Wave-front sensing

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Wave-front sensing is the first key step in Adaptive Optics

- You can't correct well if you can't measure well
- Goals for WFS:
  - high accuracy (low or no bias)
  - low noise
  - ease, low cost of implementation and use
  - robust to changes in source

Course outline

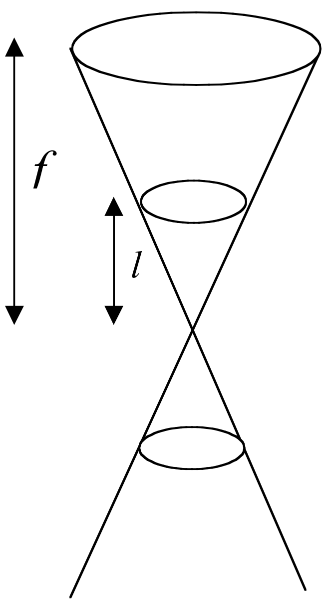
- Basic AO and WFS concepts
- Optical WFS techniques
  - Curvature, Shack-Hartmann, Pyramid
- Advanced topics:
  - aliasing
  - in-depth analysis of the centroid algorithm
  - using scenes with Shack-Hartmann sensor
  - future problems: laser GS

What are we trying to measure?

- We are trying to measure the phase aberration (usually) along the optical path of a reference.
- Commonly measure a derivative of the phase at discrete locations across the pupil.

- Other optical options:
  - measure magnitude of zernike modes
  - measure phase directly

Curvature Sensing



$$\frac{I_2 - I_1}{I_2 + I_1} \approx z\nabla^2\Phi$$

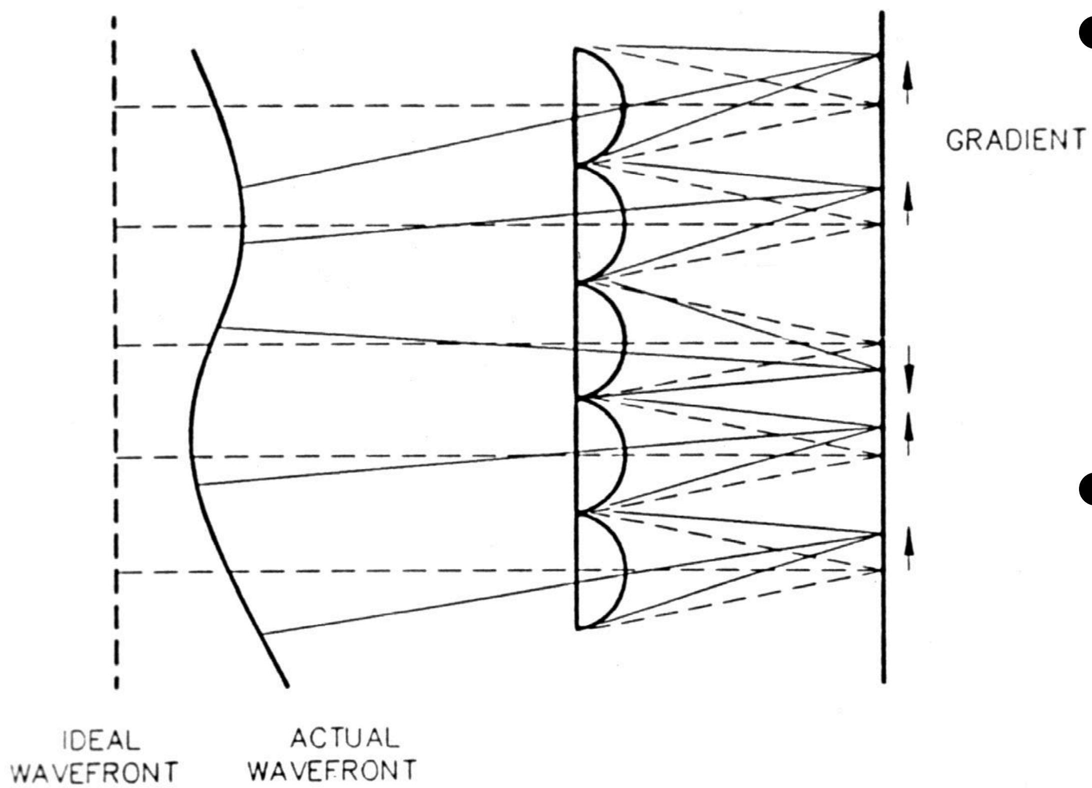
- Curvature sensing uses two defocused images in the focal plane
- Estimate the curvature with normalized difference of intensities

