Wave-front Reconstruction

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What is wave-front reconstruction?

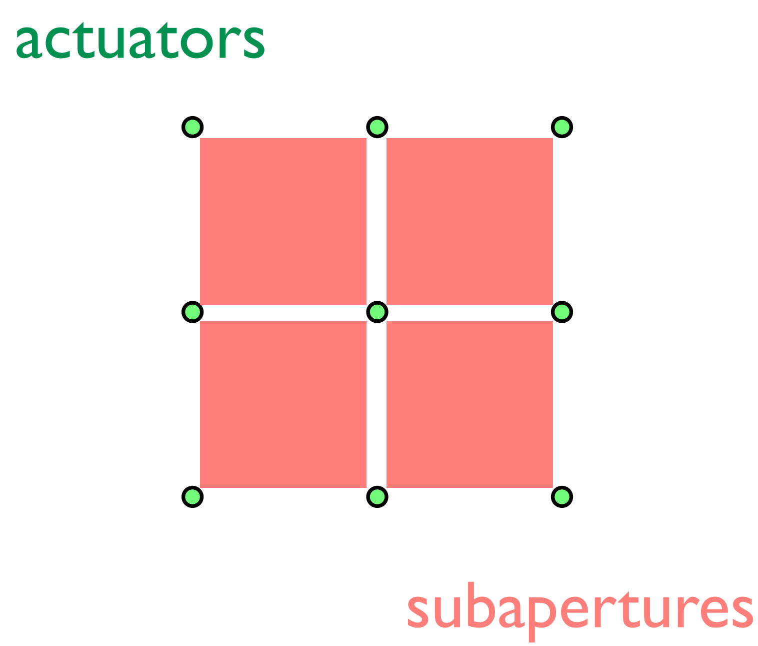
- We normally have only phase slope measurements - we want phase itself
- Second key signal processing step
  - high accuracy
  - low noise
  - speed
  - adaptability

What this talk is going to cover

- A survey of methods:
  - traditional matrix reconstruction
  - new, fast methods
    - Fourier Transform Recon, Local/heirarchic control, Sparse matrix, Conjugate Gradient technique
  - future directions
- A practical guide to real WF Recon
  - advantages/disadvantages
  - common problems

The WFS and actuators map into the phase

- Most common geometry is the 'Fried' geometry
- Others include
  - 'psuedo'hex
  - hexagonal



The matrix method relates sensors to actuators

- We can model (or measure) the relationship from actuators to sensors

$$s = \mathbf{M}a$$

- Then invert this matrix to go from sensor measurements to actuator commands

$$\hat{a} = \mathbf{M}^{-1}s$$

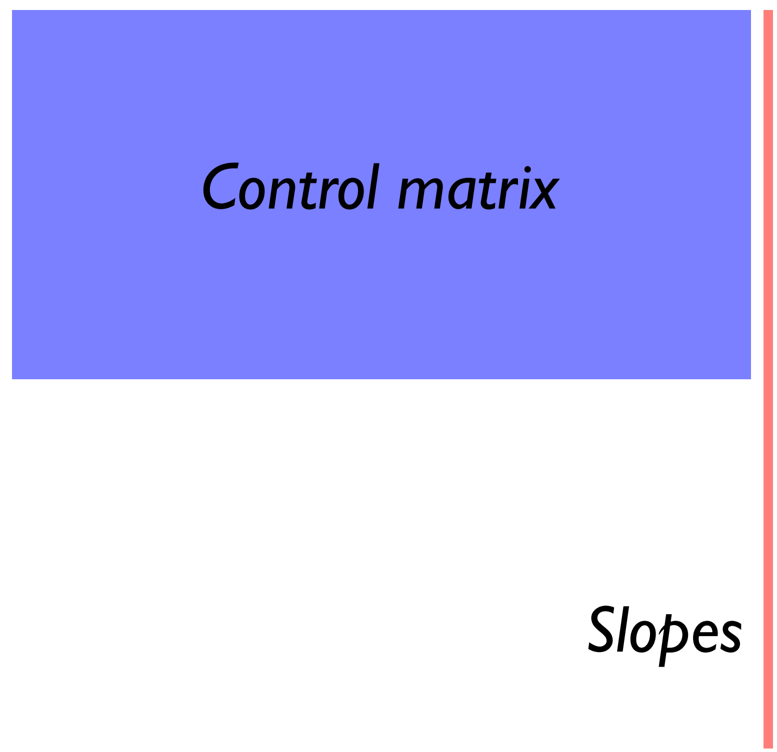
This is a pseudoinverse

Step 1: matrix inversion

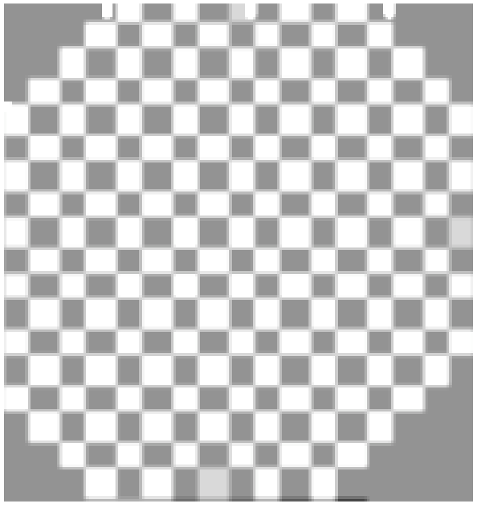
- The M matrix is singular - modes such as piston and waffle are unsensable = no real inverse
- Can use a standard method such as Singular Value Decomposition
- This step can take a long time: $O(n^3)$

