Phase Retrieval: Hubble and the James Webb Space Telescope

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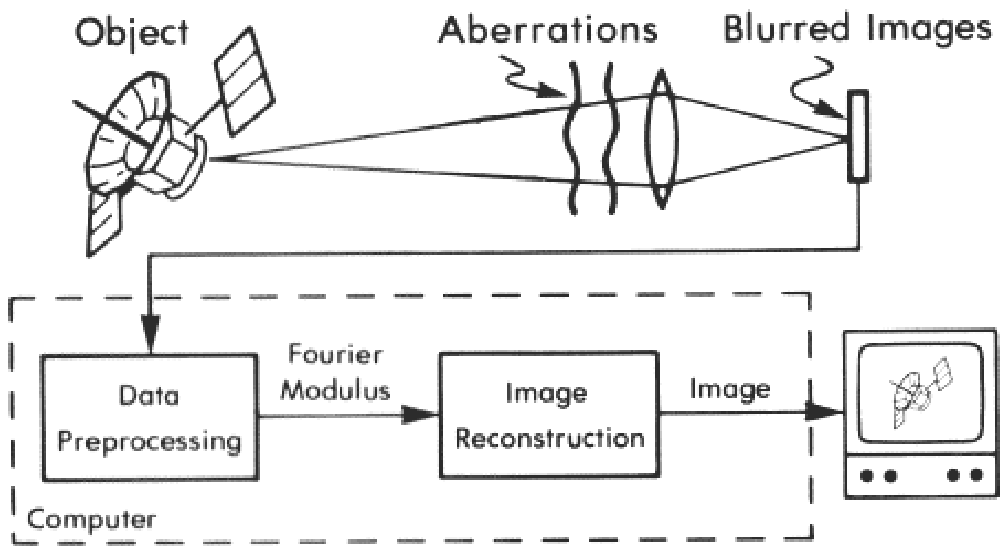
Outline

- Hubble Space Telescope (HST)
  - Phase retrieval algorithms
    - Iterative transform
    - Gradient-search parameter fitting
- James Webb Space Telescope (JWST)
  - Background
  - Phase retrieval for wavefront sensing and control

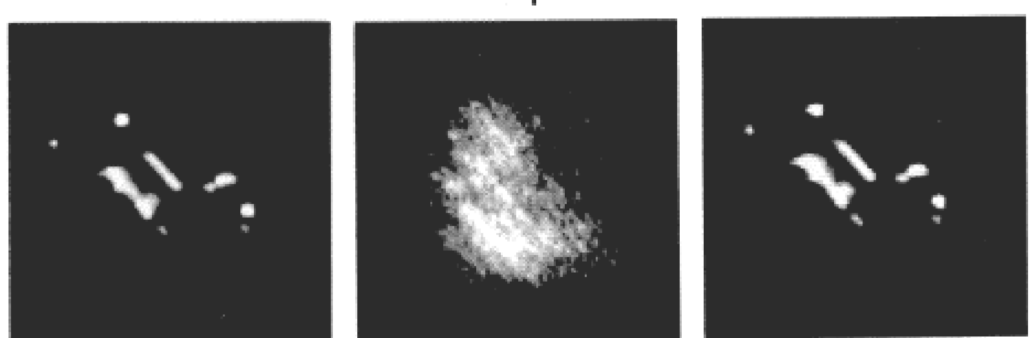
Early work was performed while Jim Fienup was at ERIM, Ann Arbor, MI

Phase Retrieval for Image Reconstruction from Stellar Speckle Interferometry Data

IMAGING WITH PHASE ERRORS



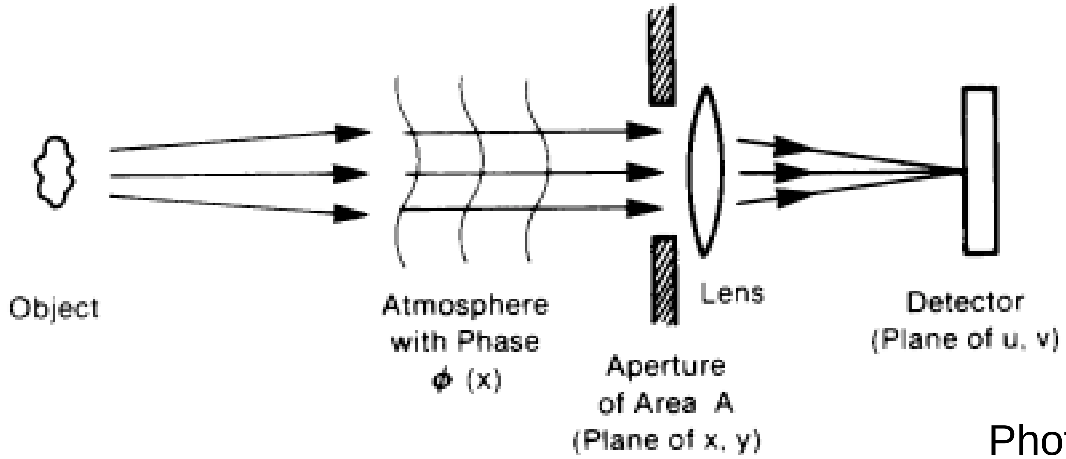
Example:



Object | Blurred Image | Reconstructed Image

Reference: J.R. Fienup, "Phase Retrieval Algorithms: A Comparison," Appl. Opt. 21, 2758-2769 (1982).

