Local Wavefront Reconstruction: Simulations, Experiments and Predictions

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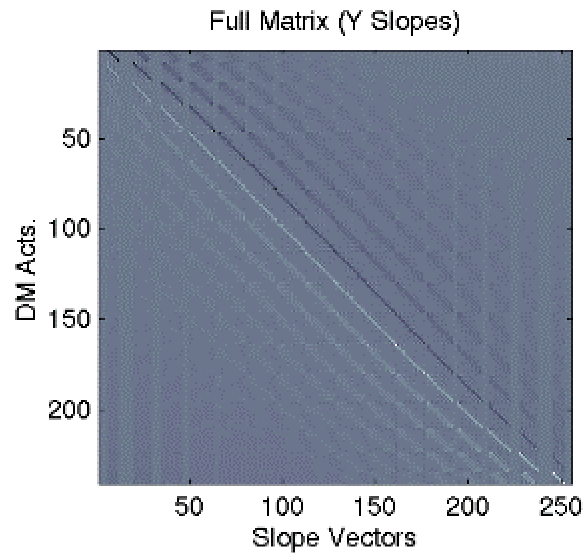
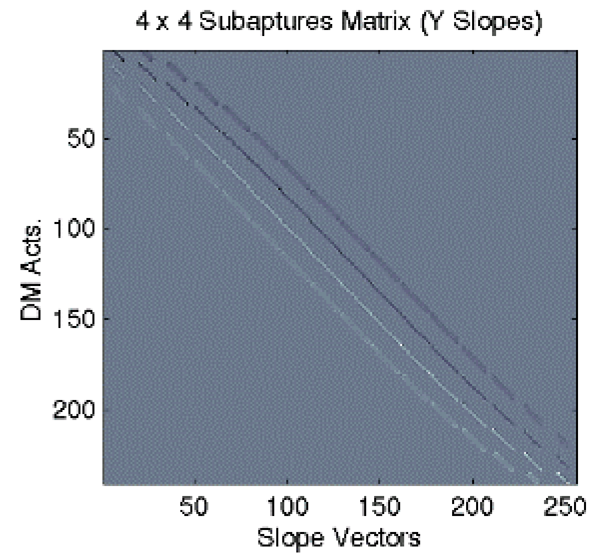
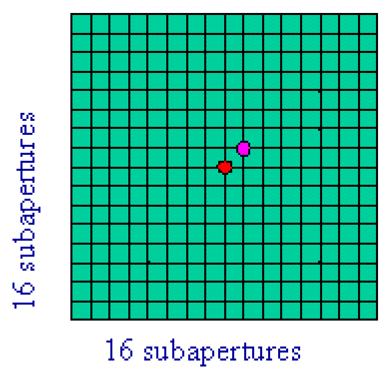
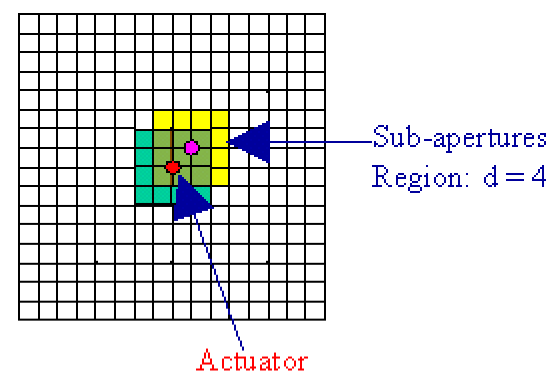
Sparse Matrices: Rationale and Methodology









- The Matrix is <u>fully populated</u>
- Reconstructor calculations for a sqrt($\boldsymbol{n \times n}$) SH sensors ~ $\boldsymbol{2n^2}$

- The Matrix has a <u>banded structure</u>
- Reconstructor calculations for a sqrt($\boldsymbol{n \times n}$) SH sensors ~ $\boldsymbol{2kn}$, ($\boldsymbol{k = d \times d}$)
- Need a smart indexing scheme