Step 2: Matrix application

- Matrix M is $2N_{sen}$ by N_{act} big
- Full VMM requires $O(4N_{sen}N_{act})$ operations
- This can, however, be fully parallelized



Quirk of the matrix 1: waffle



- Waffle is an unsensed mode into which noise propagates
- Significant waffle can degrade the PSF

You can get ride of waffle by projecting it out

- Use waffle-removal matrix W on actuator commands

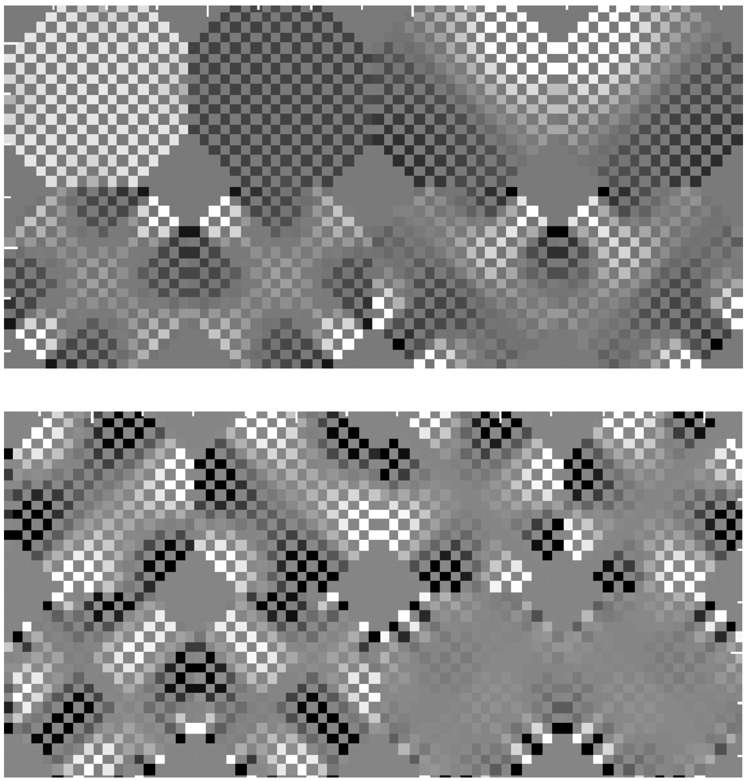
$$\tilde{a} = \mathbf{W}\hat{a}$$

- New control matrix is now simply

$$\mathbf{W}\mathbf{M}^{-1}$$

- Can remove any undesirable mode this way

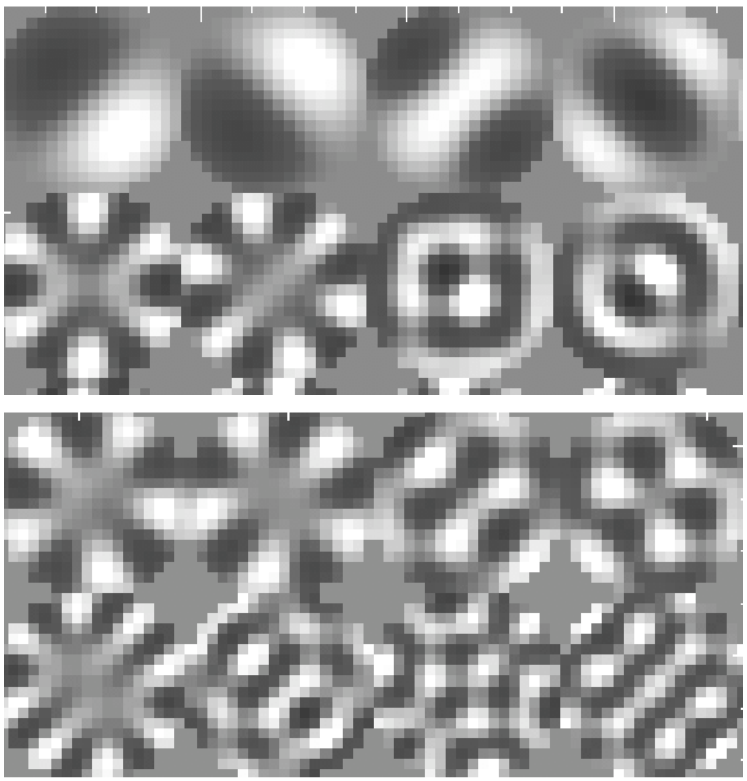
Quirk of the matrix 2: local waffle



- Local waffle exists in almost every mode and contains no pure waffle

Data from Don Gavel

Solution: regularize the matrix



- Pick a new modal basis such that waffle is concentrated in a few 'high-order' modes
- Project out these modes to remove local waffle

