Tomographic reconstruction for multi-guidestar adaptive optics

Don Gavel, Jose Milovich
LLNL

Fundamental questions for the architecture of AO for extremely large telescopes

- Number of DMs, and their conjugate locations

  Tokovinin & LeLouarn, "Isoplanatism in MCAO," JOSA A, Oct 2000

- Number of actuators per DM

  Pareto-optimal solution (Gavel, DeKany, Baumann, Nelson)

- Number and placement of *laser* guide stars

  -This talk-

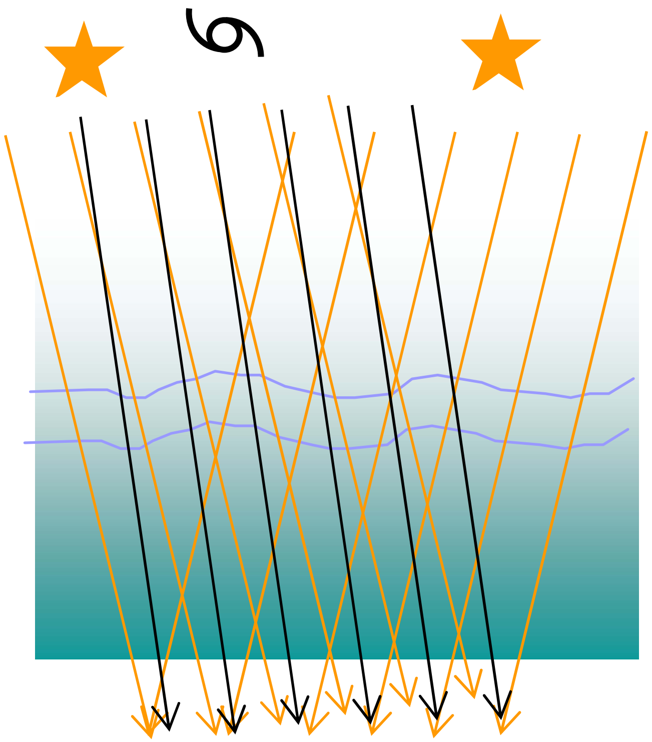
- Brightness of guide stars

  Tokovinin & Viard, "Limiting precision…," JOSA A, Apr 2001

- Controller bandwidth

Tokovinin's minimum variance wavefront reconstruction algorithm

Tokovinin & Viard, JOSA-A, 18, 4, 2001



1. **Measure wavefronts from guidestars at field angles θ_i, i=1…#gs**

$$\phi_i(x) = \int_0^\infty \Delta n(x - \theta_i z, z)\,dz$$

2. **Estimate wavefront as it would appear coming from direction θ**
   using a minimum variance estimator in the spatial frequency domain

$$\Phi(k_x) = \sum_{\#gs} g(k_x, \theta_i)\Phi_i(k_x)$$

3. **Apply wavefront correction for science object at field angle θ**

Error analysis of Tokovinin's algorithm

Wavefront error spatial frequency k_x, science direction θ = Filtered integral of index variation, Δn, along path + Noise on each WFS, filtered by the estimator, g_k

$$\varepsilon(k_x,\theta) = \int P(k_x,z,\theta)\Delta n(k_x,z)dz - \sum_{l=0}^{\#gs} g_l(k_x,\theta)\nu_l(k_x)$$

$$P(k_x,\theta,z) = \left[e^{-ik_x\theta z} - \sum_{l=1}^{\#gs} g_l(k_x,\theta) M(k_x) e^{-ik_x\theta_l z} \right]$$

Wavefront sensor operator, *M*

- **Choose feedback gains $g_l(k_x,\theta)$ to minimize the variance of wavefront error, for given $C_n^2(z) = \langle \Delta n^2(z) \rangle$ and $\sigma_n^2 = \langle v_l^2 \rangle$**