Fourier Intensity Wavefront Sensor

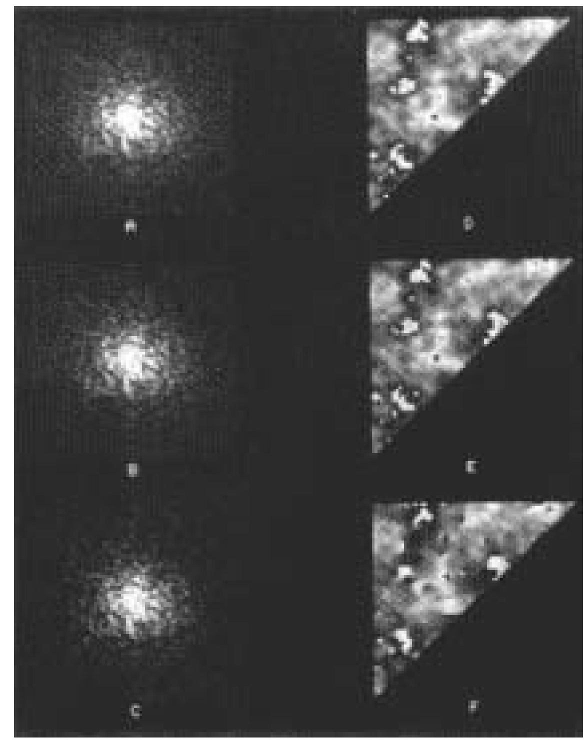


Measure atmospheric phase to drive adaptive optics

No Noise

1.1×10^5 Phot./Coh.Cell

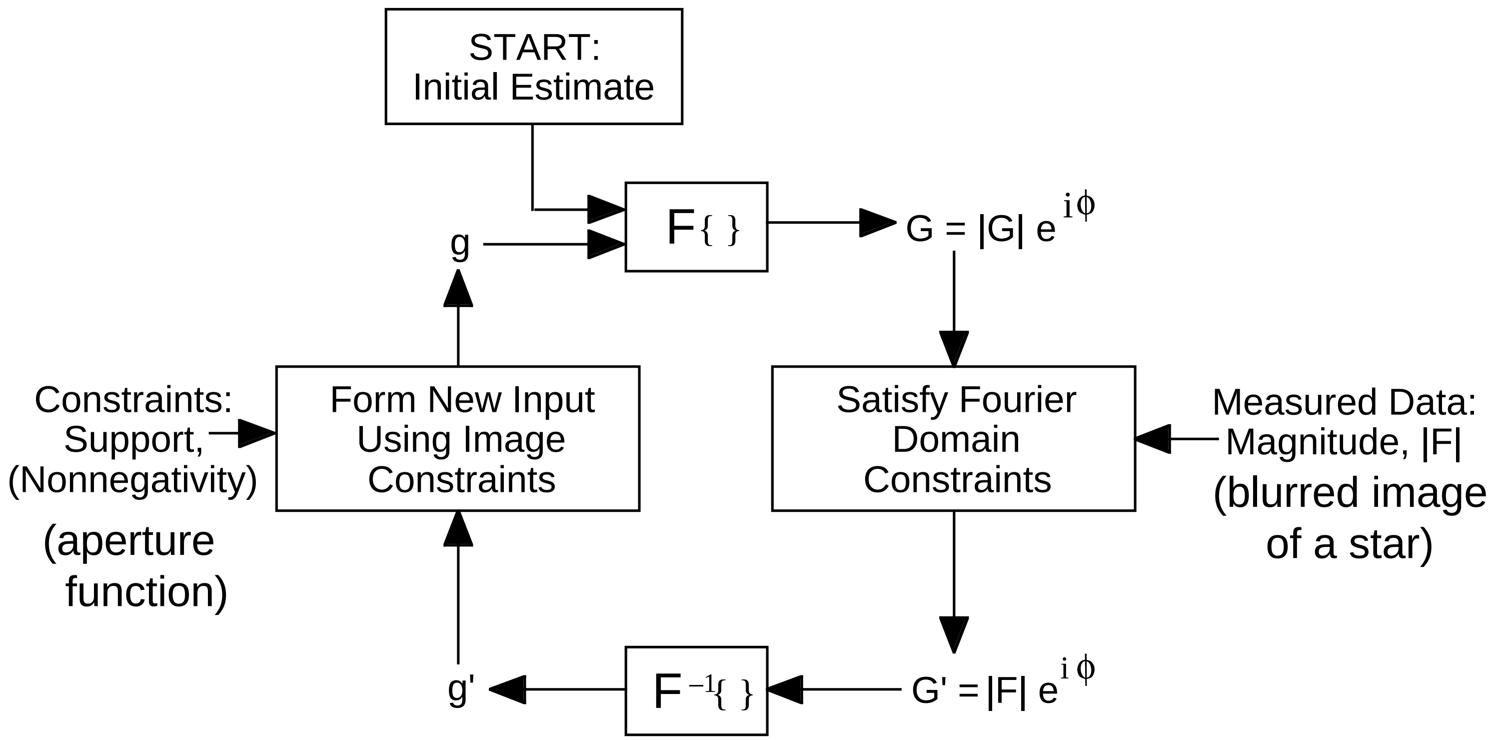
1.1×10^3 Phot./Coh.Cell



Focal Plane Intensity — Reconstructed Phase

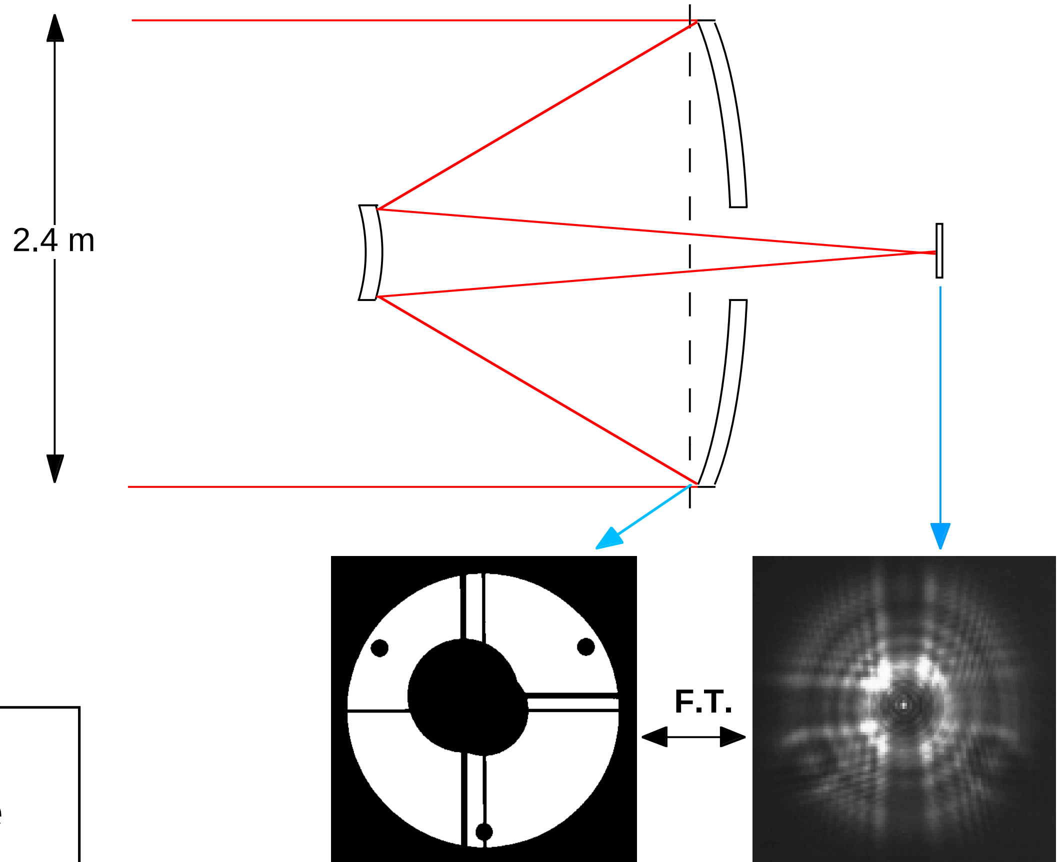
Reference: J.N. Cederquist, J.R. Fienup, C.C. Wackerman, S.R. Robinson and D. Kryskowski, "Wave-Front Phase Estimation from Fourier Intensity Measurements," J. Opt. Soc. Am. A 6, 1020-1026 (1989).

Iterative Transform Algorithm



Enforcing magnitude constraints in both domains
is the “Error Reduction” or “Gerchberg-Saxton” algorithm

Determine HST Aberrations from PSF



(Hubble Space Telescope)

<u>Measurements & Constraints</u>:

Pupil plane: known aperture shape

phase error fairly smooth function

Focal plane: measured PSF intensity

Wavefronts in pupil plane and focal plane are related by a Fourier Transform

Reference: J.R. Fienup, J.C. Marron, T.J. Schulz and J.H. Seldin, "Hubble Space Telescope Characterized by Using Phase Retrieval Algorithms," Appl. Opt. <u>32</u> 1747-1768 (1993).

Benefits of Phase Retrieval

Knowing aberrations precisely allows for:

- Design correction optics to fix the HST
  - WF/PC II
  - COSTAR
- Optimize alignment of secondary mirror of HST OTA
- Monitor telescope shrinkage (desorption) and focus