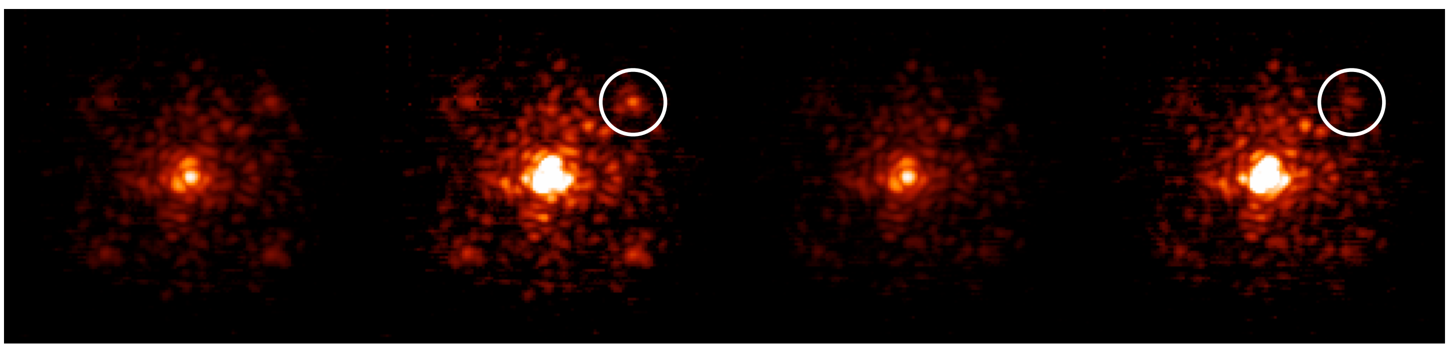
Data from Don Gavel

Can regularize with Bayesian reconstructor

- Modes unlike the atmosphere (e.g. local waffle) are penalized

Keck images

Old reconstructor New reconstructor



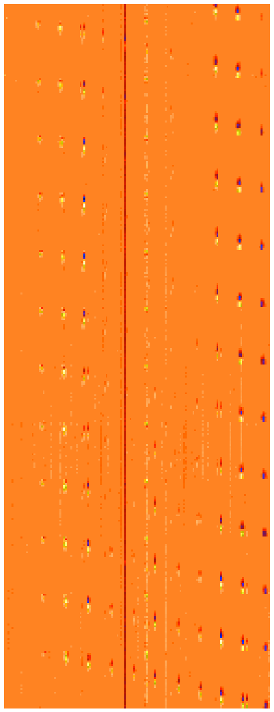
Data from Marcos van Dam

Quirk of the matrix 3: model or measurement?

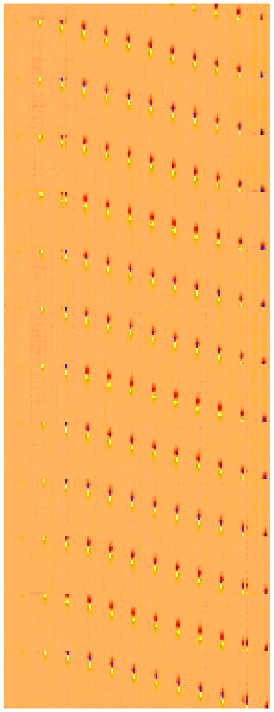
- Matrix M can be developed a priori based on a model of the system
  - how accurate does your system have to be aligned?
- Matrix M can be determined experimentally by the ‘push actuator; method
  - reflects current state of system

Measuring the system matrix can lead to noise corruption

Bad case



Good case



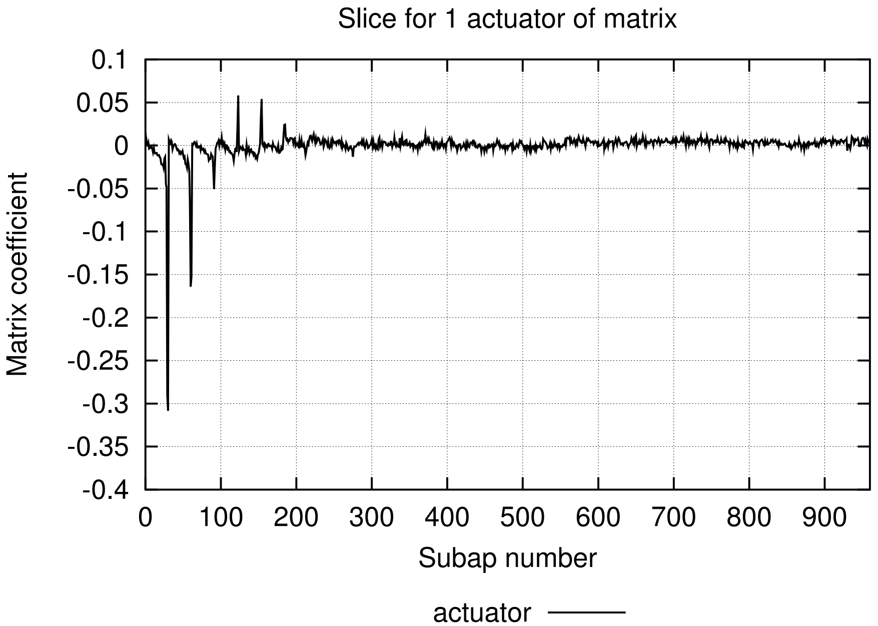
- Push-actuator/ read WFS method can be noisy
- This leads to a worse control matrix

Data from Steve Jones & Julianna Lin

Mini-break!