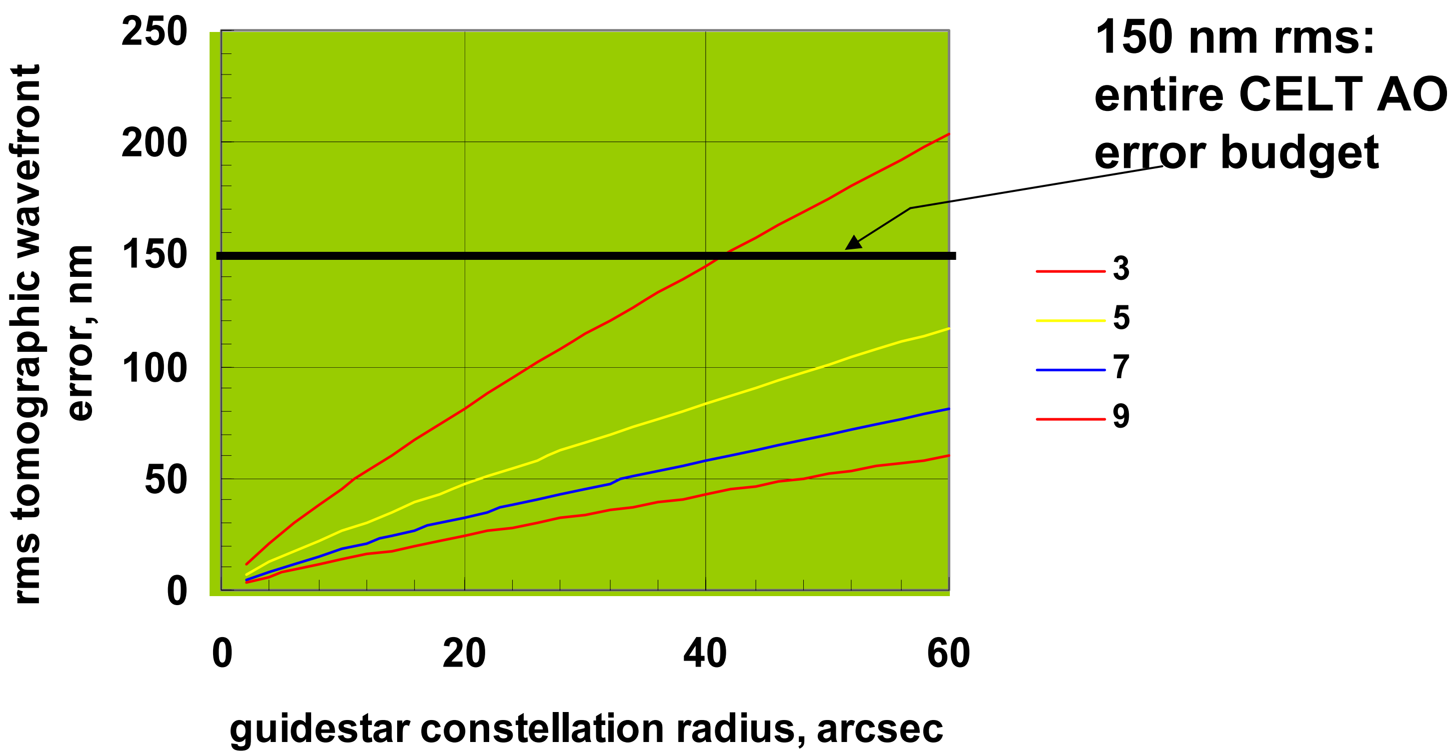
The algorithm $g_l(k_x, \theta)$ assumes

- infinite aperture
- plane waves

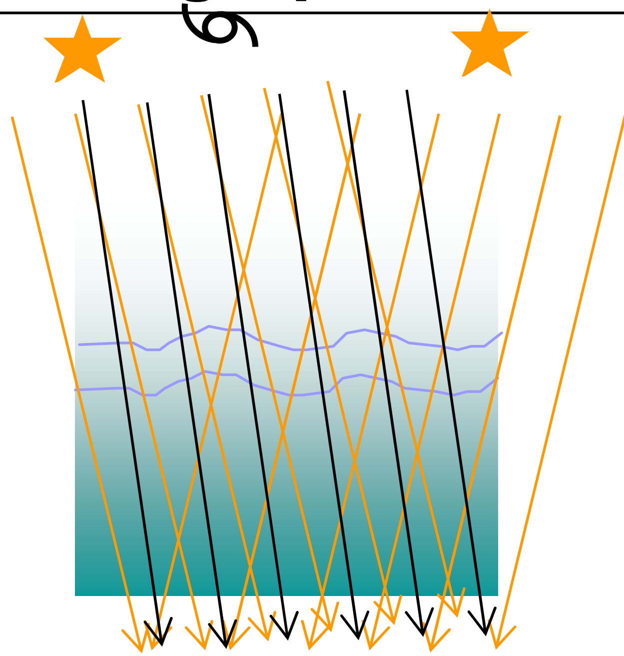
Tomographic reconstruction error
(assumes infinite aperture and plane waves)