- Compute analytic point-spread functions for image deconvolution
  - Noise-free
  - Depends on λ, $\Delta\lambda$, camera, field position
  - Is highly space-variant for WF/PC
  - Eliminates requirement to measure numerous PSF's

In addition, reconstruction of pupil function allows determination of alignment between OTA and WF/PC

Phase Retrieval Basics

Focal plane field → $F(u,v)$; Pupil plane field → $f(x,y)$

Fourier transform: $$F(u,v) = \iint_{-\infty}^{\infty} f(x,y) e^{-i2\pi(ux+vy)} dx dy$$
$$= |F(u,v)| e^{i\psi(u,v)} = \mathcal{F}[f(x,y)]$$

Inverse transform: $$f(x,y) = \iint_{-\infty}^{\infty} F(u,v) e^{i2\pi(ux+vy)} du dv = \mathcal{F}^{-1}[F(u,v)]$$

Phase retrieval problem:

Given $|F(u,v)|$ and some constraints on $f(x,y)$,
Reconstruct $f(x,y)$, or equivalently retrieve $\psi(u,v)$

Phase Retrieval Techniques

Minimize error metric by

- Cut & try (Holtzman)
- Iterative transform algorithm (Gerchberg-Saxton/Misell/Fienup)
- Gradient search (steepest descent, conjugate gradient, . . .)
- Damped least squares (Newton-Raphson)
- Neural network
- Linear programming
- Prescription retrieval
- Phase diversity
- etc.

- Other groups doing phase retrieval
  - Rick Lyon *et al.* Hughes Danbury Optical Systems
  - Chris Burrows (Space Telescope Science Institute)
  - Mike Shao (JPL)
  - Francois Roddier (U. Hawaii), . . .

