want to get the PSF for.
Computation of $\langle \varepsilon_i \varepsilon_j \rangle$

- If we had a perfect WFS looking at the residual phase, it would measure $\varepsilon_i(t)$ and thus computing $\langle \varepsilon_i \varepsilon_j \rangle$ would be straightforward.

- We only have: $\hat{\varepsilon}_i(t) = \varepsilon_i(t) + n_i(t) + r_i(t)$
  
  Actual WFS measurement  | Ideal WFS measurement  | Noise on WFS measurement  | Spatial aliasing

- If the correction bandwidth is high (fast system, bright guide source):
  
  $\langle \varepsilon_i \varepsilon_j \rangle = \langle \hat{\varepsilon}_i \hat{\varepsilon}_j \rangle - \langle n_i n_j \rangle + \langle r_i r_j \rangle$
Computation of the spatial aliasing contribution $\langle r_i r_j \rangle$

- Spatial aliasing is due to finite sampling of the WFS
- Spatial aliasing depends on the type of WFS
- Spatial aliasing depends only on $r_0$ and can be computed by modeling
- For a curvature WFS, $\sigma_{\text{alias}}^2 \approx 1.1 \sigma_{\text{fitting}}^2$
- For a SH WFS, $\sigma_{\text{alias}}^2 \approx 0.3 \sigma_{\text{fitting}}^2$
Computation of the WFS noise contribution \( \langle \hat{e}_i \hat{e}_j \rangle - \langle n_in_j \rangle \)

- WFS noise depends on type of WFS (photon noise, read-out noise, background noise, flat-field, dark current)
- For any WFS operating in closed loop, we have \( \sigma_{\hat{e}_i}^2 \approx \sigma_{n_i}^2 \)
- Better to compute \( \langle \hat{e}_i \hat{e}_j \rangle \) where \( \hat{e}_i = \hat{e}_i - n_i \) (noiseless measurement)

- For curvature sensing, only photon noise (APDs) so:

\[
\begin{align*}
w &= \frac{n_1 - n_2}{n_1 + n_2} \\
W &= \frac{N_1 - N_2}{N_1 + N_2} \\
n_1 &= \text{Poisson}(N_1) \\
n_2 &= \text{Poisson}(N_2) \\
\sigma_w^2 &= \frac{\sigma^2(n_1 - n_2)}{\langle n_1 + n_2 \rangle}
\end{align*}
\]

- \( n_1 + n_2 \) must be large enough
Computation of $r_0$

- Use external seeing monitor
  or:
  - Derive it from the DM command variances (include spatial aliasing and WFS noise).
  - Watch for outer scale effect on lowest modes
\begin{align*}
\text{\& \&} & \quad \text{Real (measured) PSF} \\
\text{\& \&} & \quad \text{Estimated PSF} \\
\text{\& \&} & \quad |\text{Real PSF} - \text{Estimated PSF}| \\
\text{\& \&} & \quad \text{G.S.: } m_R = 10.4 \\
\text{\& \&} & \quad \text{SR}_{\text{r}} = 62.2\% \quad \text{SR}_{\text{e}} = 63.6\% \\
\text{\& \&} & \quad r_0 = 15.4\text{cm (}@0.5\mu\text{m}\text{) H2 band} \\
\text{\& \&} & \quad |\text{Real (measured) OTF} - |\text{Estimated OTF}| \\
\text{\& \&} & \quad |\text{Real OTF} - |\text{Estimated OTF}| \\
\end{align*}
\[ r_0 = 18.5 \text{cm} \]

**H band**

G.S.: \( m_R = 15.6 \)

- SRr = 16.3\%, SRe = 21.6\%

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\[ \text{Real (measured) PSF} \]

\[ \text{Estimated PSF} \]

\[ |\text{Real PSF} - \text{Estimated PSF}| \]

\[ \text{Real (measured) OTF} \]

\[ |\text{Estimated OTF}| \]

\[ |\text{Real OTF} - |\text{Estimated OTF}| | \]
Real-time statistics gathering at CFHT

- Statistical data gathered by the AO control computer (SPARC)
- One sample every 30 ms at 1 kHz
- Accumulation starts when exposure starts
- Accumulation stops when exposure stops
- Statistics saved in .psf file
- .psf file analyzed on the fly and structure function is computed (and r0 displayed)
- dphXXXXXX.fits saved with XXXXXXXo.fits

```c
#define PSF Vector of the mean correction modes:
  double PSF_mean_mode_vector = 21 1
#define PSF Matrix of the covariances of the correction modes:
  double PSF_covariance_mode_matrix = 21 21
#define PSF Vector of the mean actuator controls:
  double PSF_mean_act_ctrl_vector = 21 1
#define PSF Matrix of the covariances of the actuator controls:
  double PSF_covariance_act_ctrl_matrix = 21 21
#define PSF Vector of the mean APD sum counters:
  double PSF_mean_sum_vector = 19 1
#define PSF Vector of the variances of the APD sum counters:
  double PSF_variance_sum_vector = 19 1
#define PSF Vector of the mean APD difference counters:
  double PSF_mean_diff_vector = 19 1
#define PSF Vector of the variances of the APD difference counters:
  double PSF_variance_diff_vector = 19 1
#define PSF Number of count samples during exposure:
  double PSF_count = 1 1
#define PSF Vector of the means of the mode gains:
  double PSF_mean_mode_gain_vector = 21 1
#define PSF Vector of the variances of the mode gains:
  double PSF_variance_mode_gain_vector = 21 1
#define PSF mean optical gain during exposure:
  double PSF_mean_optical_gain = 1 1
#define PSF variance optical gain during exposure:
  double PSF_variance_optical_gain = 1 1
```
**Procedure @ CFHT**

**Scientific observations**
- **Scientific camera**
  - 388083o.fits
  - dph388083o.fits
- **AO RTC**

**Calibrations observations**
- **AO RTC**
  - dph388000o.fits
  - 388000o.fits
- **Scientific camera**

**Off-line**
- **Standard reduction**

**On-line**

**DPH2PSF software**
http://www.cfht.hawaii.edu/IDL, C

- psf388083o.fits
  - ...

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WFCS process allocation & major data paths

- Statistics & Display
- Optimer
- RP Buffer
- Buffers
- RTC & Aux Processes
- WFS Control
- RP Generator
- WFS Control
- SBC 1
- SBC 2
- CPU X
- CPU Y
- Ethernet
- Private bus
- VME Bus
- T/T
- DM
- Altair
- DHS