$$\sigma^2 \cong \left(\frac{\Theta\delta}{r_0}\right)^{5/3} e(\theta)$$

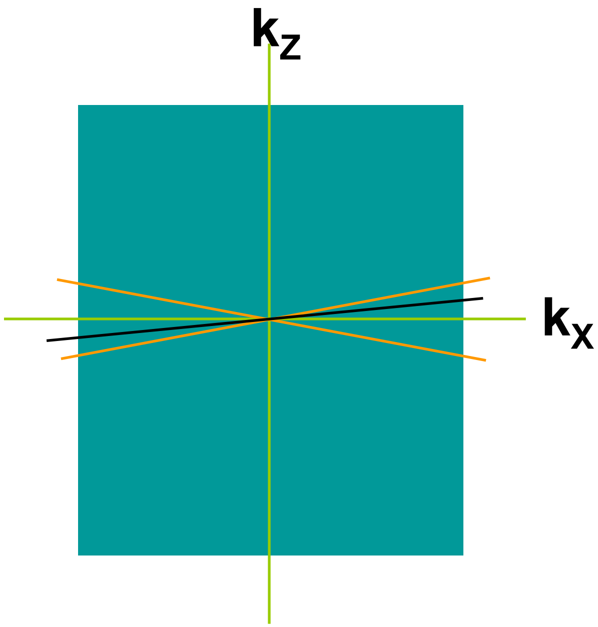
Θ = constellation radius
r_0 = transverse coherence distance (Fried's parameter)
δ = effective layer thickness
***e* = field-dependent factor (≤1 inside constellation)**



Fourier transform interpretation of tomographic wavefront reconstruction



$$\phi(x) = \int_0^\infty \Delta n(x - \theta z, z)\, dz$$



$$\Phi(k_x) = \Delta N(k_x, -k_x \theta)$$

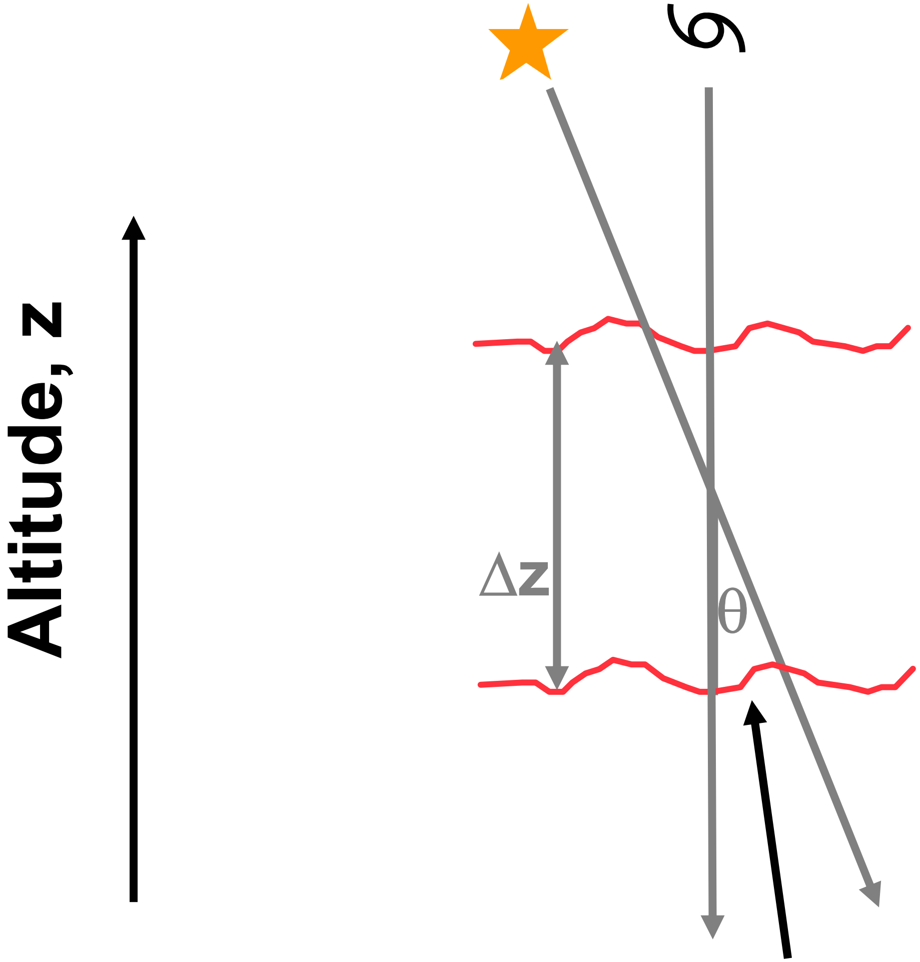
Fourier slice **theorem in tomography (Kak, 1988)**

