

Phase Retrieval: Hubble and the James Webb Space Telescope

James R. Fienup

Robert E. Hopkins Professor of Optics

University of Rochester
Institute of Optics

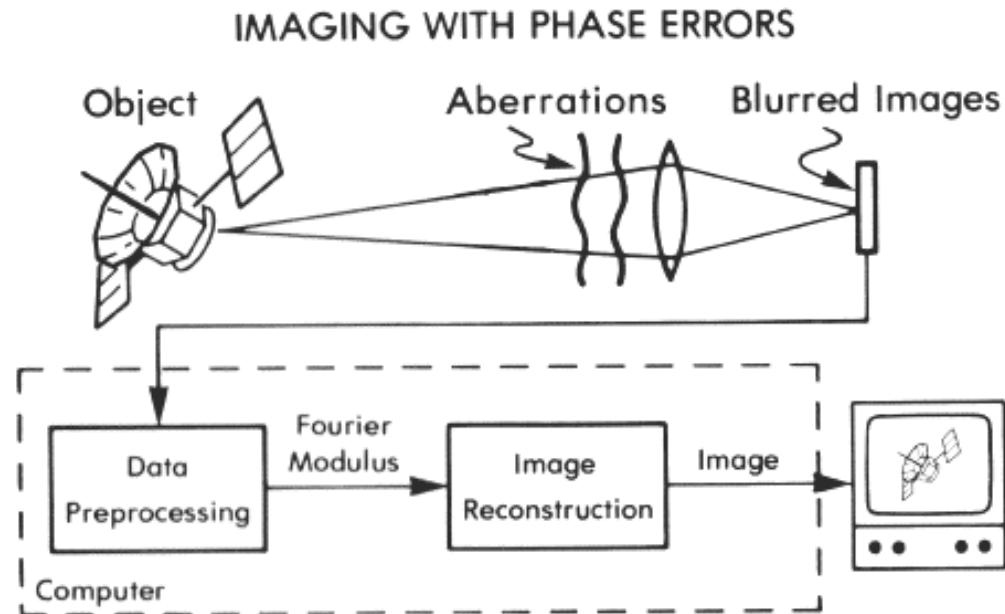
fienup@optics.rochester.edu

March 2003

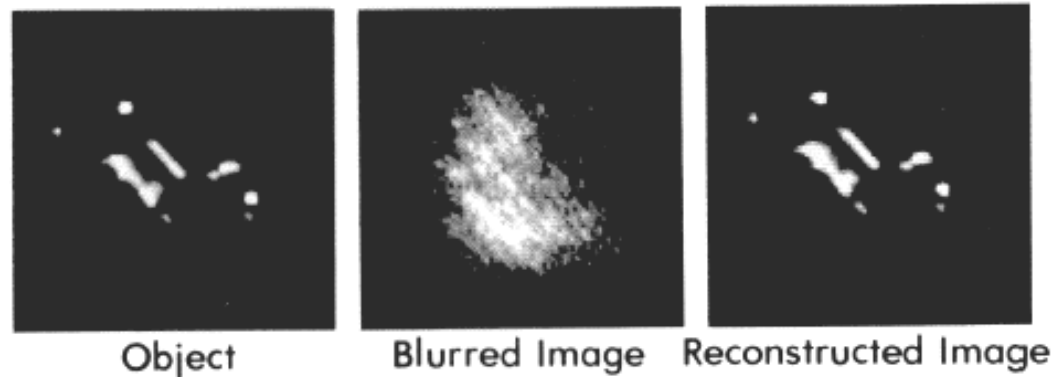
- Hubble Space Telescope (HST)
 - Phase retrieval algorithms
 - Iterative transform
 - Gradient-search parameter fitting
- James Webb Space Telescope (JWST)
 - Background
 - Phase retrieval for wavefront sensing and control

Early work was performed while Jim Fienup was at ERIM, Ann Arbor, MI

Phase Retrieval for Image Reconstruction from Stellar Speckle Interferometry Data

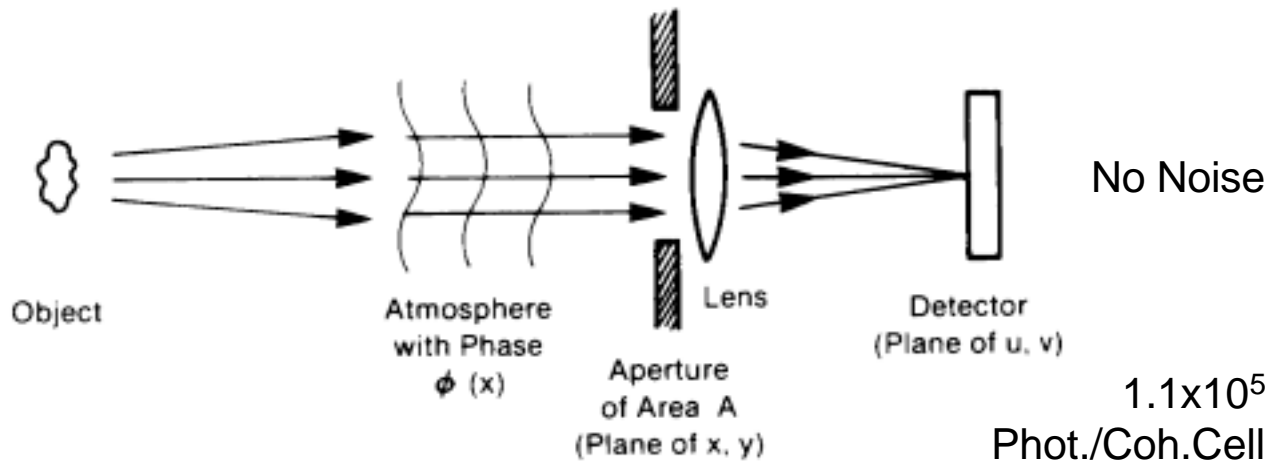


Example:



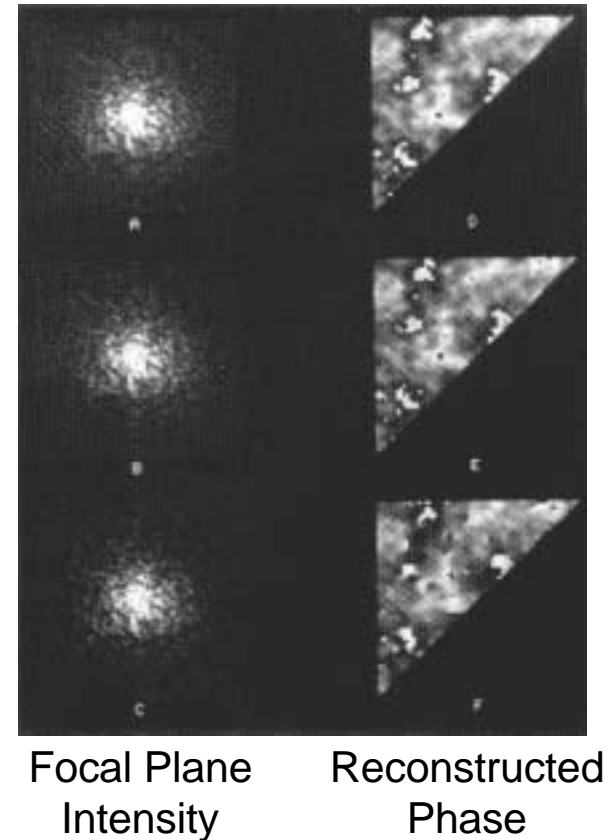
Reference: J.R. Fienup, "Phase Retrieval Algorithms: A Comparison," *Appl. Opt.* 21, 2758-2769 (1982).

Fourier Intensity Wavefront Sensor



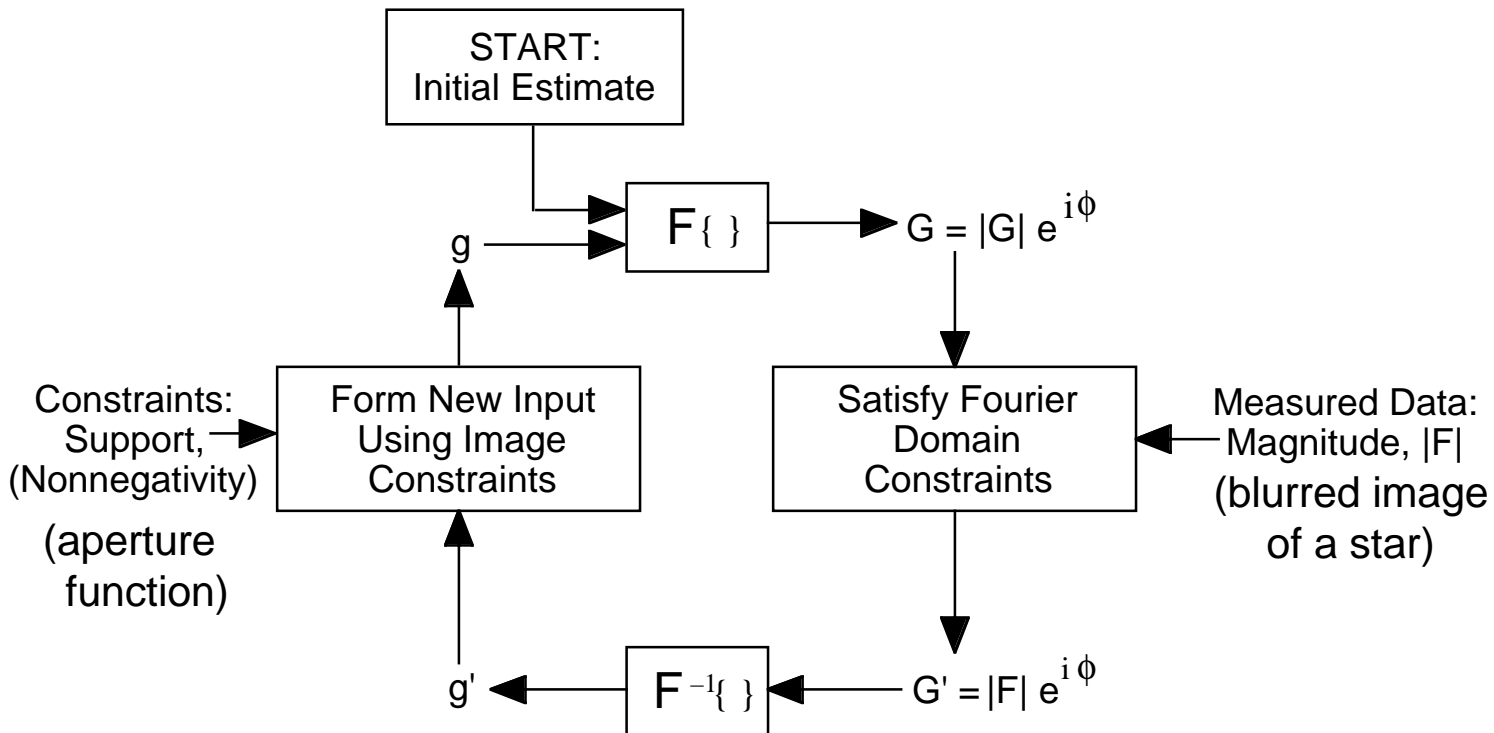
Measure atmospheric phase to drive adaptive optics