Sparse Matrices: Rationale and Methodology

***Computation Challenge for WF Reconstructor*:**

- For wavefront sensor with sqrt(*n)* actuator/sensor across the diameter of telescope the full reconstruction matrix A^+ has a size **~** n
- WFS reconstruction calculation => ~ $2n^2$ multiplications and additions
- n is proportional to telescope diameter D^2: big challenge for telescopes such as CELT

***Sparse WF Reconstruction Matrix*:**

- A^+ has a diagonal band structure because each DM actuator is mostly influenced by the WFS slopes measured by the *nearest surrounding subapertures.* This is the base for our sparsification.
- Sparsification Method:
  - **Truncation:** *(a working approach)* Truncated a full reconstruction matrix corresponding to the set of sensors s that are within influence region size d of each actuator
  - **Localized least square (LLS):** *(a better approach)* Reconstruct the phase for each actuator using the nearest subapertures (within influence region size d) using the least square method (localized pseudo-inverse)
  - **Hierarchic controller:** *(the best approach)* Add extra layer(s) reconstruction on top of LLS to sense the global modes in a reduced (averaged over several subapertures) sensor data set
- The "sparsed" matrix will have **~**$2kn$ non-zero components, with $k = d$ _ $d << n$ and k is a constant depends on the actuator influence region size not telescope size D
- The reconstruction calculations scale as **~** n instead of n^2
- Hierarchic controller adds more calculations but scales only as **~** $n^{4/3}$

Sparse Matrices: Rationale and Methodology

***Sparse WF Reconstruction Matrix*:**

- The sparsed matrix (either truncation or LLS) behaves like a low pass filter which will always lose gains for global modes with spatial frequency $\nu < d/D$
- The sparse matrices may cause a higher gain for certain characteristic spatial frequency around $\nu = d/D$
- Hierarchic controller recovers most gains of the global modes without much increase of computation burden because the global modes are recovered on a much reduced data set in order of ***D/d***

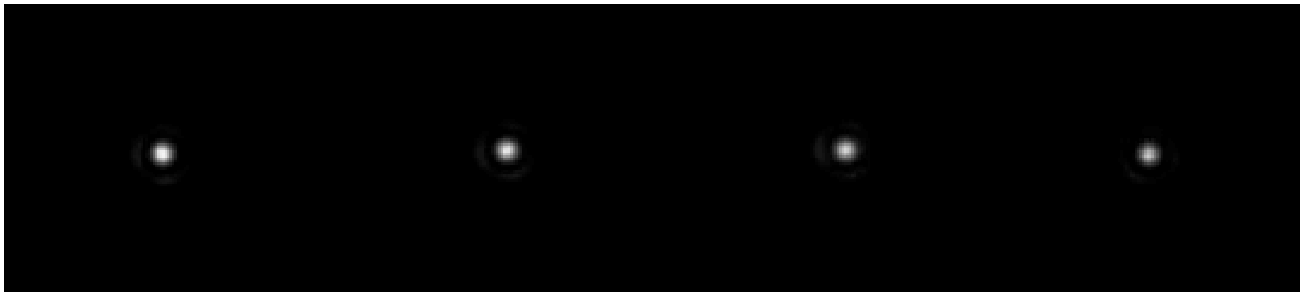
Experiment and Simulation on PALAO

- Experiment have been done for all three types of matrices on PALAO
  - Truncated sparse matrices with influence region size from $d = 2$ to $d = 12$
  - Localized least square (LLS) sparse matrices with influence region size from $d = 2$ to $d = 12$
  - Hierarchic controller based on a $d = 4$ LLS sparse matrix and a ***4 _ 4*** global reconstructor
- Matrices maintain the full matrix format in the PALAO reconstructor => **did not** take the advantage of the computation saving but make the performance comparison easy
- Sparse matrix closed-loop wavefront sensor data is recorded at 100 Hz together with PHARO K-band images

- Simulations have been done for all three types of matrices using PALAO open loop data

Full and Sparse Matrices Closed-loop K-band Images

Full | d = 4 | d = 2 | d = 4 (hier.)