- **Each wavefront sensor measures the integral of index variation along the ray lines**
- **The line integral along z determines the k_z=0 Fourier spatial frequency component**
- **Projections at several angles sample the k_x,k_y,k_z volume**

Consequences of finite number and field of guide stars

- **Wider spacing of guide stars gives less dense sampling in k-space. $\delta z \delta k_z$~constant so thin δz layers result in wide δk_z lines. Wider δk_z lines (thin layer of turbulence) means fewer samples (guide stars) are required.**
- **Unfortunately, this does not extend to multiple thin layers. E.g. for two thin δz layers spaced apart by Δz, the k-space line is only $1/\Delta z$ wide, not $1/\delta z$. So it doesn't change the situation much from turbulence over the entire Δz region.**
- **Because of the angle of the sample lines in k-space, k_z sampling interval increases with k_x, so higher k_x spatial frequencies are less well measured. The cut of is near $k_x<1/[\Delta z(\theta-\theta_{gs})]$**

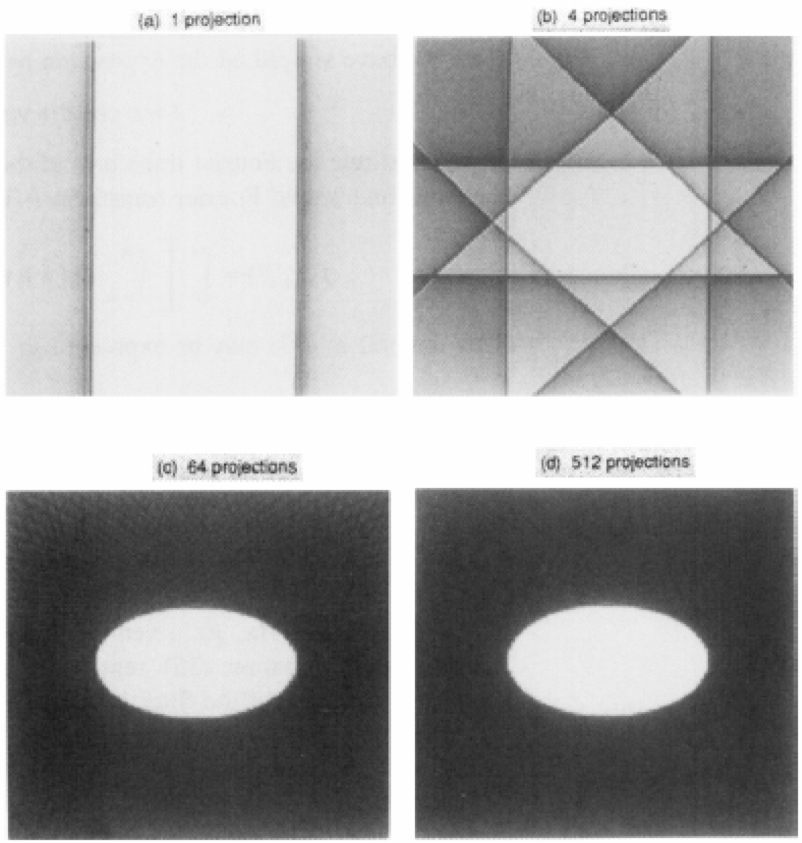
$k_x < 1/[\Delta z(\theta - \theta_{gs})]$ requirement spatial interpretation