Phase Retrieval by Optimization

- Model optical system
  - Known parameters (constraints)
  - Unknown parameters (to retrieve)
- Compute model of data
- Compare model of data with actual measured data
  - Compute error metric
- Minimize error metric over space of unknown parameters
  - using nonlinear optimization algorithms

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Error Metrics

Focal plane:

$$E = \sum_{u} W(u)[|G(u)| - |F(u)|]^2$$

(Equivalent to a maximum likelihood algorithm for the case of additive Gaussian noise)

$$E = \sum_{u} W(u)[|G(u)|^2 - |F(u)|^2]^2$$

$$E = \sum_{u} W(u)\{|G(u)| - |F(u)| - |G(u)| \ln(|F(u)|/|G(u)|)\}$$

Aperture plane:

$$E = \sum_{x} [|g(x)| - |f(x)|]^2$$

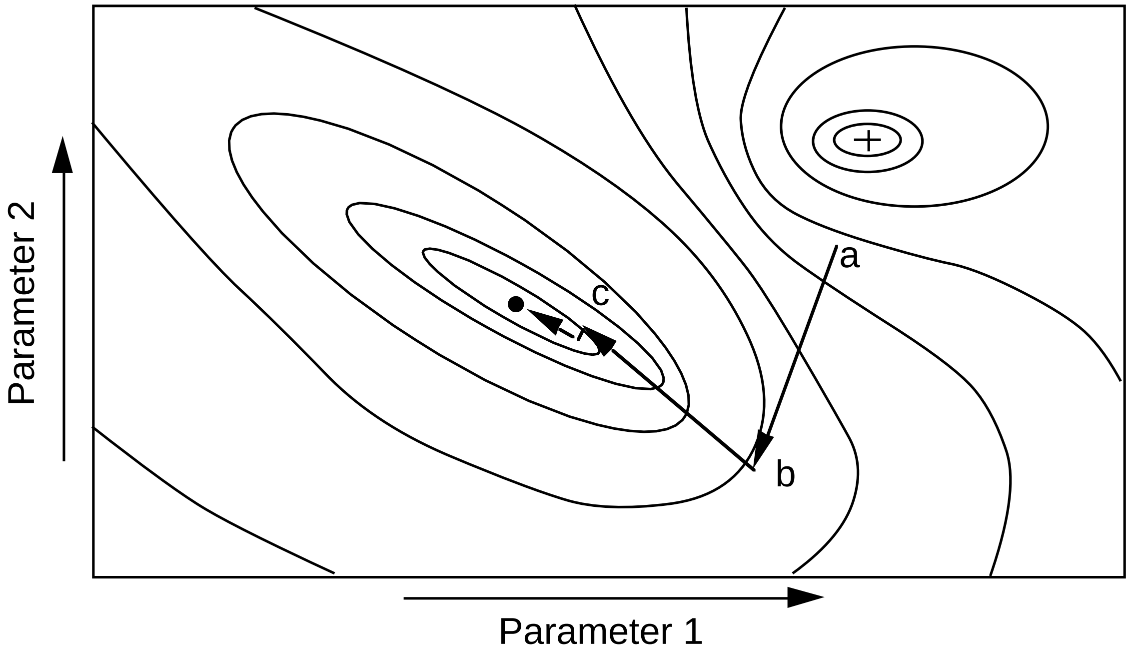
$$E = \sum_{x} \{|g(x)| - |f(x)| - |g(x)| \ln(|f(x)|/|g(x)|)\}$$

$$E = \sum_{x \in S} [|g(x)|]^2$$

Techniques Employing Gradients

Minimize Error Metric, e.g.: $E = \sum_{u} W(u)\left[|G(u)| - |F(u)|\right]^2$

Contour Plot of Error Metric



<u>Repeat three steps</u>:

1. Compute gradient: $\dfrac{\partial E}{\partial p_1}, \dfrac{\partial E}{\partial p_2}, \ldots$

2. Compute direction of search

3. Perform line search

Gradient methods:

(Steepest Descent)
Conjugate Gradient
BFGS/Quasi-Newton
…

Analytic Gradients

$$E = \sum_u W(u)\left[|G(u)| - |F(u)|\right]^2$$

For point-by-point phase map, θ(x), $\dfrac{\partial E}{\partial \theta(x)} = 2\,\mathrm{Im}\left\{ g(x)\, g^{W*}(x) \right\}$

For Zernike polynomial coefficients, $\dfrac{\partial E}{\partial a_j} = 2\,\mathrm{Im}\left\{ \sum_x g(x)\, g^{W*}(x)\, Z_j(x) \right\}$

where

$$g(x) = m_o(x)\, e^{i\theta(x)}, \qquad \theta(x) = \sum_{j=1}^{J} a_j Z_j(x), \qquad G(u) = P[g(x)],$$

$$G^W(u) = W(u)\left[|F(u)| \frac{G(u)}{|G(u)|} - G(u) \right], \text{ and } \quad g^W(x) = P^{\dagger}[G^W(u)] .$$

$P[\bullet]$ can be a single FFT or multiple-plane Fresnel transforms with phase factors and obscurations

Analytic gradients very fast compared with calculation by finite differences

J.R. Fienup, "Phase-Retrieval Algorithms for a Complicated Optical System," Appl. Opt. 32, 1737-1746 (1993).

J.R. Fienup, J.C. Marron, T.J. Schulz and J.H. Seldin, "Hubble Space Telescope Characterized by Using Phase Retrieval Algorithms," Appl. Opt. 32 1747-1768 (1993).

Hubble Telescope Retrieval Approach

- Pupil (support constraint) was known imperfectly
- Phase was relatively smooth and dominated by low-order Zernike's
  — Use boot-strapping approach

Skipping this step gave worse results

1. With initial guess for pupil, fit Zernike polynomial coefficients
   (parametric phase retrieval by gradient search)
2. With initial guess for Zernike polynomials, estimate pupil by ITA
   (retrieve magnitude, given an estimate of phase)

   3. Redo steps 1 and 2 until convergence (2 iterations)

4. Estimate phase map by ITA, starting with Zernike polynomial phase
   (nonparametric phase retrieval by G-S or gradient search)
5. Refit Zernike coefficients to phase map