Figure from Marcos van Dam

More details of curvature sensing

- Amount of defocus to use depends on the phase aberration
  - can determine this for atmosphere
  - can change in closed-loop if designed correctly
- Acquisition is done in practice with time-multiplexing of a single detector to both image planes
- Requires 2 pixels per subaperture
- Usually used for low-order correction on dim stars

Shack-Hartmann sensing

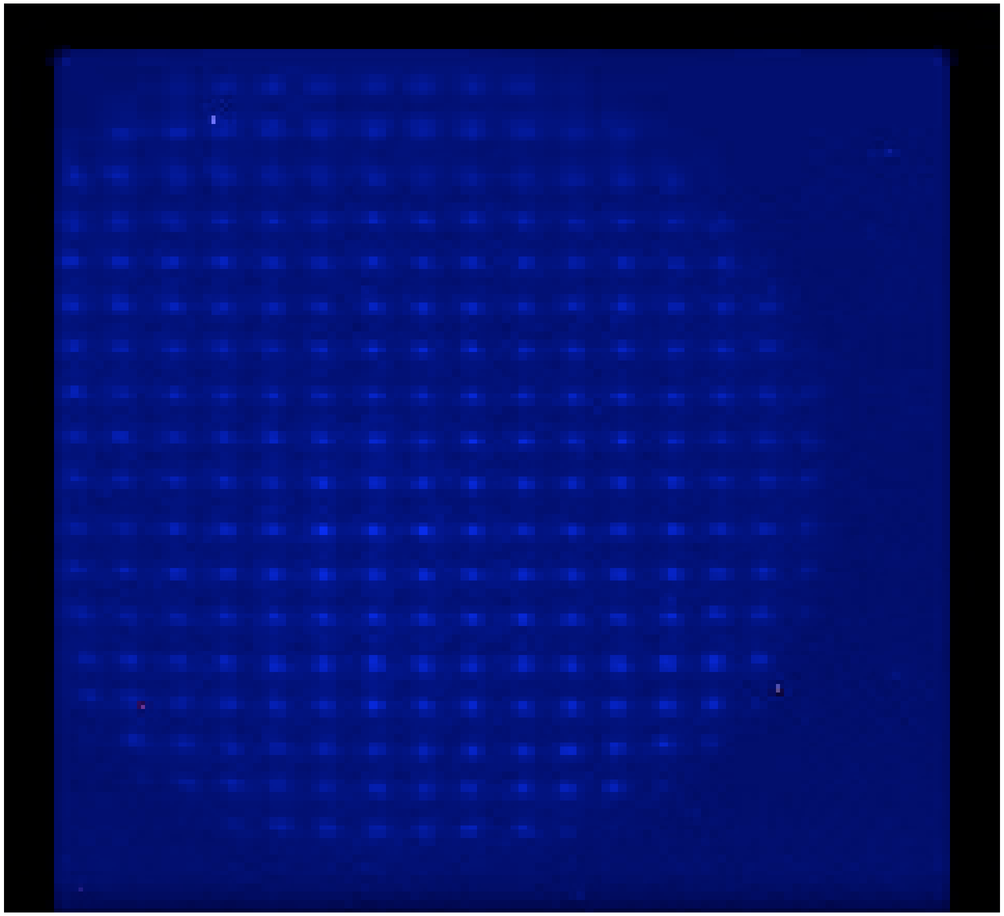


- Sensor measures the average phase gradient across the subaperture
- This is Fourier optics - the spot shifts based on linear phase term

Figure from Gary Chanan

Shack-Hartmann forms an array of spots

- Note illumination changes near edge
- Actual sampling of spots/spot size is variable and system dependent



WFS image from Abdul Awwal

More details for Shack-Hartmann

- Need to have a reference location that the spots shift away from (may not be null)
- Spots diffraction-limited by subaperture size or the seeing
- Requires at least 4 pixels per subperture