$\theta\, \Delta z < r_0$

Back-projection algorithm

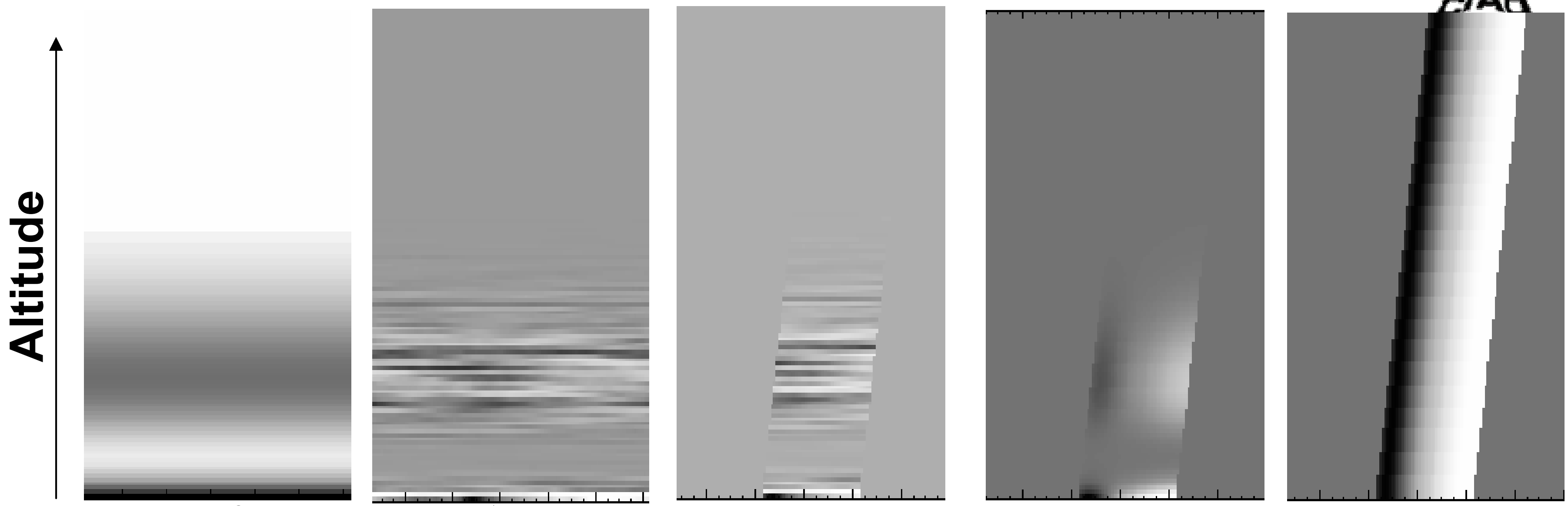
Kak, 1988



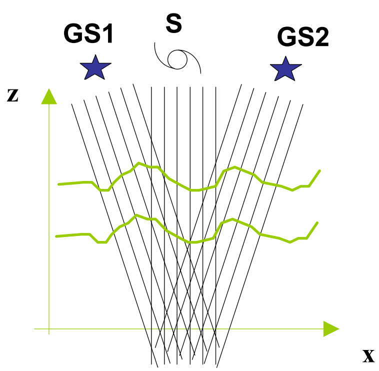
Example tomographic reconstruction of an ellipsoidal target using back projections

- **Concepts are borrowed from tomography**
- **The Fourier-slice reconstruction algorithm is reformulated in terms of projection, filtered back-projection steps (mathematically equivalent to Fourier-slice in the infinite aperture plane wave case).**
- ***Advantages* are: finite aperture, “fan rays”, finite # of probe directions are brought out explicitly.**
- **Back propagation is mathematically equivalent to Tokovinin’s algorithm if we take into account $C_n^2(z)$ profile when back-propagating through the atmosphere. (Proven for ngs=1 case, “statistical addition” must be used for ngs>1)**