   6. Redo steps 2 - 5

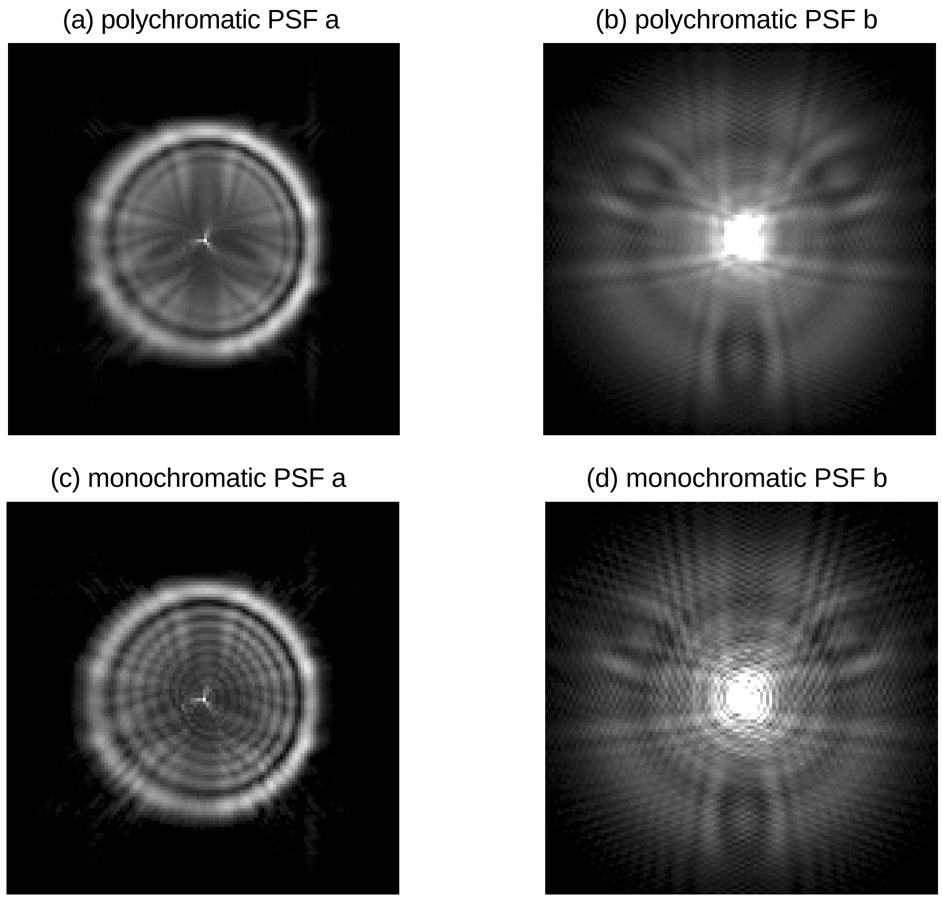
Overcoming Model Limitations

- Restrictions on early algorithms
  - Narrow-band light $\Delta\lambda/\lambda_c \ll 1$
    - Restricted retrieval to images through narrow-band filters only
  - Nyquist-sampled data
    - Restricted retrieval to images from HST through filters with
      - $\lambda_c > 0.500$ µm for Planetary Camera
      - $\lambda_c > 1.667$ µm for Wide-Field Camera (none existed)

- Consequence: Could not use many of the available images of stars

- Solution: Generalized phase retrieval algorithm
  Using physical model with wide-band light and undersampling
  - Computationally efficient analytic expression for gradient
  - Allows simultaneous fitting of multiple images with diverse phase
    - Phase-diverse phase retrieval

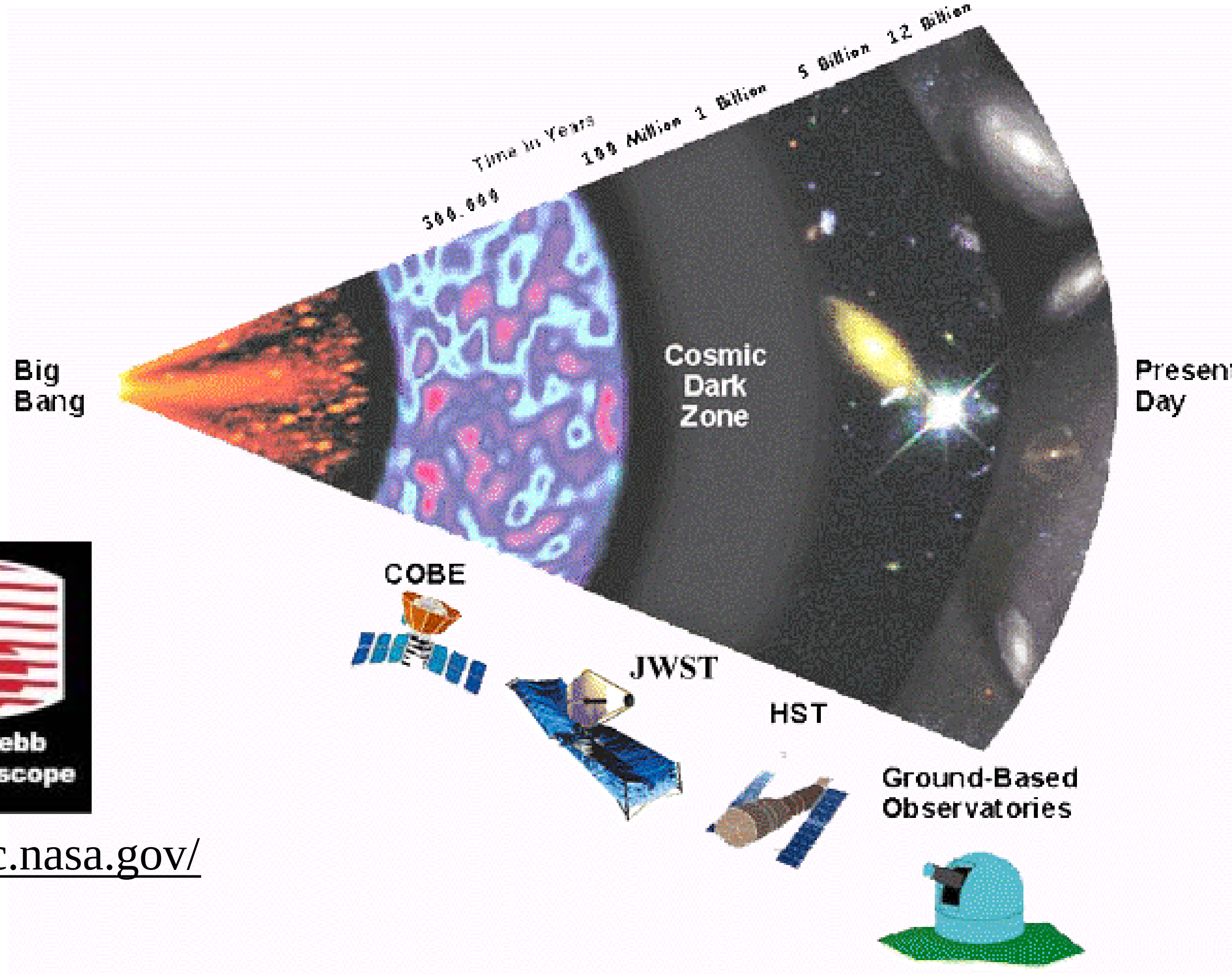
Monochromatic/Polychromatic Simulated Star Images