The control matrix looks sparse...

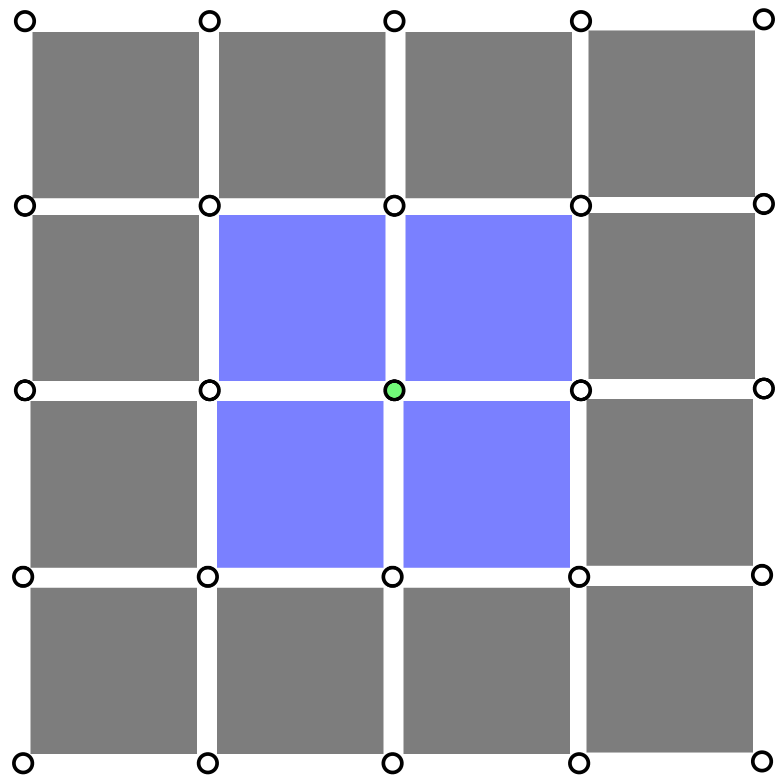
- A single actuator result is dominated by the subaps close by
- Subaps farther away contribute, but at a much lower level



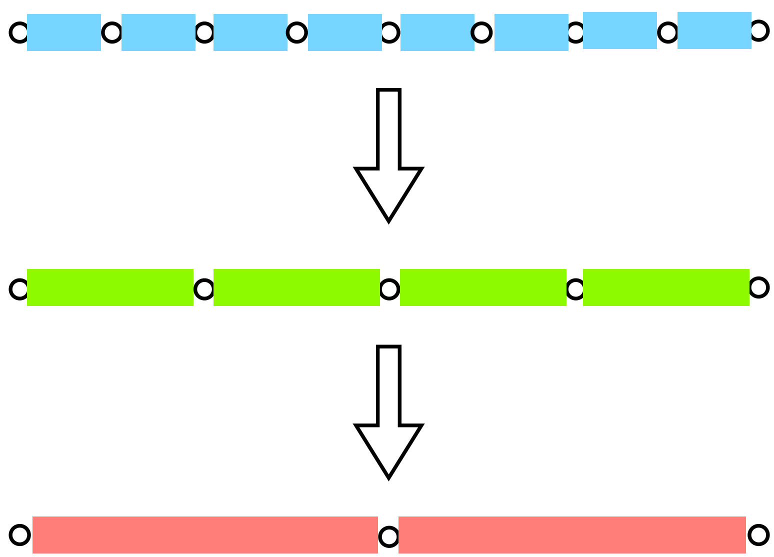
Local control uses only nearby slopes

- Can truncate the number of subapertures used in M^{-1}
- Can do a local LS 'mini-matrix':