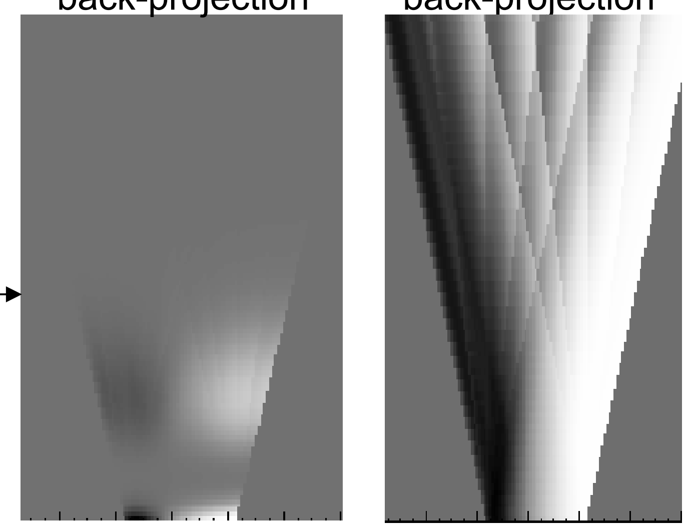
Example NGS finite aperture reconstruction using tomographic back-projection*



C_n^2 (Turbulence Strength) | Δn (Index variations) | Projection | C_n^2 weighted back-projection | Uniformly weighted back-projection



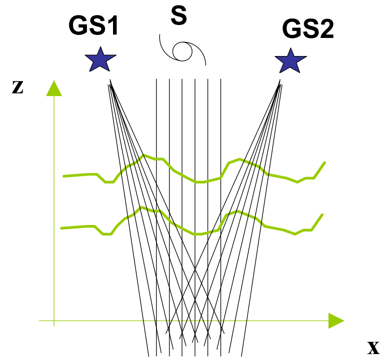
5-guidestar reconstructions →

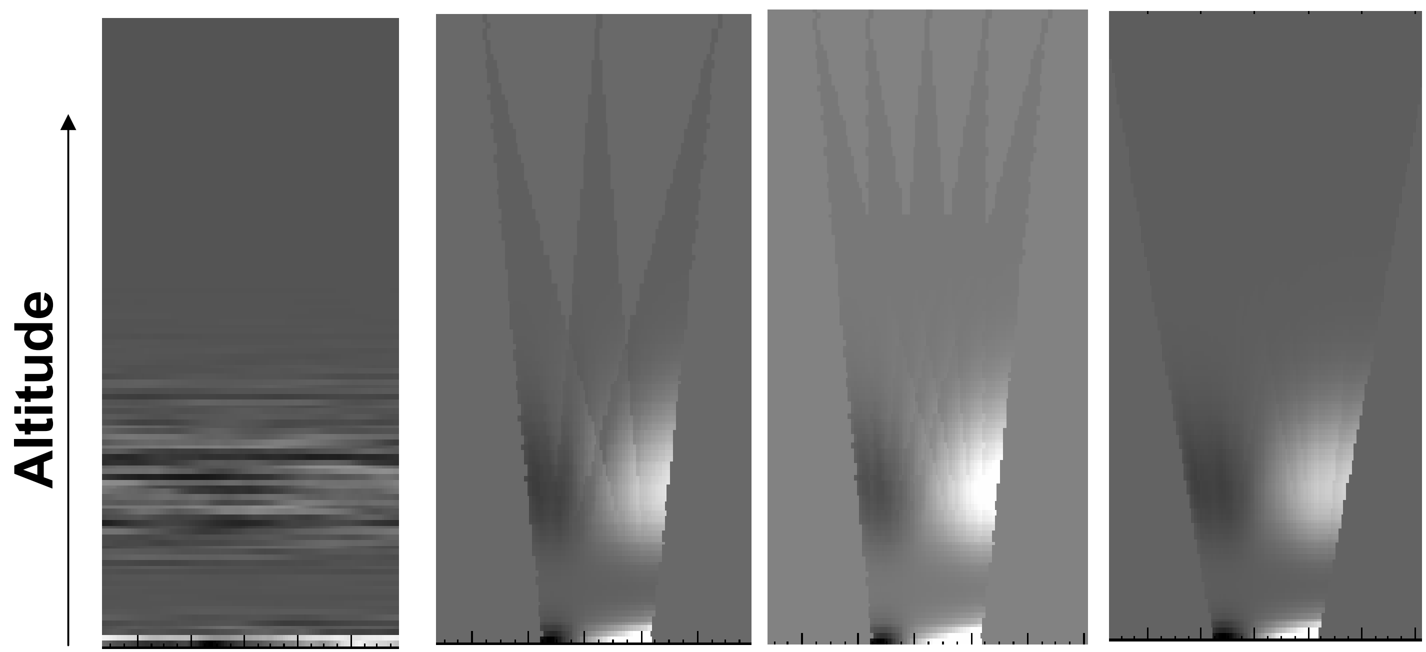


*numerical implementation of Fourier-slice

note: Tokovinin & Viard's algorithm is equivalent to a C_n^2 weighted back-projection