Pyramid sensing

- Pyramid sensor divides the field in the *focal plane* into 4 quadrants
- Reimage into pupil plane and compare intensities of same pixel in all subpupils

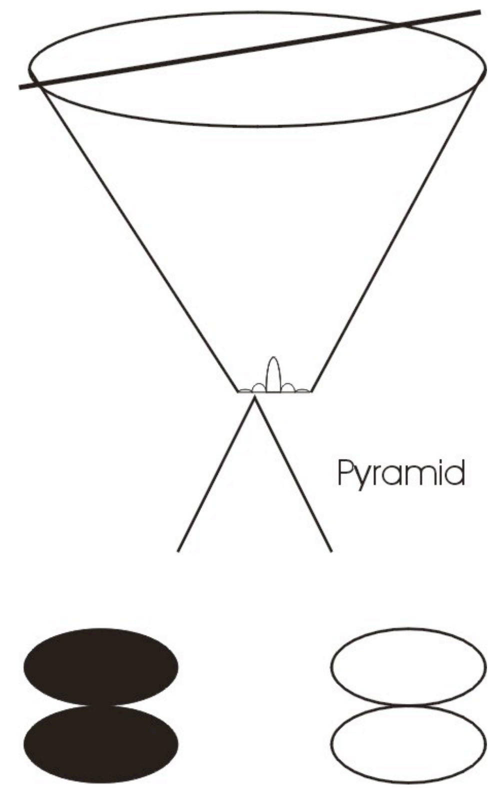
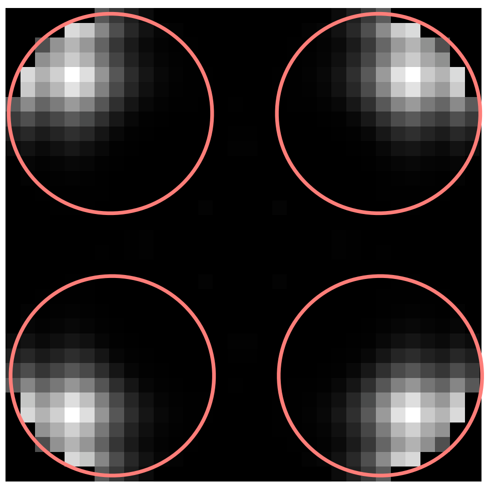


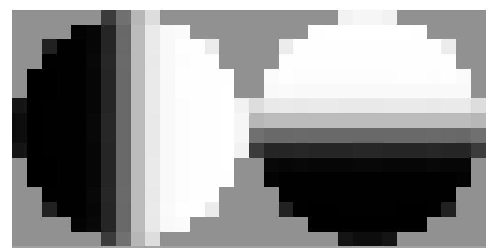
Figure from Richard Clare

Linear range of pyramid is very small

- Consider a focus phase term, with linear slope



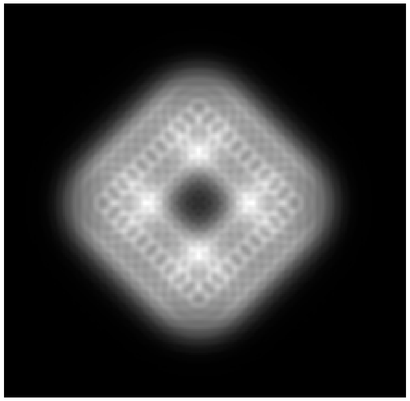
4 pupil images



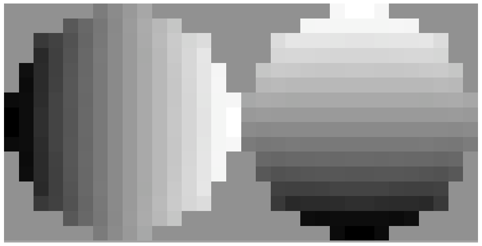
X- and y-slopes.
Note the lack of linearity