$$O(n^{4/3})$$

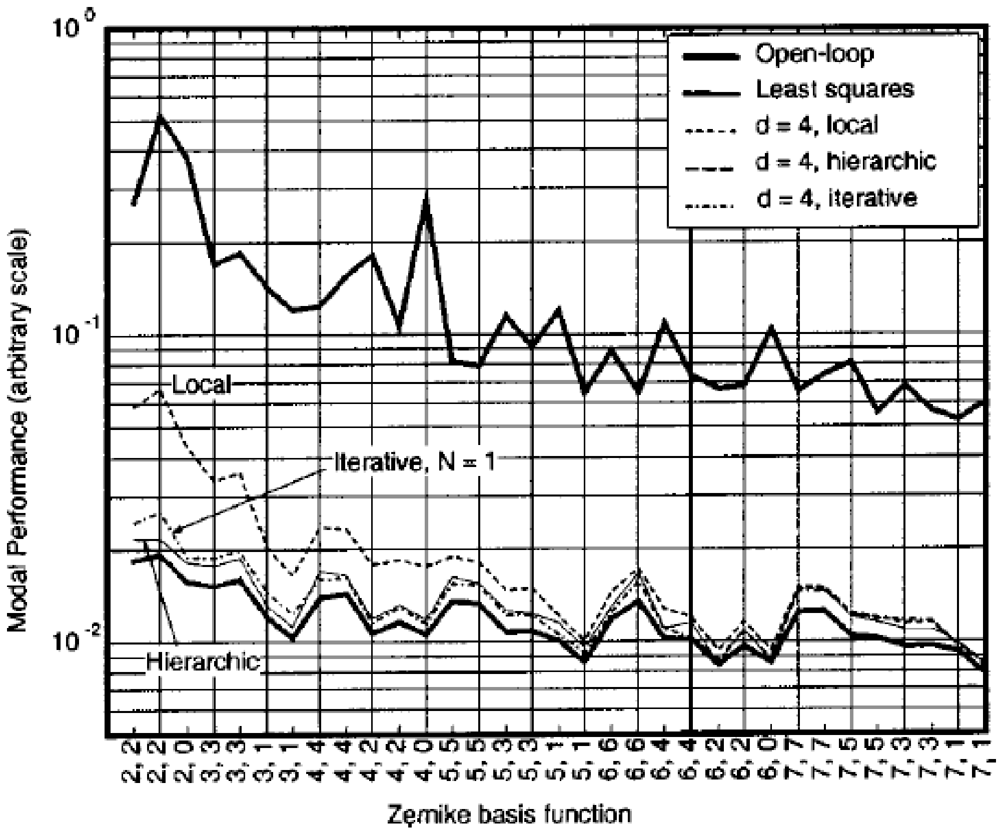


Being local means missing global



- Many modes are truly global: local control has the most error on these
- Can ameliorate with extra layers of estimation

Local control in Palomar simulation shows lost modes



- Trade-off: does speed-up due to computation outweigh loss of correction?

Figure from MacMartin, JOSA A 20, p1084-1093

Approximate the matrix as sparse

- Two advantages to sparse matrices:
  - can invert faster
  - can multiply control matrix faster
- Direct matrix inversion time reduced to $O(n^{3/2})$
- Need to approximate the traditional problem formulation in a way that works with sparse matrices

New formulation for the matrix minimum variance estimation (MVU)

- Incorporates (approximated) knowledge of the phase aberration

$$\phi = \mathbf{H}_x x - \mathbf{H}_a a \qquad s = \mathbf{G}_x x + n$$

Open-loop case

- Minimizes the residual error in either a specific direction or a given field

$$\sigma^2 = \phi^T \mathbf{W} \phi$$

Equations from Ellerbroek, JOSA A 20, p1803-1816

New formulation for the matrix minimum variance estimation (MVU)

- Result is two steps: estimation, then fitting

$$\hat{a} = \mathbf{F}\mathbf{E}s$$

- Estimation matrix uses phase, noise covariance and measurement method

$$\mathbf{E} = < xx^T > \mathbf{G}_x^T \left(\mathbf{G}_x < xx^T > \mathbf{G}_x + < nn^T > \right)^{-1}$$

- Fitting uses DM model, field of view

$$\mathbf{F} = \left(\mathbf{H}_a^T \mathbf{W} \mathbf{H}_a \right)^{-1} \left(\mathbf{H}_a^T \mathbf{W} \mathbf{H}_x \right)$$

Equations from Ellerbroek, JOSA A 20, p1803-1816

ExAO performance for Sparse matrix

- Sparse matrix computation is feasible of ExAO and maintains open-loop error as system size increases

Table 4. SRA Wave-Front Fitting Error versus Order of Correction for a Conventional NGS AO System[a]

Aperture Diameter (m)	Order of Correction	DM Actuators	CMMR σ^2 (μm^2)	SRA σ^2 (μm^2)	SRA Computation Time (s)	SRA Memory Requirements (Mbytes)
8	16×16	257	0.01334	0.01336	0.25	0.72
16	32×32	921	—	0.01392	1.17	3.81
24	48×48	1981	—	0.01416	3.60	NA
32	64×64	3461	—	0.01440	8.94	19.30