SR = 0.60
FWHM = 94 mas

SR = 0.55
FWHM = 94 mas

SR = 0.49
FWHM = 97 mas

SR = 0.53
FWHM = 90 mas

- Bright star SAO64701: M_V=5.9 mag K0
- Reconstruction matrices used are indicated on top of each closed-loop images
- Estimated Strehls and FWHM sizes are indicated at the bottom of each images

Wavefront Error of Sparse Matrices

Experiment:

- Total wavefront error consists of <u>system error</u> and <u>sparse matrix error</u>

$$\sigma_{total}^2 = \sigma_{system}^2 + \sigma_{sparse}^2$$

- AO system error is calculated from the full matrix PHARO image Strehls

$$S \approx \exp\left[-\left(\frac{2\pi}{\lambda_K}\sigma_{system}\right)^2\right]$$

- Wavefront error due the sparse matrices are:

$$\sigma_{sparse}^2 = 4 \cdot \sum_j \frac{\left(a_j^{Full} - a_j^{Sparse}\right)^2}{n_{act}}$$

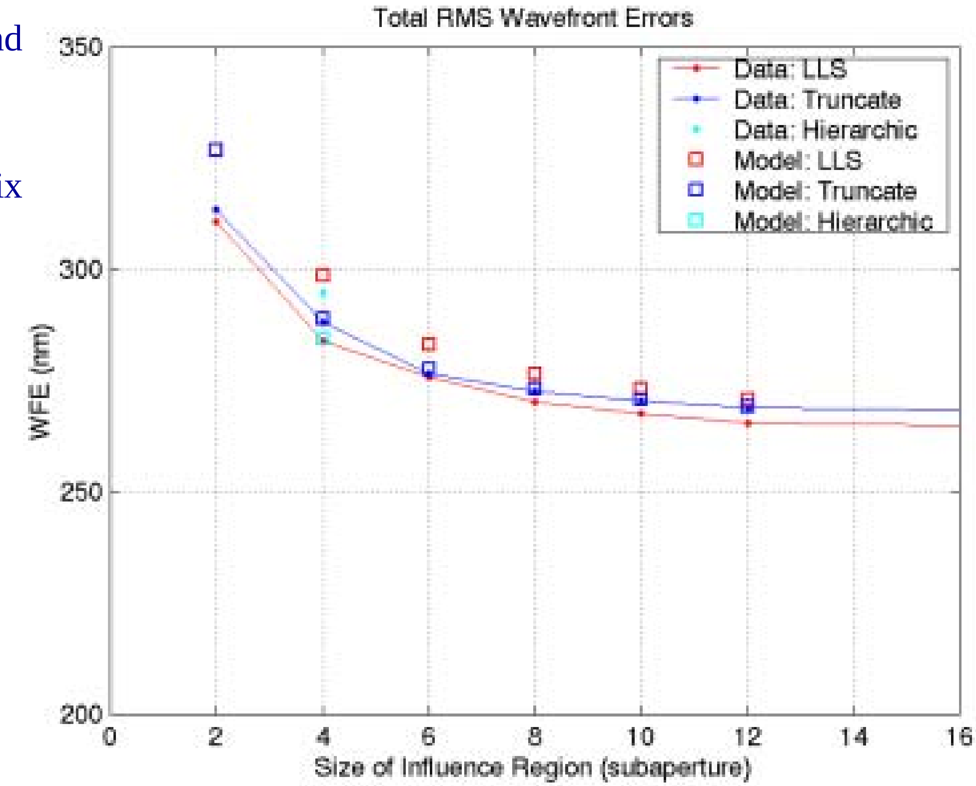
where a_j is the actuator command reconstructed from the recorded WFS data:

$$\vec{a}^{Full} = A^{Full} \cdot \vec{s}^{Sparse}$$

$$\vec{a}^{Sparse} = A^{Sparse} \cdot \vec{s}^{Sparse}$$

Model Simulation:

- Open loop data is used for model simulations
- Model simulates the closed-loop performance with the same sparse matrices
- Model uses the estimated AO system error of the full matrix to calibrate the model
- ***100 Hz*** recoded data lack of the excitation of higher frequency modes



Matrix Sparseness

DM Actuator Residuals PSD: LLS Matrices

- *Upper panels*: The radially averaged PSD of the recorded closed-loop DM actuator residuals for different sparse matrices

- *Lower panels:* Same PSDs normalized by the PSD of full matrix case