Modulation of pyramid leads to wider linear range

- Move the pyramid (or the spot) around in an optimal pattern



Modulation pattern of field intensity



X- and y-slopes. Wider linear range

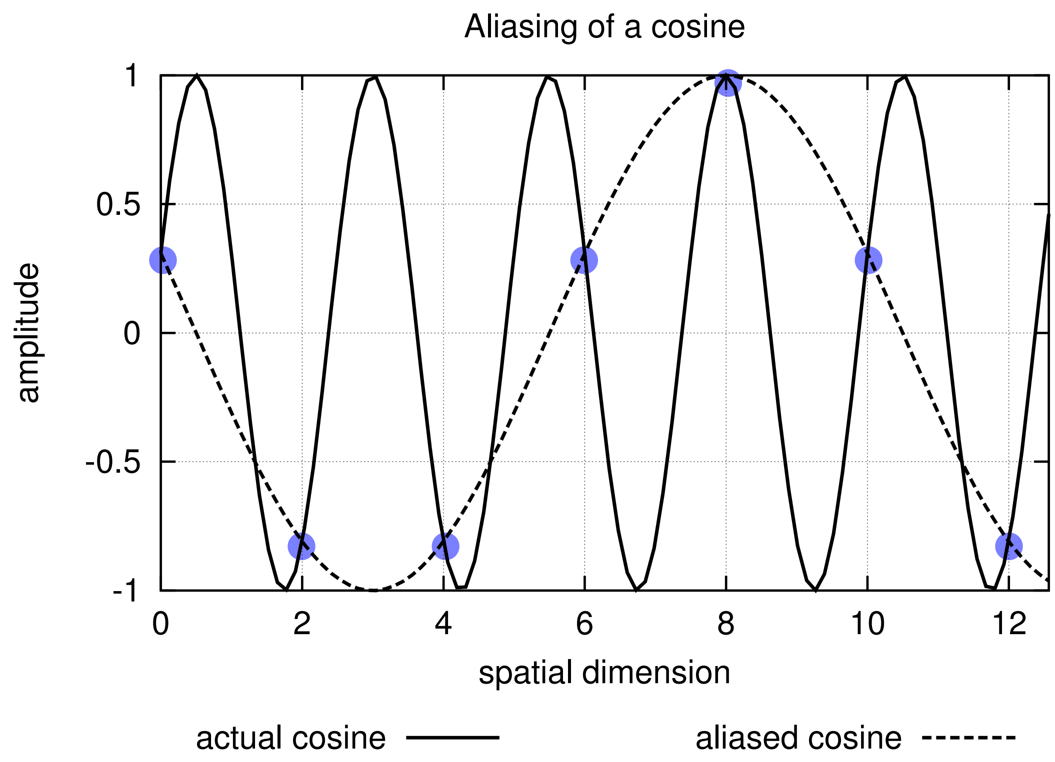
How do these compare?

- Computation:
  - All require small amount of computation on pixels
  - Slope sensors w/ DM require phase reconstruction
  - Curvature sensor w/ bimorph applies curvatures directly (in theory)
- Signal-to-noise
  - Which is better is quite contentious.... and unresolved
  - Must consider read noise of detector

Mini-break!

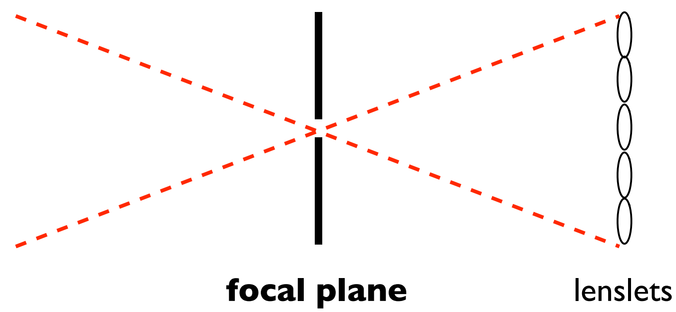
Aliasing limits AO performance by fooling the system