1.1×10^3 Phot./Coh.Cell



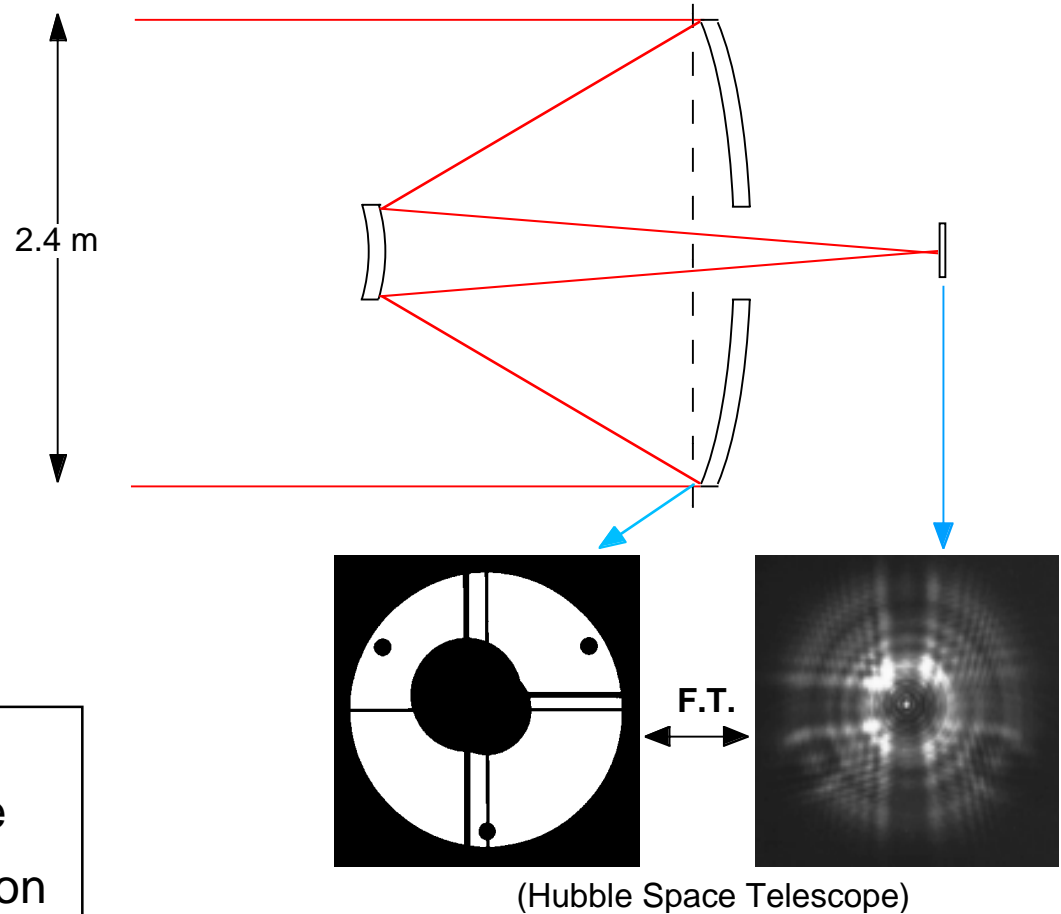
Reference: J.N. Cederquist, J.R. Fienup, C.C. Wackerman, S.R. Robinson and D. Kryskowski, "Wave-Front Phase Estimation from Fourier Intensity Measurements," J. Opt. Soc. Am. A 6, 1020-1026 (1989).

Iterative Transform Algorithm



Enforcing magnitude constraints in both domains is the "Error Reduction" or "Gerchberg-Saxton" algorithm

Determine HST Aberrations from PSF



Measurements & Constraints:

Pupil plane: known aperture shape
 phase error fairly smooth function
 Focal plane: measured PSF intensity

Wavefronts in pupil plane and focal plane
 are related by a Fourier Transform

Reference: J.R. Fienup, J.C. Marron, T.J. Schulz and J.H. Seldin, "Hubble Space Telescope Characterized by Using Phase Retrieval Algorithms," *Appl. Opt.* **32** 1747-1768 (1993).

Knowing aberrations precisely allows for:

- Design correction optics to fix the HST
 - WF/PC II
 - COSTAR
- Optimize alignment of secondary mirror of HST OTA
- Monitor telescope shrinkage (desorption) and focus
- Compute analytic point-spread functions for image deconvolution
 - Noise-free
 - Depends on λ , $\Delta\lambda$, camera, field position
 - Is highly space-variant for WF/PC
 - Eliminates requirement to measure numerous PSF's

In addition, reconstruction of pupil function allows determination of alignment between OTA and WF/PC

Focal plane field

Pupil plane field

$$\begin{aligned} \text{Fourier transform: } F(u, v) &= \int \int_{-\infty}^{\infty} f(x, y) e^{-i2\pi(ux + vy)} dx dy \\ &= |F(u, v)| e^{i\psi(u, v)} = F[f(x, y)] \end{aligned}$$

$$\text{Inverse transform: } f(x, y) = \int \int_{-\infty}^{\infty} F(u, v) e^{i2\pi(ux + vy)} du dv = F^{-1}[F(u, v)]$$

Phase retrieval problem:

Given $|F(u, v)|$ and some constraints on $f(x, y)$,
Reconstruct $f(x, y)$, or equivalently retrieve $\psi(u, v)$

Minimize error metric by

- Cut & try (Holtzman)
- Iterative transform algorithm (Gerchberg-Saxton/Misell/Fienup)
- Gradient search (steepest descent, conjugate gradient, . . .)
- Damped least squares (Newton-Raphson)
- Neural network
- Linear programming
- Prescription retrieval
- Phase diversity
- etc.

- Other groups doing phase retrieval
 - Rick Lyon *et al.* Hughes Danbury Optical Systems
 - Chris Burrows (Space Telescope Science Institute)
 - Mike Shao (JPL)
 - Francois Roddier (U. Hawaii), . . .

- Model optical system
 - Known parameters (constraints)
 - Unknown parameters (to retrieve)
- Compute model of data
- Compare model of data with actual measured data
 - Compute error metric
- Minimize error metric over space of unknown parameters
 - using nonlinear optimization algorithms

Focal plane:

$$\longrightarrow E = \sum_u W(u) [|G(u)| - |F(u)|]^2$$

(Equivalent to a maximum likelihood algorithm for the case of additive Gaussian noise)

$$E = \sum_u W(u) [|G(u)|^2 - |F(u)|^2]^2$$

$$E = \sum_u W(u) \{ |G(u)| - |F(u)| - |G(u)| \ln(|F(u)|/|G(u)|) \}$$

Aperture plane:

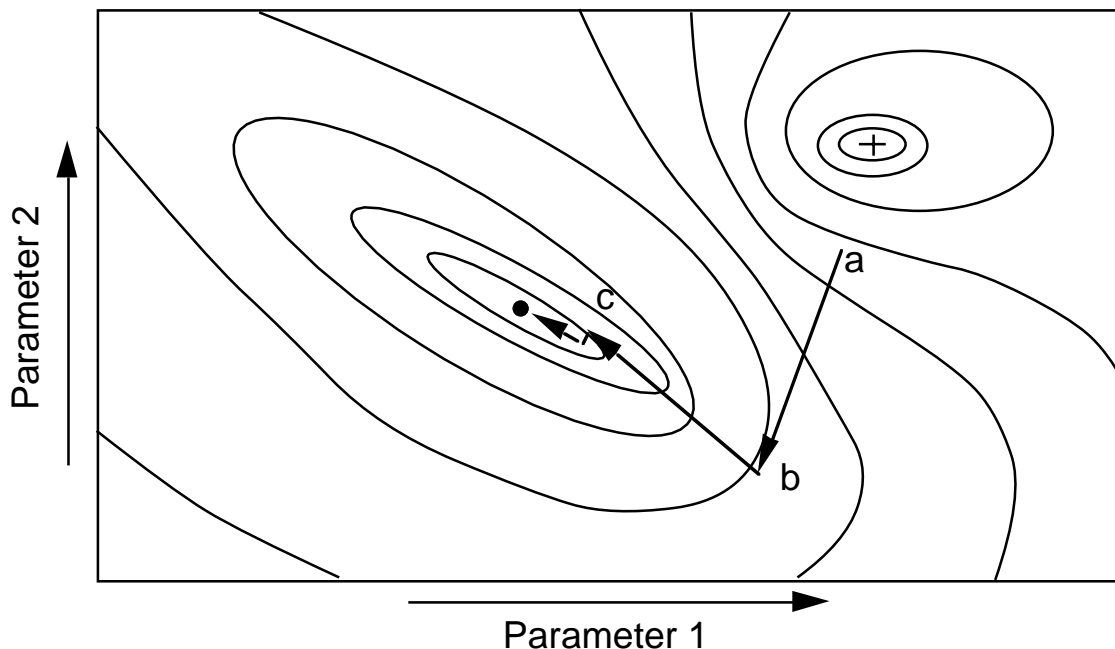
$$E = \sum_x [|g(x)| - |f(x)|]^2$$

$$E = \sum_x \{ |g(x)| - |f(x)| - |g(x)| \ln(|f(x)|/|g(x)|) \}$$

$$E = \sum_{x \in S} [|g(x)|]^2$$

Minimize Error Metric, e.g.: $E = \sum_u W(u)[|G(u)| - |F(u)|]^2$

Contour Plot of Error Metric



Repeat three steps:

1. Compute gradient:

$$\frac{\partial E}{\partial p_1}, \frac{\partial E}{\partial p_2}, \dots$$

2. Compute direction of search

3. Perform line search

Gradient methods:

(Steepest Descent)

Conjugate Gradient

BFGS/Quasi-Newton

...

$$E = \sum_u W(u) [|G(u)| - |F(u)|]^2$$

For point-by-point phase map, $\theta(x)$, $\frac{\partial E}{\partial \theta(x)} = 2 \operatorname{Im}\{g(x) g^{W*}(x)\}$

For Zernike polynomial coefficients, $\frac{\partial E}{\partial a_j} = 2 \operatorname{Im}\left\{\sum_x g(x) g^{W*}(x) Z_j(x)\right\}$

where

$$g(x) = m_o(x) e^{i\theta(x)}, \quad \theta(x) = \sum_{j=1}^J a_j Z_j(x), \quad G(u) = \mathbf{P}[g(x)],$$

$$G^W(u) = W(u) \left[|F(u)| \frac{G(u)}{|G(u)|} - G(u) \right], \quad \text{and} \quad g^w(x) = \mathbf{P}^\dagger[G^W(u)].$$

$\mathbf{P}[\cdot]$ can be a single FFT or multiple-plane Fresnel transforms with phase factors and obscurations

Analytic gradients very fast compared with calculation by finite differences

J.R. Fienup, "Phase-Retrieval Algorithms for a Complicated Optical System," *Appl. Opt.* 32, 1737-1746 (1993).

J.R. Fienup, J.C. Marron, T.J. Schulz and J.H. Seldin, "Hubble Space Telescope Characterized by Using Phase Retrieval Algorithms," *Appl. Opt.* 32 1747-1768 (1993).

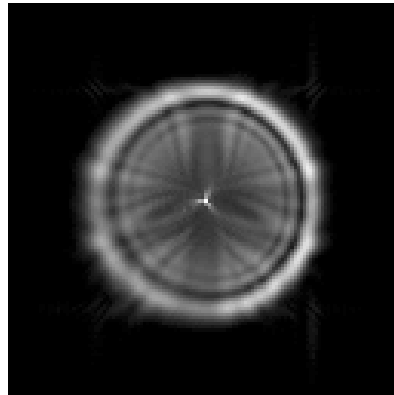
- Pupil (support constraint) was known imperfectly
 - Phase was relatively smooth and dominated by low-order Zernike's
 - Use boot-strapping approach
1. With initial guess for pupil, fit Zernike polynomial coefficients
(parametric phase retrieval by gradient search)
 2. With initial guess for Zernike polynomials, estimate pupil by ITA
(retrieve magnitude, given an estimate of phase)
 3. Redo steps 1 and 2 until convergence (2 iterations)
 4. Estimate phase map by ITA, starting with Zernike polynomial phase
(nonparametric phase retrieval by G-S or gradient search)
 5. Refit Zernike coefficients to phase map
 6. Redo steps 2 - 5

*Skipping this
step gave
worse results*

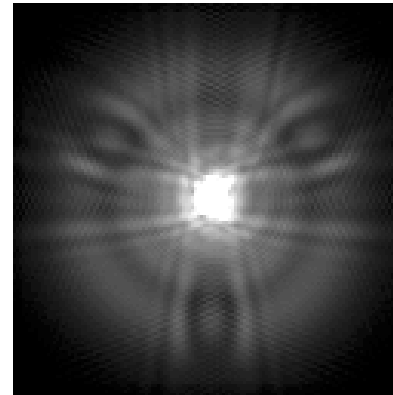
- Restrictions on early algorithms
 - Narrow-band light $\Delta\lambda/\lambda_c \ll 1$
 - Restricted retrieval to images through narrow-band filters only
 - Nyquist-sampled data
 - Restricted retrieval to images from HST through filters with
 - $\lambda_c > 0.500 \mu\text{m}$ for Planetary Camera
 - $\lambda_c > 1.667 \mu\text{m}$ for Wide-Field Camera (none existed)
- Consequence: Could not use many of the available images of stars
- Solution: Generalized phase retrieval algorithm
 - Using physical model with wide-band light and undersampling
 - Computationally efficient analytic expression for gradient
 - Allows simultaneous fitting of multiple images with diverse phase
 - Phase-diverse phase retrieval