- –0.30 μm rms Spherical, small amounts of others;
- 2x2 pixel integration;
- WF/PC F555W filter, $\{\lambda_l\} = \{472.5, 516.0, 562.5, 609.0, 656.0\}$ nm

$$\{S_l\} = \{\ 0.78,\ \ 0.91,\ \ 0.82,\ \ 0.50,\ \ 0.18\}$$

James Webb Space Telescope
(Next Generation Space Telescope)



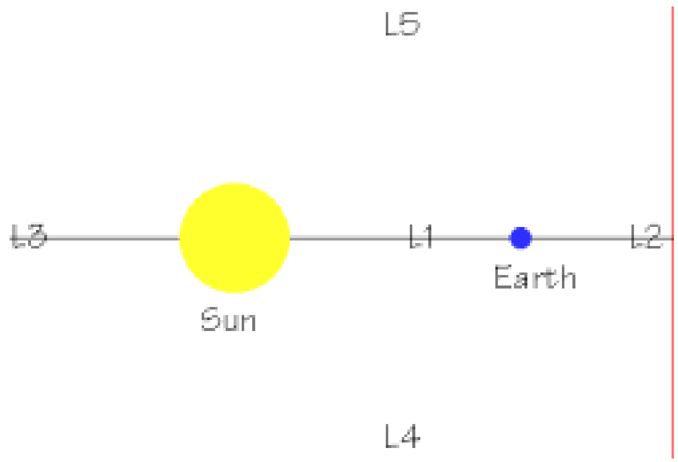
James Webb Space Telescope

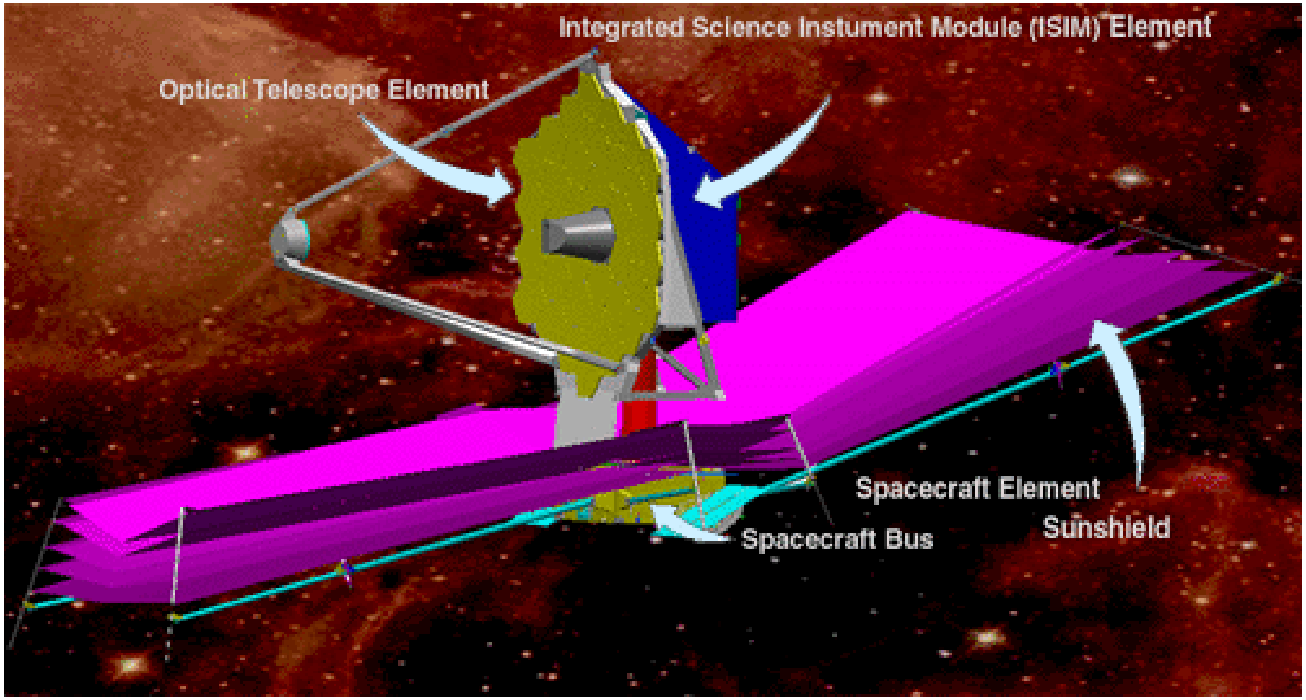
http://ngst.gsfc.nasa.gov/

See farther back towards the beginnings of the universe
Light is red-shifted into infrared