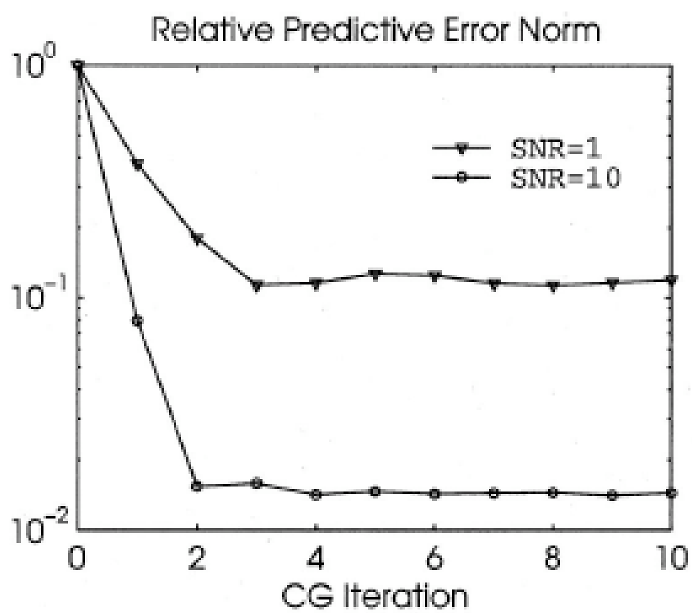
[a] This table summarizes the performance of the SRA for a conventional NGS AO scenario where the only significant source of wave-front error is the finite spatial resolution of the DM actuators and the WFS subapertures. The NGS is coincident with the evaluation direction, and the noise equivalent angle for the WFS is an almost negligible 0.02 arc sec. The DM actuator spacing is held constant, so the AO order of correction is proportional to the telescope aperture diameter. The SRA estimation error is virtually identical with the minimum-variance CMMR for the case of an order 16×16 AO system and increases fairly gradually with increasing telescope aperture diameter. The computation times and the memory requirements for the SRA grow much less rapidly than the $O(n^3)$ and $O(n^2)$ scaling laws that apply for the case of the CMMR.

Figure from Ellerbroek, JOSA A 20, p1803-1816

Conjugate-gradient is an iterative way to solve the matrix equations

- Preconditioned Conjugate Gradient (PCG) method
  - *conjugate gradient*: an iterative technique for solving linear systems
  - *preconditioned*: helps method converge faster. Different preconditioners are possible.
- These are iterative - the more iterations, the closer to exact the solution

PCG is iterative, but it converges rapidly



- Predictive error norm shows rapid convergence
- Convergence rate and level varies with SNR

Figure for MVU with FFT-based preconditioner

Figure from Gilles, JOSA A 20, p1817-1822

MVU needs to be modified to work in closed loop - work in progress

- Open-loop control unstable
- 2 new attempts:
  - Closed-loop regularized
  - Pseudo-open-loop with accurate knowledge of DM response