High-frequency errors incorrectly measured as low-frequency errors



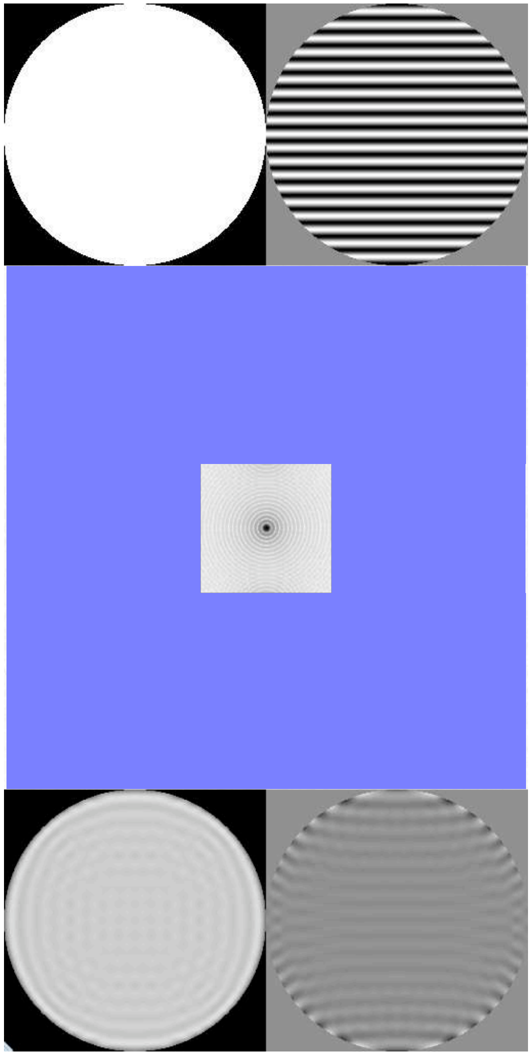
Spatially-filtered WFS implemented with a field stop

- Spatial filter is a field stop of width λ/d in focal plane
- Designed to reject phase past 1/2d in spat. freq.



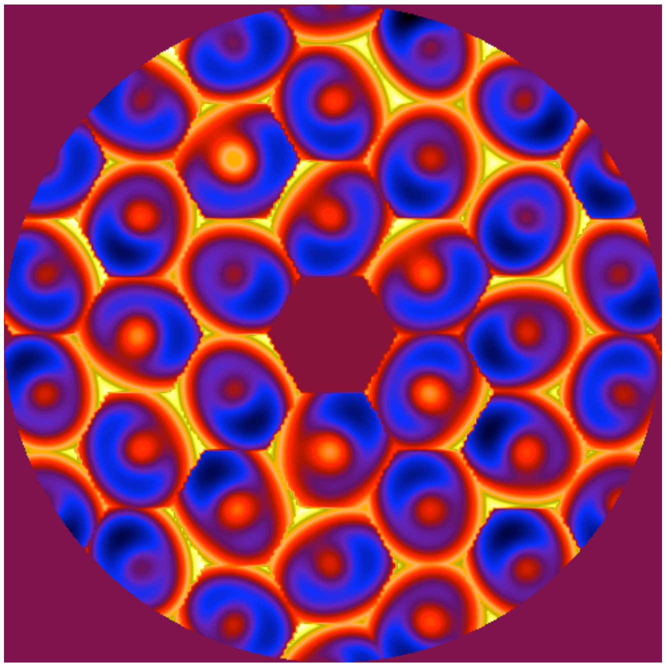
Spatially-filtered WFS suppresses high-spatial-frequency phase

- Cosine phase aberration scatters light to a specific location
- Field stop will reject this light



For discussion on PSF formation, see
Perrin et al "Structure of high Strehl..." ApJ in press