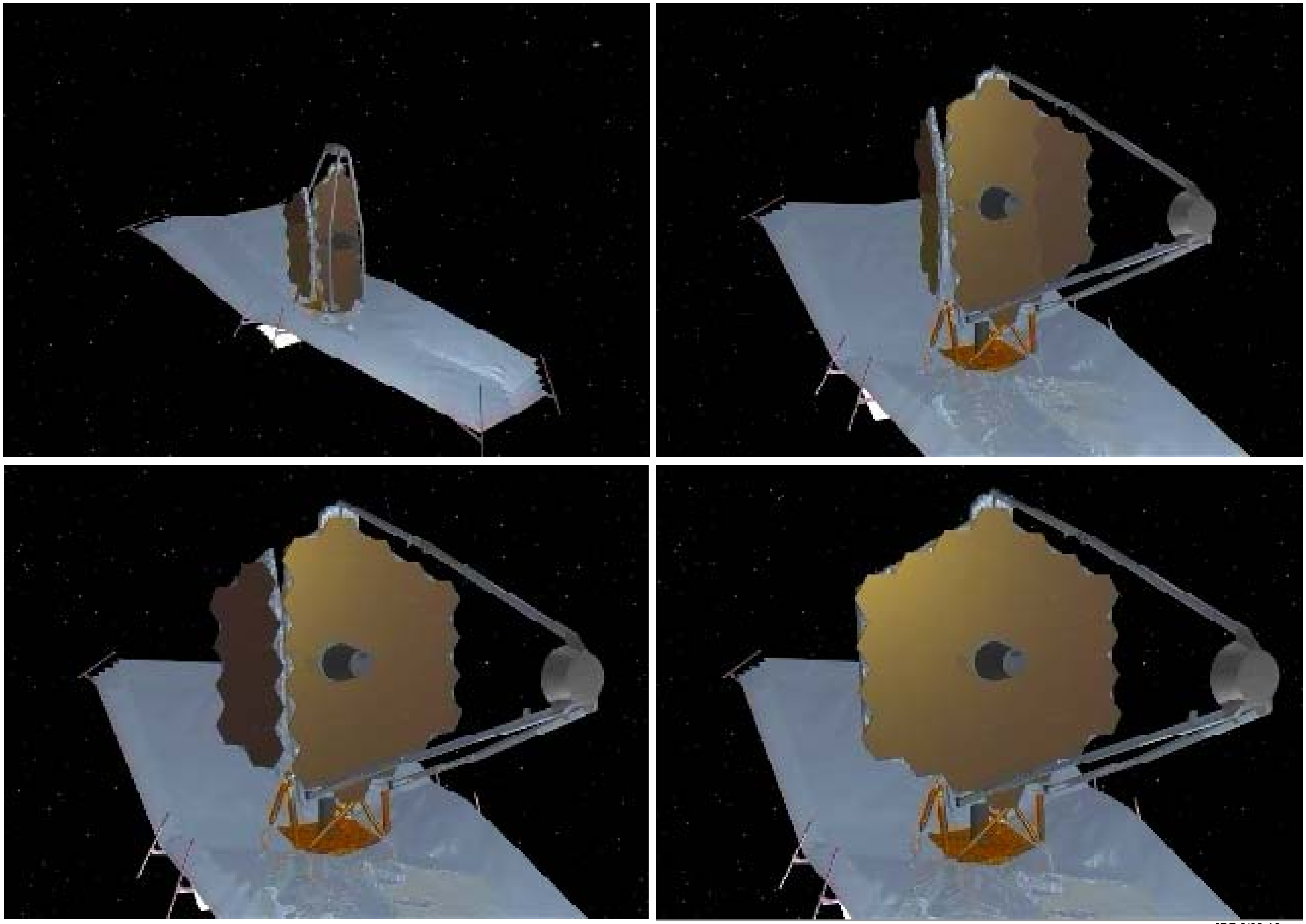
James Webb Space Telescope (JWST)

- See red-shifted light from early universe
  - 0.6 µm to 28 µm
  - L2 orbit for passive cooling, avoiding light from sun and earth
  - 6 m diameter primary mirror
    - Deployable, segmented optics
    - Phase retrieval to align segments