Monochromatic/Polychromatic Simulated Star Images

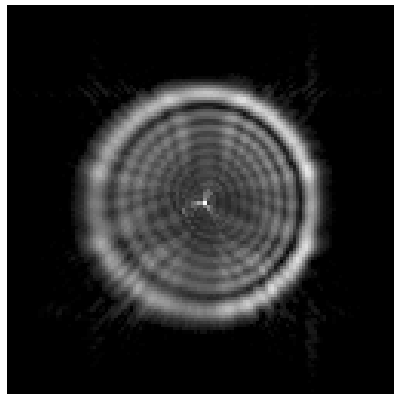
(a) polychromatic PSF a



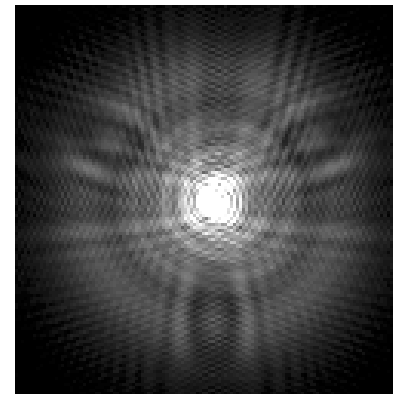
(b) polychromatic PSF b



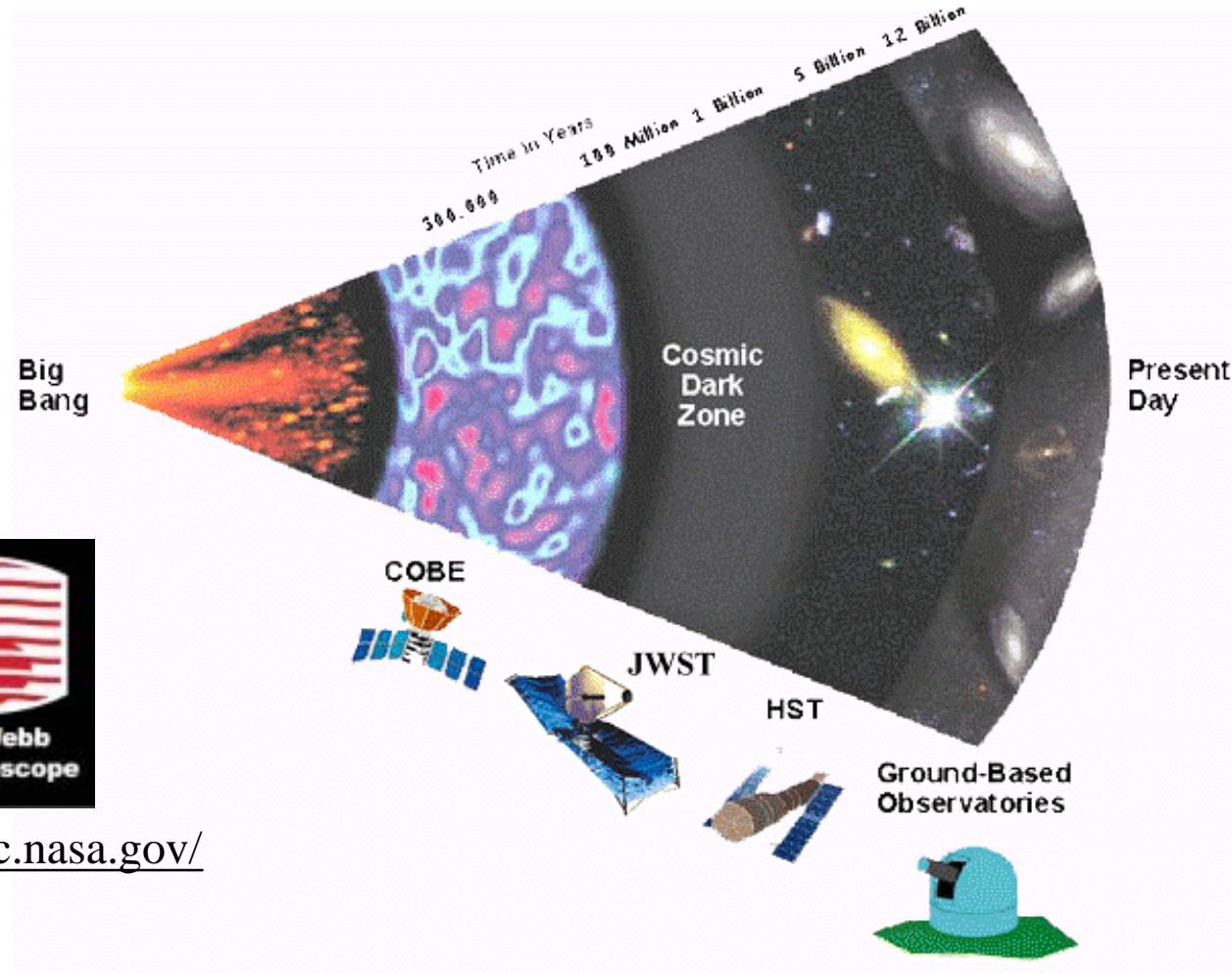
(c) monochromatic PSF a



(d) monochromatic PSF b