Cone (laser guide star) back-projections





Δn

(Index variations)

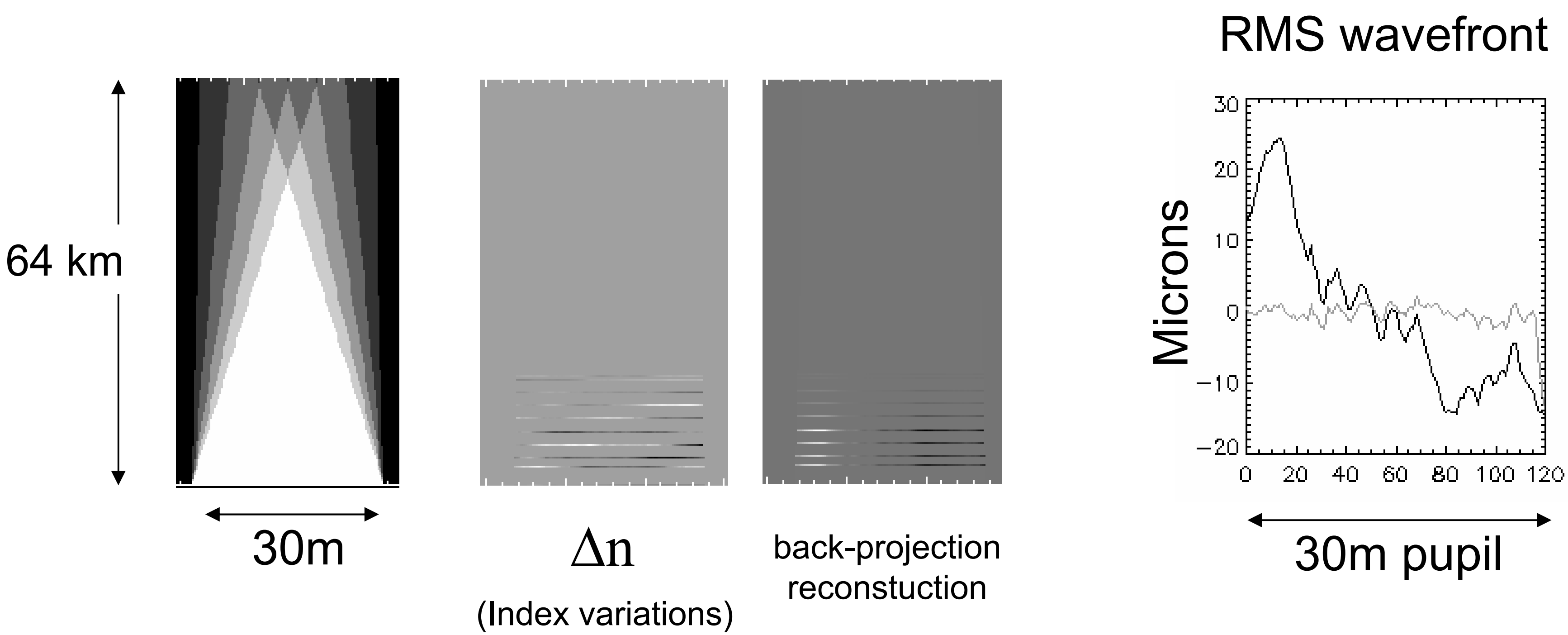
3 laser guidestar reconstuction

5 laser guidestar reconstuction

5 natural star reconstuction

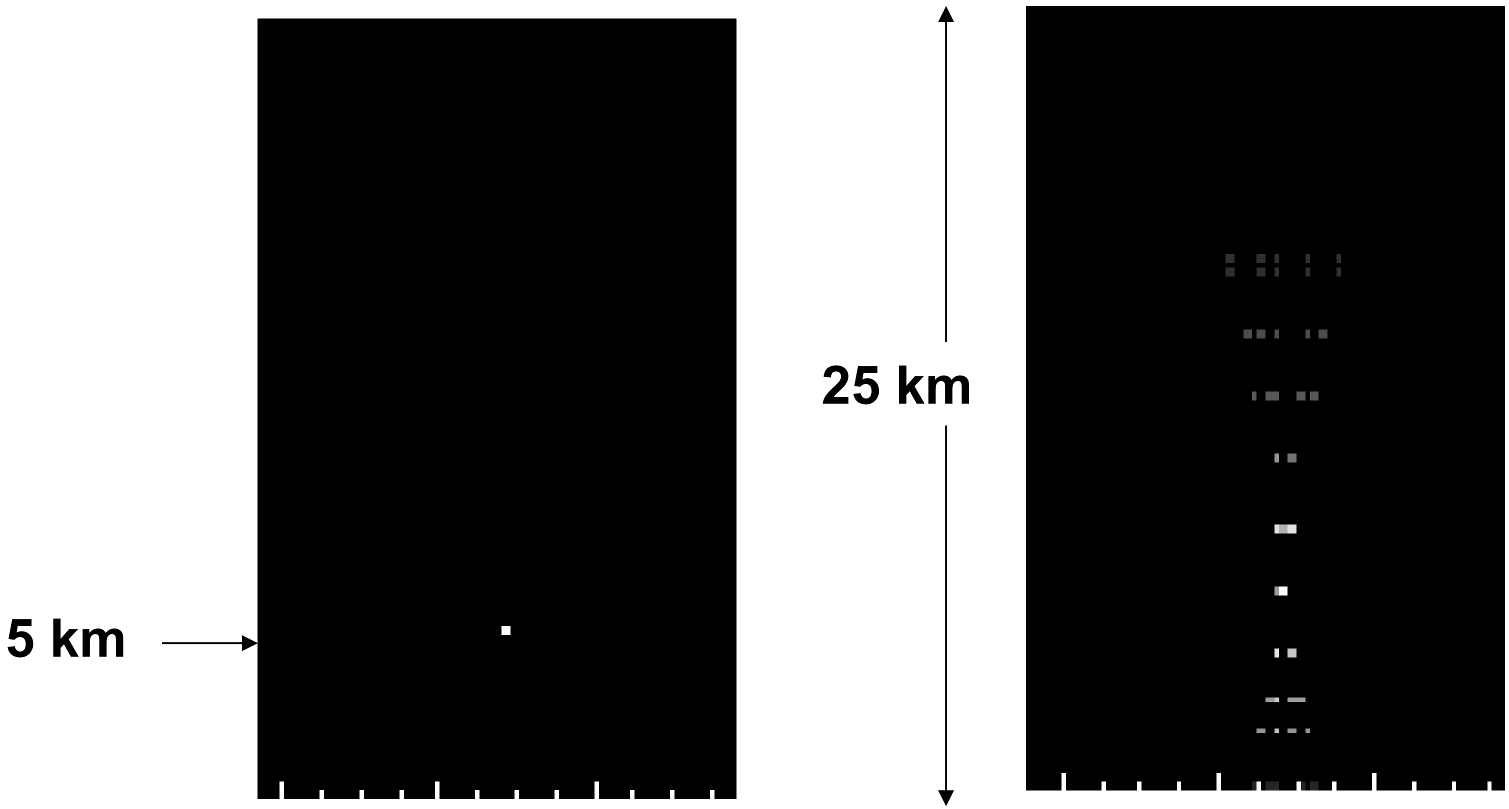
LGS tomography error predictions

30 m telescope, Mauna Kea 12-layer profile



- **Predicts anisoplanatism, cone-effect (validated in single-guidestar simulations)**
- **Provides insight into number and placement of guidestars**
- **Back-projection gives ~100x improvement over uncorrected seeing – problem: need ~1000x**

Simulation with a test-point Δn demonstrates back-projection's inherent error



'Phantom'
Single test point Δn at 5km

Reconstruction
MK Cn2 profile-weighted back-projection

Summary

- Interpretation of MCAO in terms of tomography give insight into system design and performance.
- Includes dependence on number and placement of guidestars.
- Separates the tomography effects from other error sources (e.g. guidestar brightness, DM sampling)
- Predicts “traditional” anisoplanatism and cone effect
- However: back-propagation reconstruction algorithm appears to be 5 to 10 x less accurate than regularized least-squares solutions. Implies back-propagation is not a good real-time reconstructor.