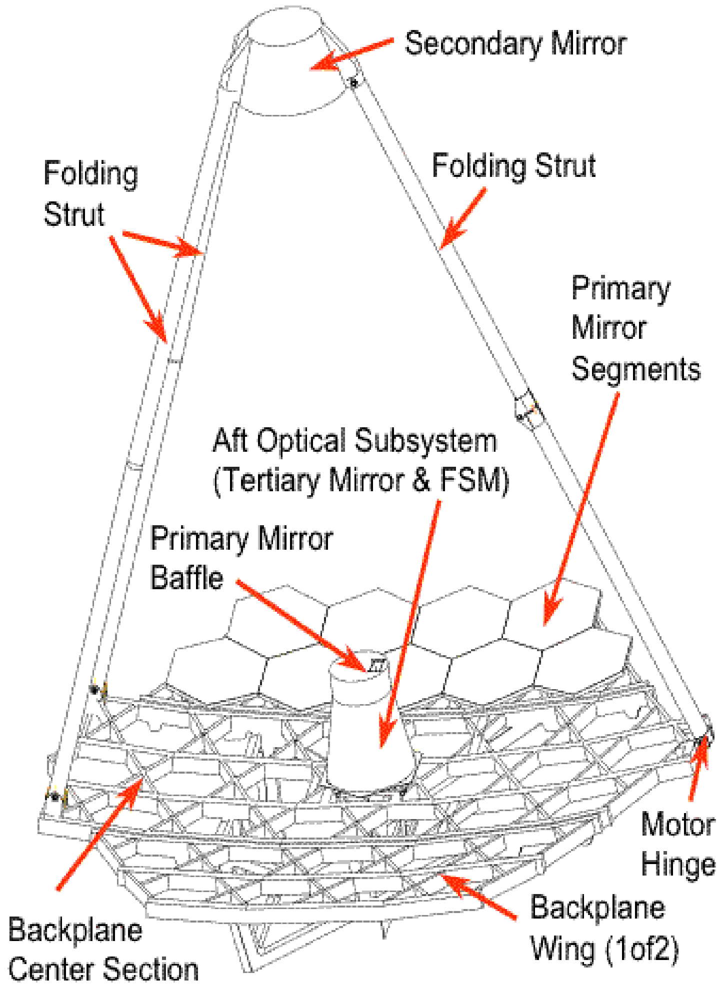


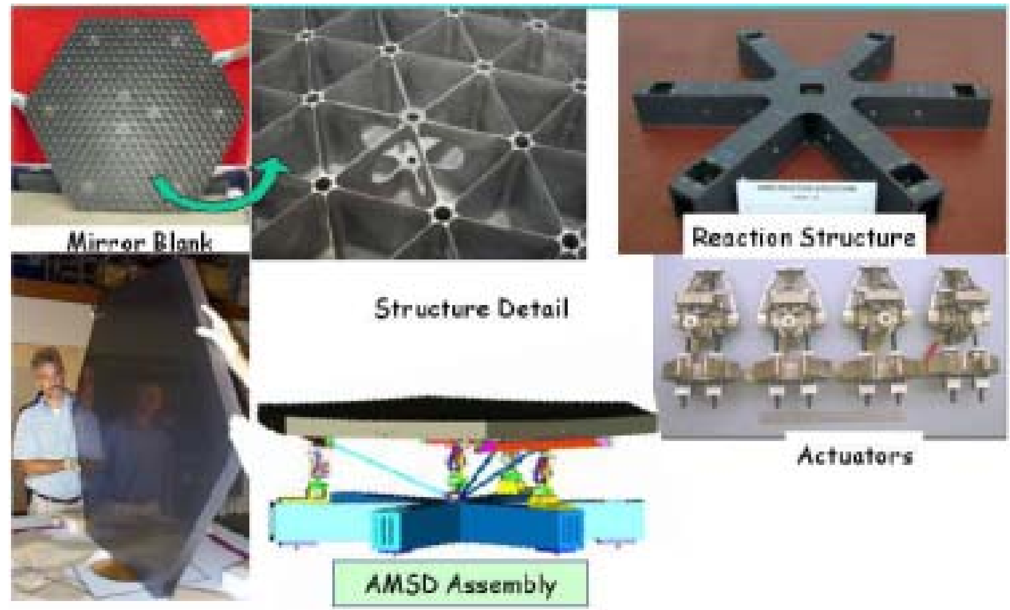


Segmented-Mirror Deployment

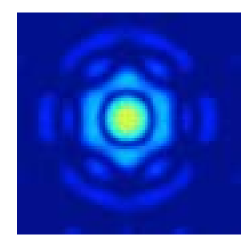


JWST Optics





Ball Advanced Mirror System Demonstrator (AMSD)



Ideal PSF

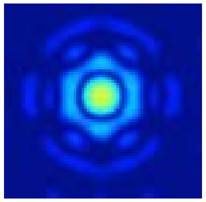
4 parameters (piston/tip/tilt/radius) x 36 segments = 144 total

JWST Team, U. Rochester

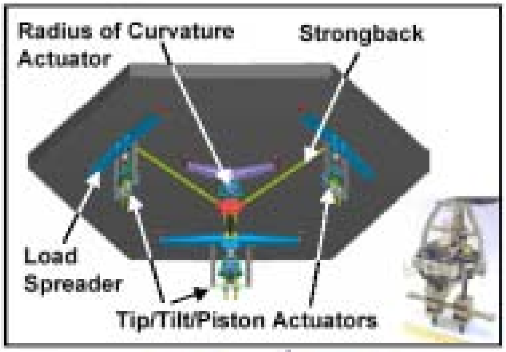
- Northrop Grumman Space Technology (TRW) — Prime Contractor
- Ball Aerospace — Optical System Development
  - Optical Telescope Element optical design and optics
  - Wave-Front Sensing & Control (WFS&C) design & algorithms
  - Mirror segment cryogenic testing
  - OTE and Observatory AI&T support
- Eastman Kodak — Telescope Integration and Test
  - OTE ground AI&T
  - Plum Brook test configuration and interfaces
  - Fabricate ULE mirrors (if option selected)
- Veridian Systems — WFS&C technology and algorithms
- University of Rochester — WFS&C alternative approaches & algor.

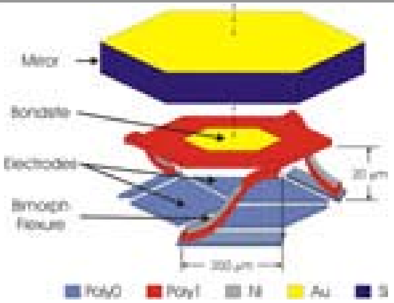
Expected UofR JWST Activities: Identify Alternative WFS Approaches

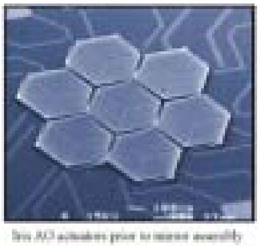
- Develop improved WFS (phase retrieval) algorithms
  - Faster, converge more reliably, less sensitive to noise, 2π jumps
  - Work with larger aberrations, broadband illumination
  - Employ alternative measurement diversities
    - Increase robustness and accuracy
  - Phase retrieval performance



Ideal PSF







- <u>Experiments with UofR JWST laboratory simulator</u>
  - Adaptive optics deformable mirror (36 hex. segments)
  - Interferometer measure wavefront independently
  - Put in misalignment, reconstruct wavefronts, compare with interferometer "truth"

Post-doc position available