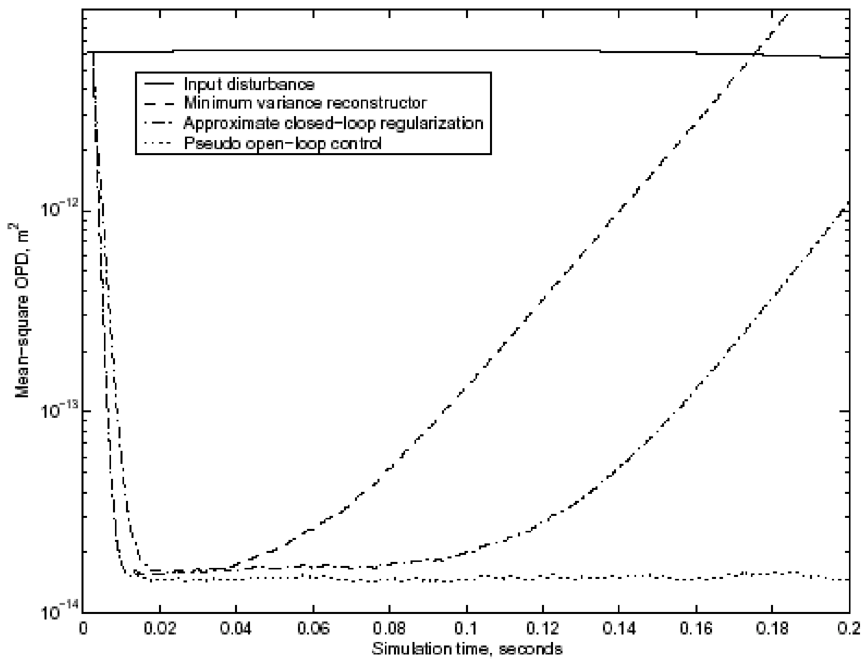


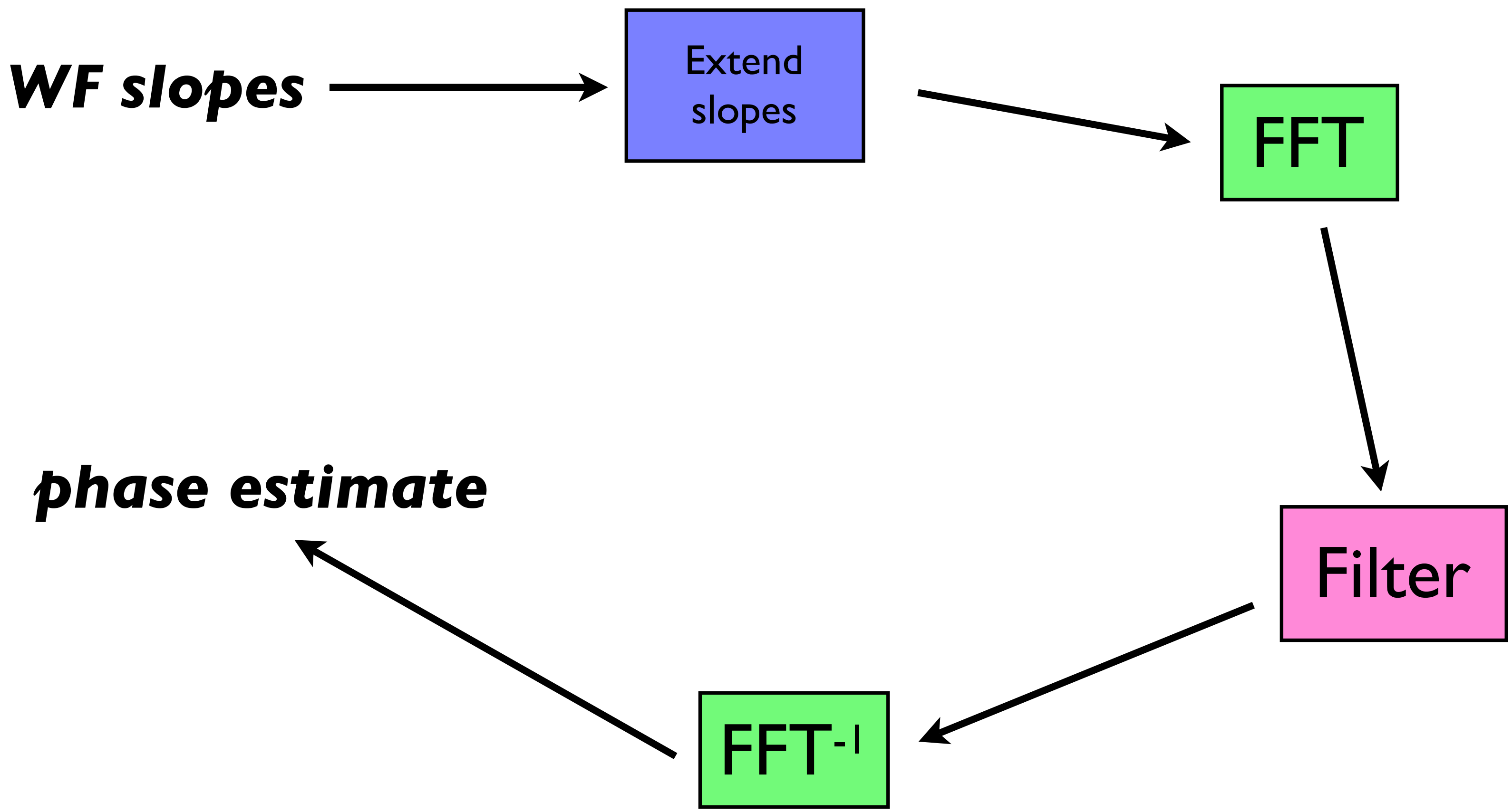
Figure from Ellerbroek, SPIE 5169 (in press)

Mini-break!

Filtering is a different approach to the problem

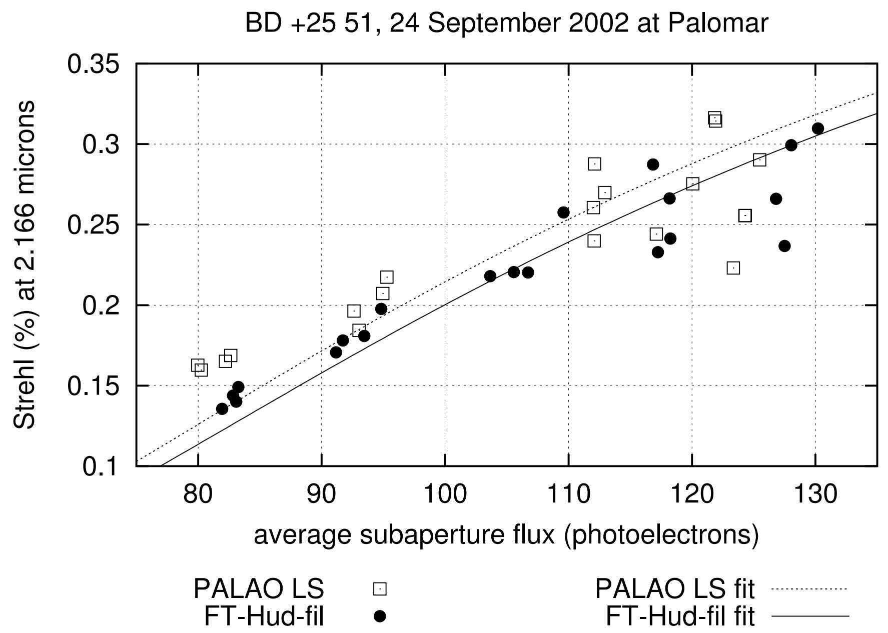
- Fourier transform reconstruction is an *inverse filter* on the subaperture measurements.
- Fast because of FFT: $O(n \log n)$

FTR requires slope extension then filtering to solve boundary problem



FTR validated on the sky at Palomar

- Implement FTR as a matrix
- On even dimmest star, there was no significant performance difference between best FTR and the LS matrix