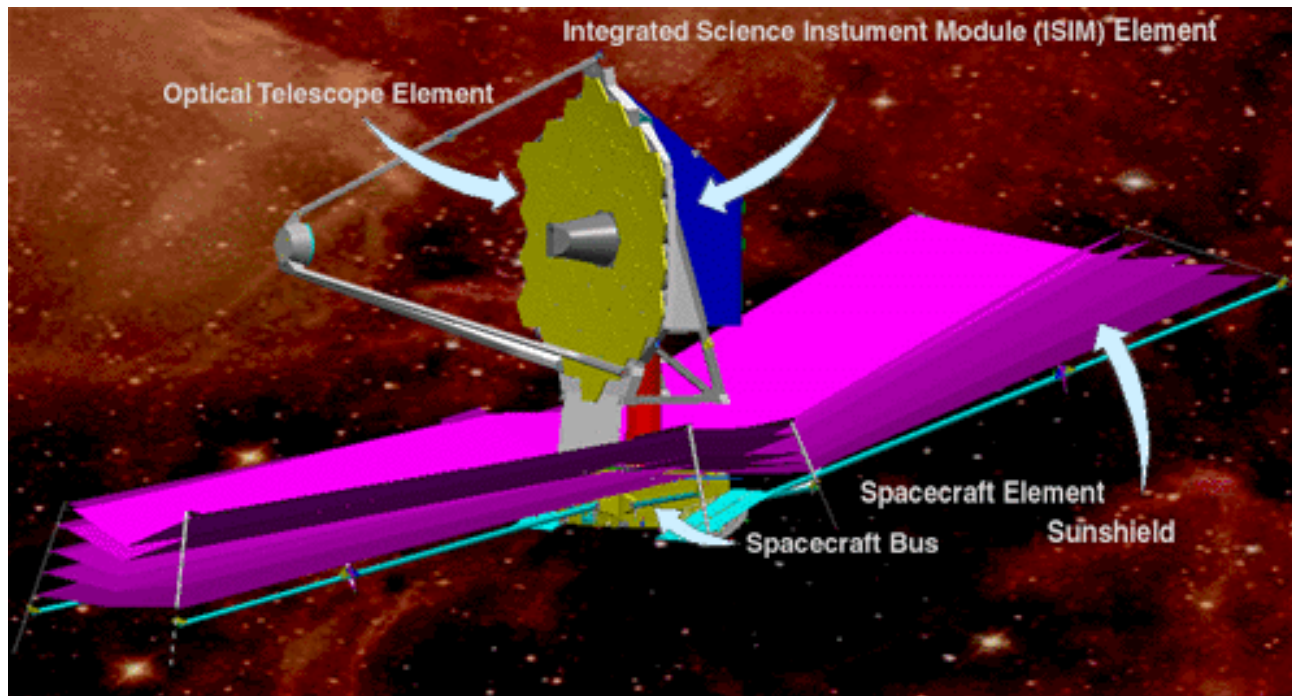
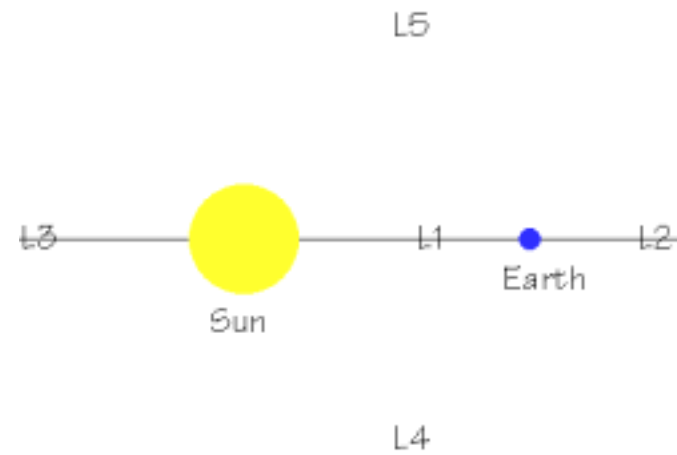
- $-0.30 \mu\text{m}$ rms Spherical, small amounts of others;
- 2×2 pixel integration;
- WF/PC F555W filter, $\{\lambda_i\} = \{472.5, 516.0, 562.5, 609.0, 656.0\}$ nm
- $\{S_i\} = \{0.78, 0.91, 0.82, 0.50, 0.18\}$