Correction of a segmented primary

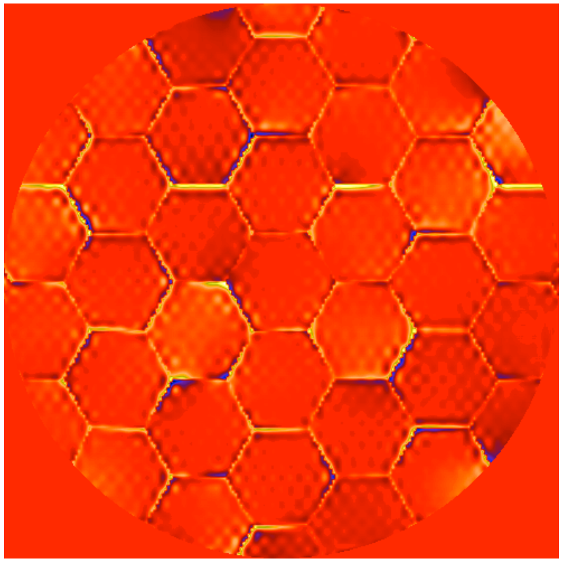


Segmented primary mirror phase error - 110 nm rms

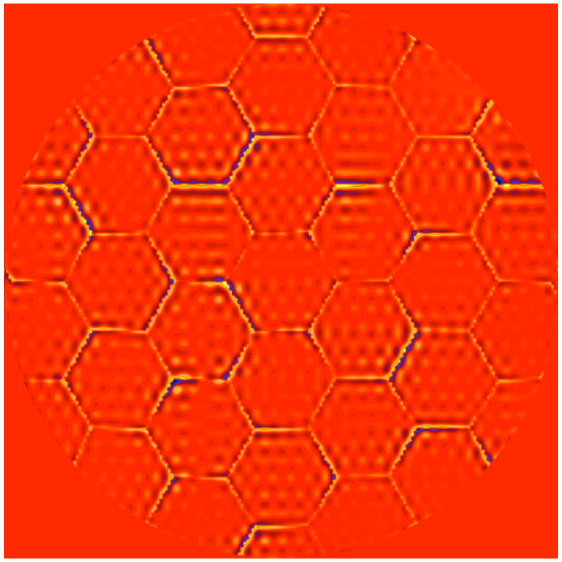
- Correct phase discontinuities between segment edges
- Model based on detailed info of Keck primary

Model details based on Chanan *Ap.Opt.* 39
personal communication with M. Troy and C. Carrano

Spatial filter cleans up segment errors



Regular AO residual
23 nm



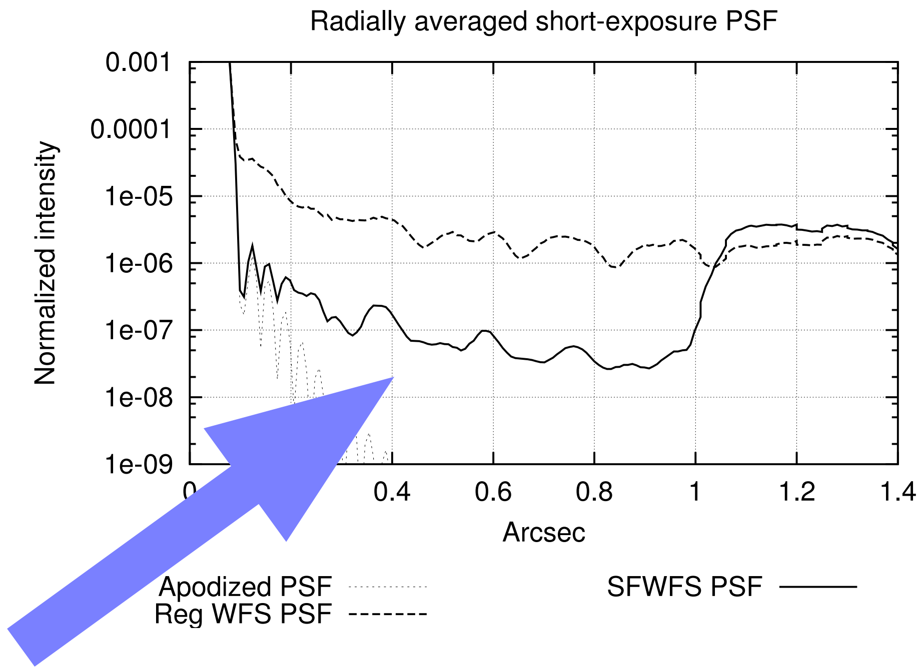
SFWFS AO residual
19 nm

This is a small difference in rms error but...