See Poyneer, Optics Letters 28, p798-800

Filtering is very useful, but it has its limitations

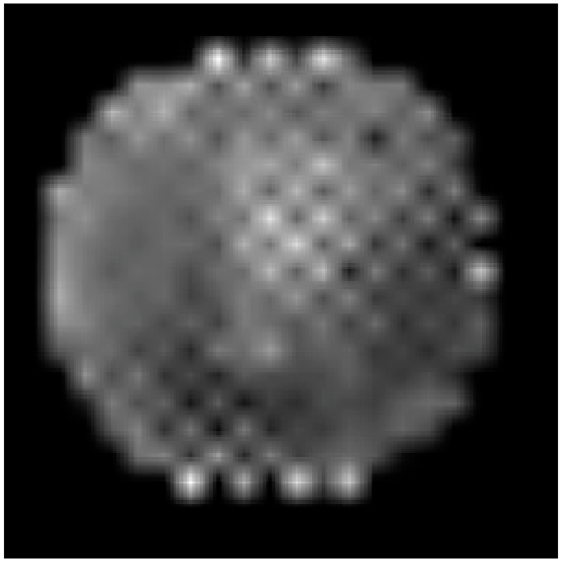
- Filter to re-align estimate
- Filter to reduce noise
- Filter to optimally correct with multiple DMs

- Can't filter to do anything spatially-variant, like de-weight subaps near the edge
- Does not work on a hexagonal grid of actuators

Local waffle removal filter works

- Design a filter to suppress local waffle

No local waffle removal

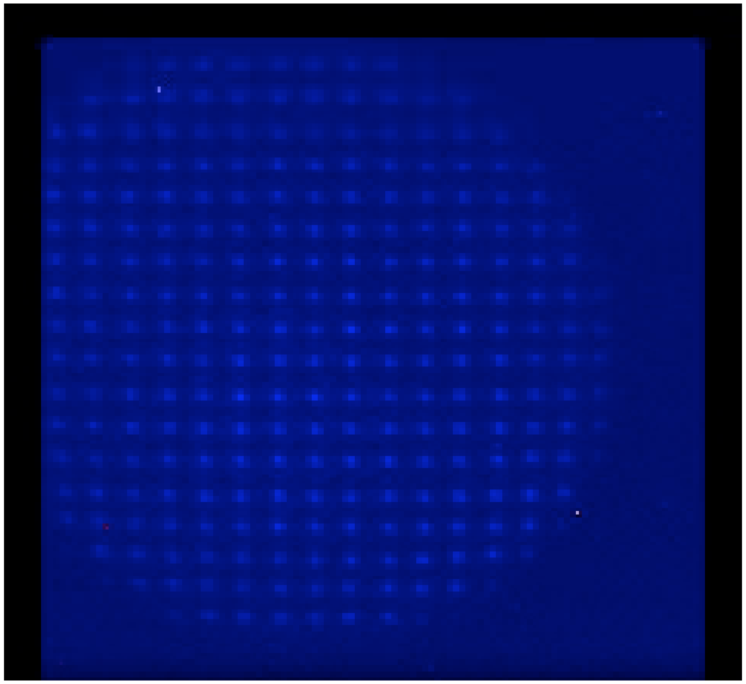


Local waffle removal

PALAO on SAO 89317: F8 m_v 10.1 star

How to deal with a changing pupil?

- The Keck pupil rotates - new control matrices are loaded in
- Vision pupil changes unpredictably
  - can re-calculate a matrix (slow)
  - FTR uses pupil mask - new mask use is $O(n)$



Data from Abdul Awwal

Where is WF Recon headed?

- Optimal Control
- Predictive Control

- And of course there's the whole MCAO/ Tomography field! (See LeLouran's talk)

Strehl-Optimal Control

- Use knowledge of WF and past reconstructions to make best estimate
  - the best estimate is the conditional mean wavefront based on all prior information
- Requires updates of covariance in closed loop
  - computationally expensive
- Simulation results for Lick show that this approach can improve Strehl compared to a fixed matrix.

ExAO may enable predictive control

- The frozen flow hypothesis: wind-blown turbulence
- If wind velocity is 20m/s, ExAO system has 20 cm subaps and 2 kHz frame rate...
  - it will take *20* time steps for phase to slide over just 1 subaperture

Conclusions

- Matrices: the established method, but care must be taken in use
- Fast new methods: a variety available, each with pluses and minuses

- Keep up to date at conferences and in journals with the latest

Papers to look up (not exhaustive!)

- FTR
  - Poyneer et al, JOSA A 19, p2100-11
  - Poyneer, SPIE 4839, p1023-1033
  - Poyneer et al, Optics Letters 28, p798-800
- Conjugate Gradient techniques
  - Gilles, Vogel & Ellerbroek, JOSA A 19, p 1817-1822
  - Gilles, Ellerbroek & Vogel, SPIE 4839, p1011-1022
  - Ellerbroek, Gilles & Vogel, SPIE 4839, p989-1000
  - Gilles, Ellerbroek and Vogel, Optics Letters (in press?)
- Local Control
  - Shi et al, SPIE 4839, p1035-1044
  - MacMartin, JOSA A 20, p1084-1093
- Matrix methods
  - Gavel, SPIE 4839, p972-980
  - Gavel & Wiberg, SPIE 4839, p890-901
  - Ellerbroek, JOSA A 19, p1803-1816
  - Ellerbroek, SPIE 5169 (in press)