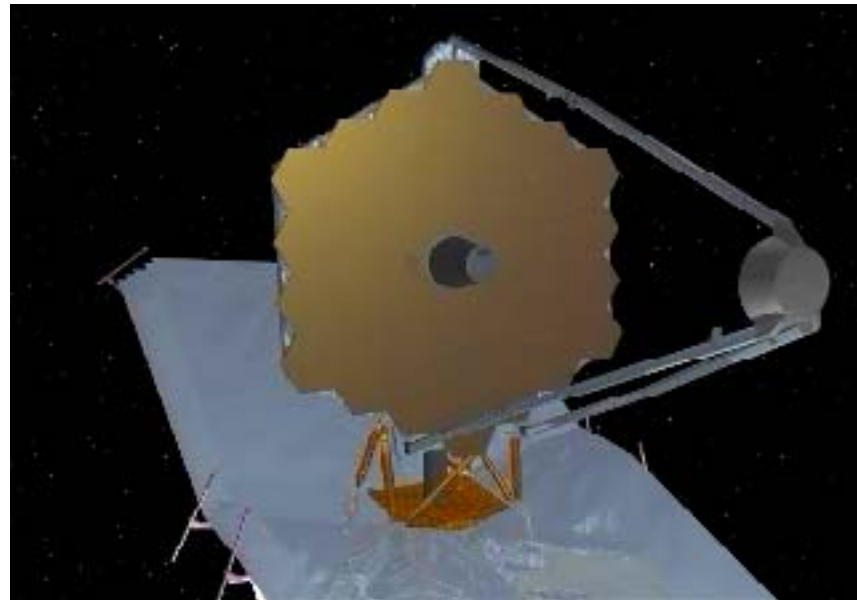
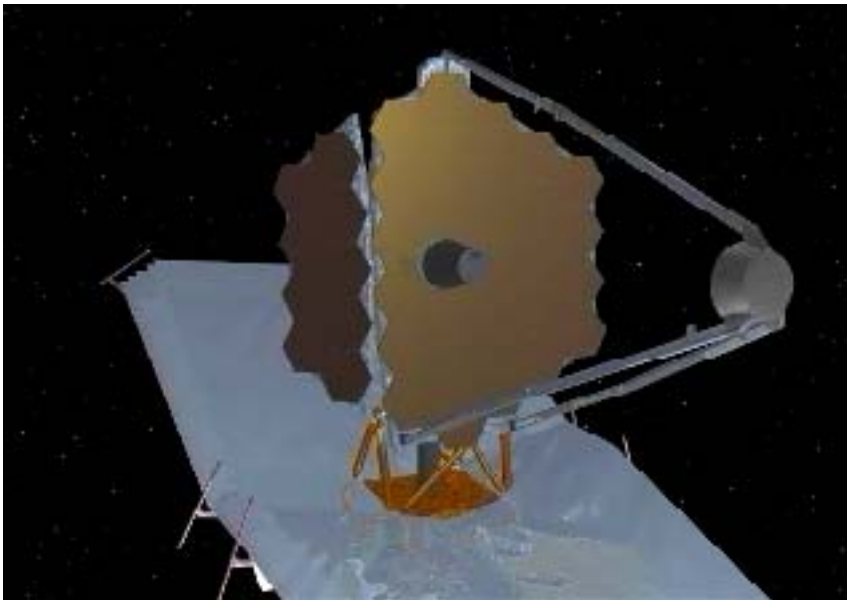
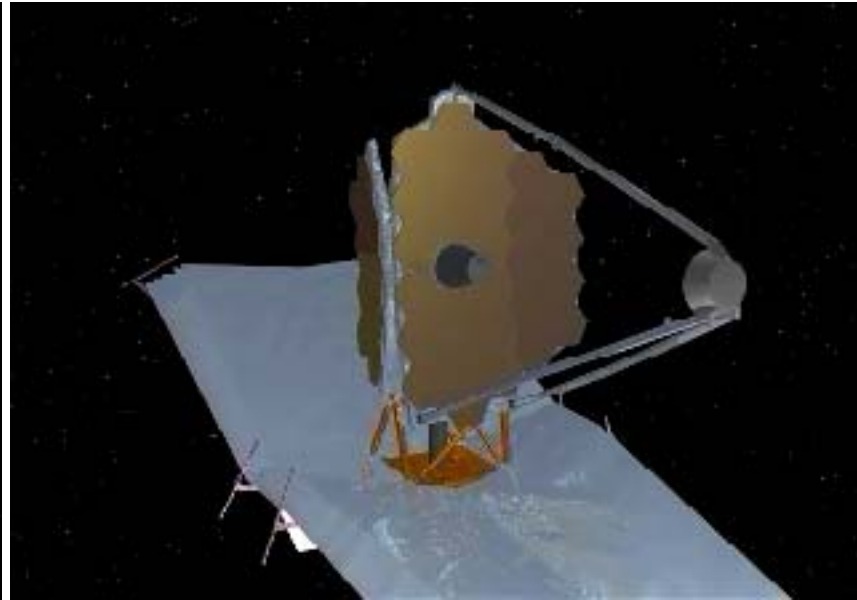
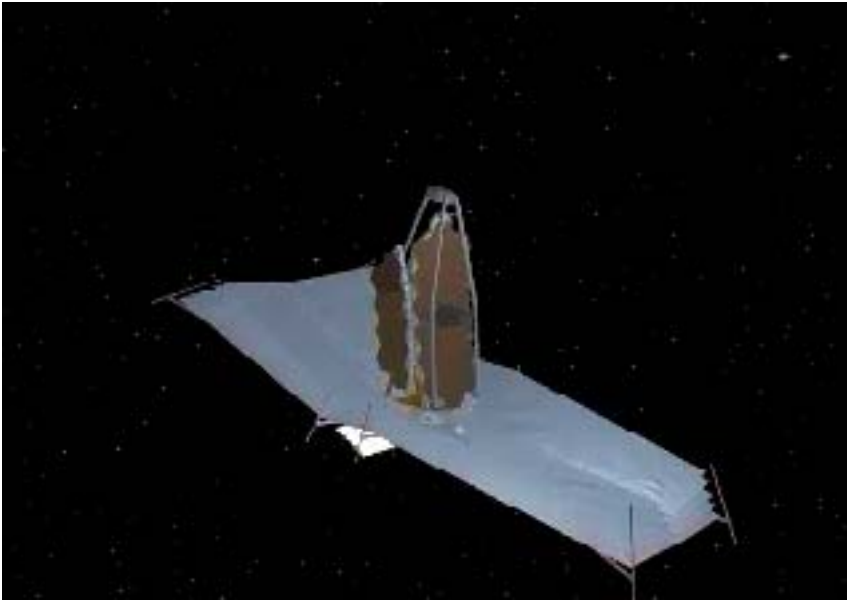