Correction of segments, ExAO noise-free simulation, D/d = 62

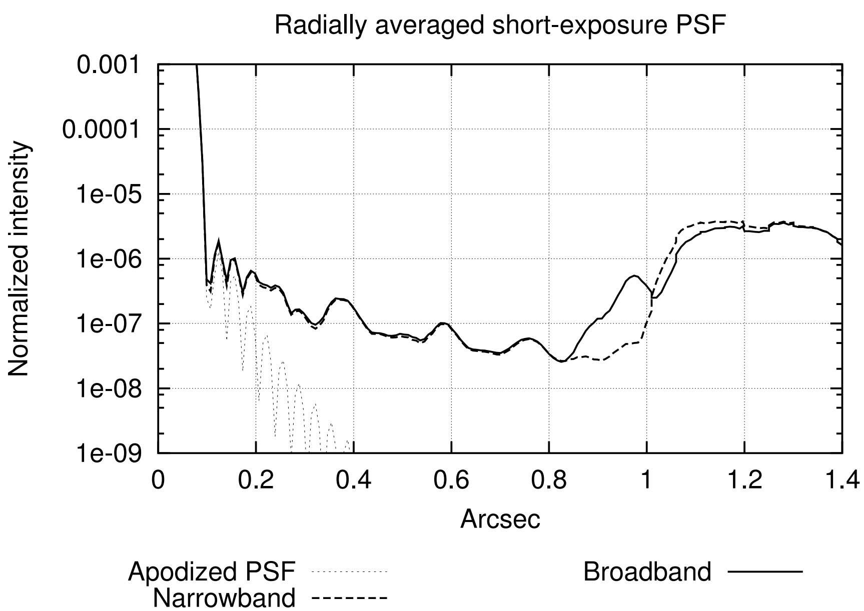
50 times reduction in PSF intensity level due to filter

- Filter produces a region out to 1 arcsec that has 50 times less light than without the filter



Correction of segments, ExAO noise-free simulation, D/d = 62

Broadband light reduces the area of improvement in PSF

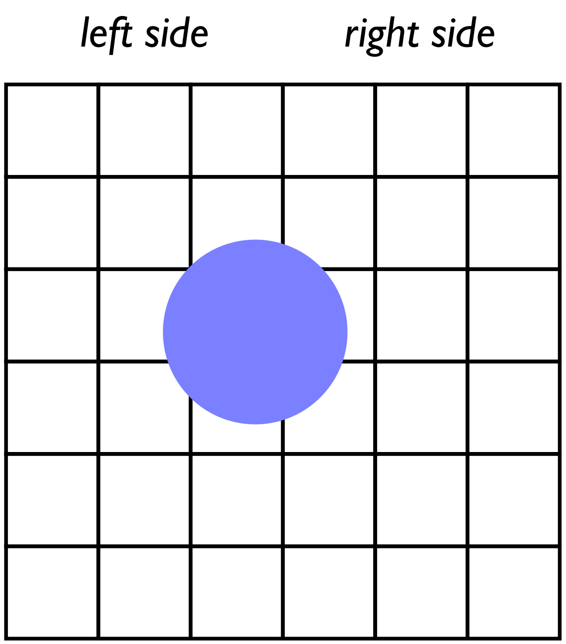


Broad-band WFS simulation used 700-900 nm light with field stop set for 800 nm

- Short λ light sees stop as too big
- Long λ light sees stop as too small
- Extra error around 1/2d due to aliasing and missing info

Shack-Hartmann generic center-of-mass formula

- The x-slope is based on the difference between linear combinations of the pixels on the right and left sides.



$$\hat{x} = \frac{r - l}{t}$$

Most CfAO systems use the Shack-Hartmann because it is inexpensive and easy to implement

For centroid on NxN pixels, performance can be analyzed

- Weight pixels by distance for center
  - NxN has center at value N/2 - 1/2

$$r = \sum_{i=N/2}^{N-1} \sum_{j=0}^{N-1} (i - N/2 + 1/2) s[i,j]$$

$$l = \sum_{i=0}^{N/2-1} \sum_{j=0}^{N-1} (-i + N/2 - 1/2) s[i,j]$$

$$t = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} s[i,j]$$

Dealing with photon and read noise

- Each pixel in the spot has an normalized expected number of counts (poisson) scaled by exposure time + white read noise

$$s[i,j] = f \times p[i,j] + n[i,j]$$

- Mean and variance are

$$m_s[i,j] = f\lambda[i,j] \qquad \sigma_s^2[i,j] = f^2\lambda[i,j] + \sigma_n^2$$

Use analysis to determine algorithm behavior

- You should fully characterize your algorithm
  - how does noise propagate, both photon and read?
  - what happens if the spot size/shape changes?
  - do you need background subtraction?