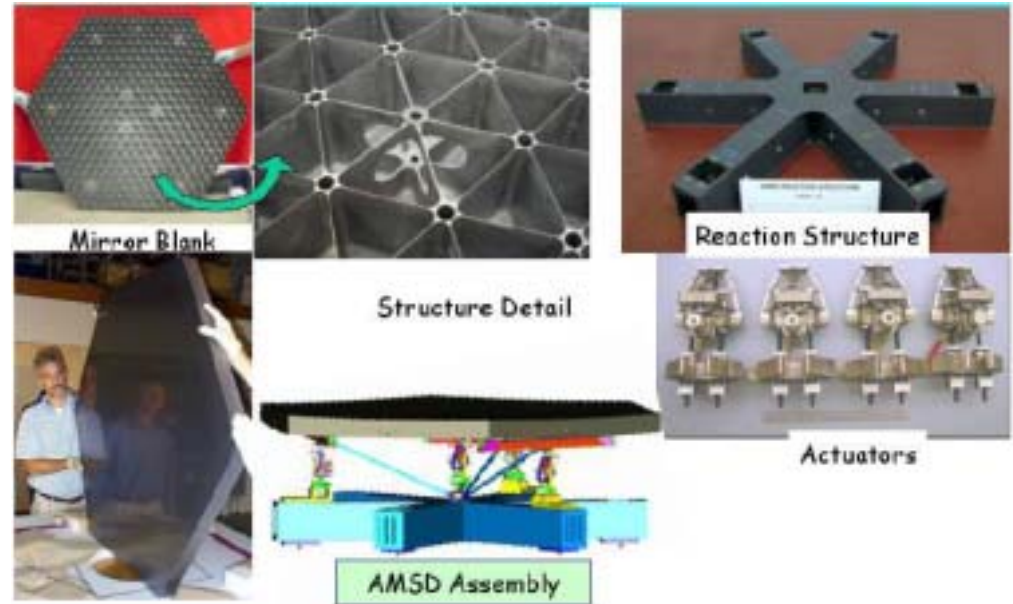
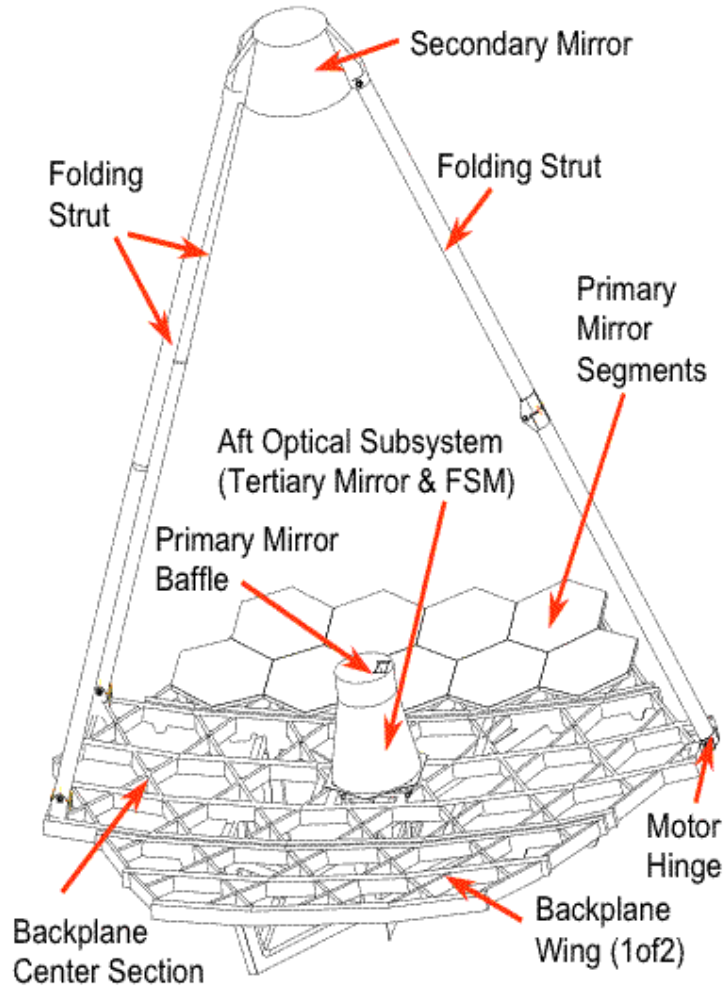
<http://ngst.gsfc.nasa.gov/>

See farther back towards the beginnings of the universe
Light is red-shifted into infrared

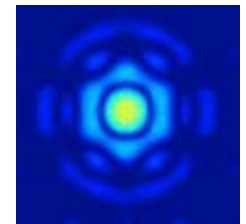
- See red-shifted light from early universe
 - 0.6 μm to 28 μm
 - L2 orbit for passive cooling, avoiding light from sun and earth
 - 6 m diameter primary mirror
 - Deployable, segmented optics
 - Phase retrieval to align segments







Ball Advanced Mirror System Demonstrator (AMSD)



Ideal PSF

4 parameters (piston/tip/tilt/radius) x 36 segments = 144 total

NORTHROP GRUMMAN

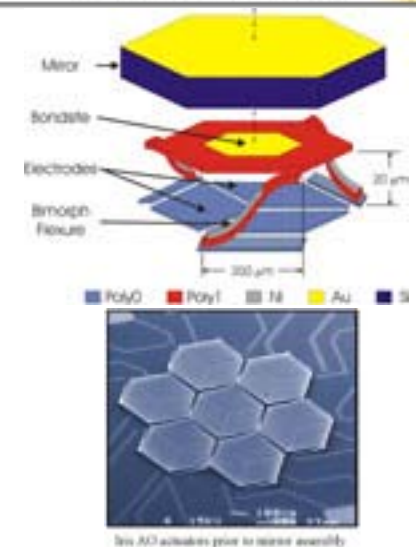
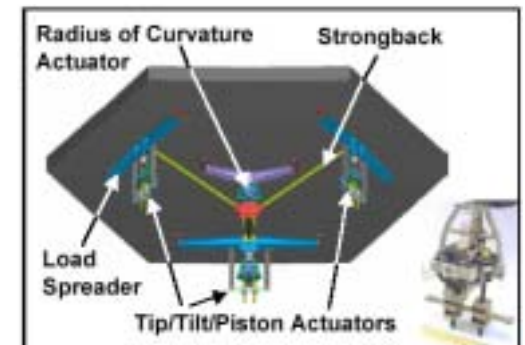


- Northrop Grumman Space Technology (TRW) — Prime Contractor
- Ball Aerospace — Optical System Development
 - Optical Telescope Element optical design and optics
 - Wave-Front Sensing & Control (WFS&C) design & algorithms
 - Mirror segment cryogenic testing
 - OTE and Observatory AI&T support
- Eastman Kodak — Telescope Integration and Test
 - OTE ground AI&T
 - Plum Brook test configuration and interfaces
 - Fabricate ULE mirrors (if option selected)
- Veridian Systems — WFS&C technology and algorithms
- University of Rochester — WFS&C alternative approaches & algor.

Expected UofR JWST Activities: Identify Alternative WFS Approaches

- Develop improved WFS (phase retrieval) algorithms
 - Faster, converge more reliably, less sensitive to noise, 2π jumps
 - Work with larger aberrations, broadband illumination
 - Employ alternative measurement diversities
 - Increase robustness and accuracy
 - Phase retrieval performance

- Experiments with UofR JWST laboratory simulator
 - Adaptive optics deformable mirror (36 hex. segments)
 - Interferometer measure wavefront independently
 - Put in misalignment, reconstruct wavefronts, compare with interferometer “truth”



Post-doc position available