Break up into two independent terms: photon and read noise

- Error due to photons is inversely related to SNR

$$\mathrm{Var}(\hat{x})_p = \frac{\sigma_r^2 + \sigma_l^2}{f m_t^2}$$

RMS error is inverse power law in SNR

- Error variance due to read noise depends on the *square* of the total number of pixels

$$\mathrm{Var}(\hat{x})_n = \sigma_n^2 \left(\frac{N^2(N^2-1)}{12 m_t^2} \right)$$

Background light reduces gain of centroid estimate

- With no background, the expected slope is

$$\mathrm{E}[\hat{x}] = \frac{m_r - m_l}{m_t}$$

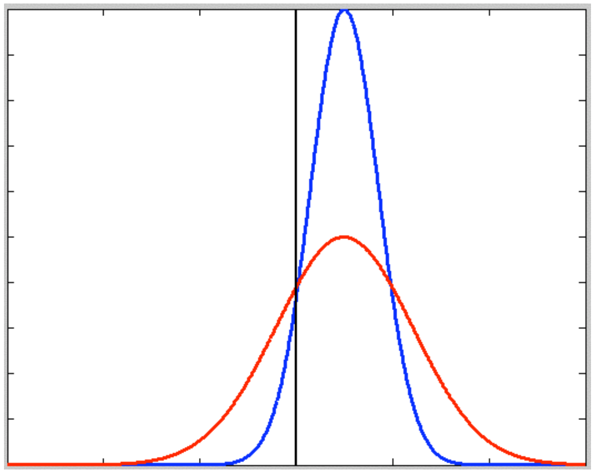
- With poisson background (mean b) added in, expected slope is now

$$\mathrm{E}[\hat{x}] = \frac{m_r - m_l}{m_t + N^2 b}$$

Incorrect background subtraction, even by a few counts, can lead to significant loses in gain and increased error

The quad-cell method suffers from gain changes with spot size

- Quad-cell is 2x2 centroid
- As spot size changes, so does gain
- Lost light off edge of pixel also biases estimate

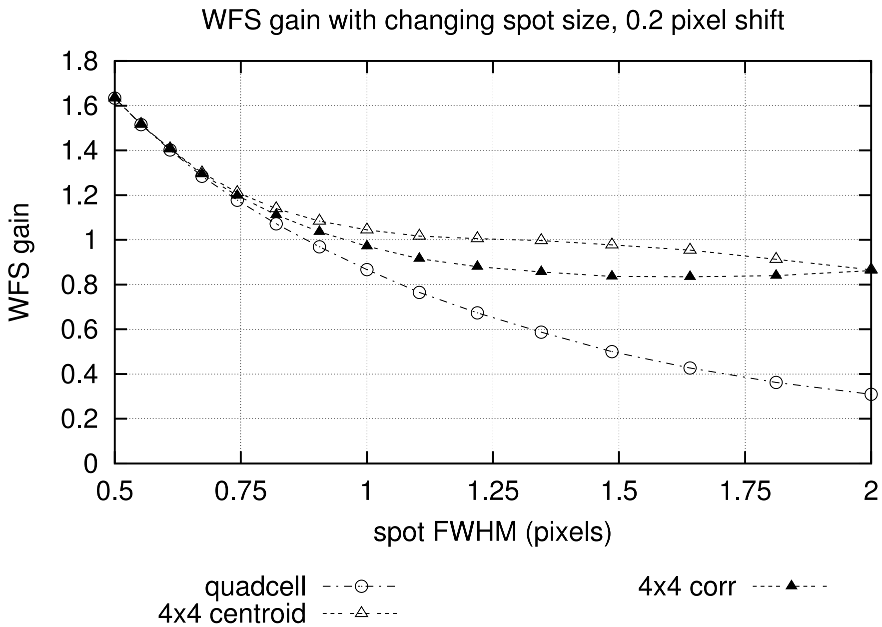


Est is 0.28

Est is 0.14

Figure from Marcos van Dam

Using 4x4 pixels ameliorates the gain problem



- 4x4 centroid has a more regular response with spot size than quad-cell.

Mini-break!

What are other algorithmic options?

- If you have very accurate knowledge of the spot profile, you can do a maximum likelihood estimate
- Can correlate the spot against a reference
  - much lower noise propagation
  - expected estimate is insensitive to background levels
  - requires more computation

Correlation has lower read noise propagation

- Error variance due to noise does not depend of number of pixels

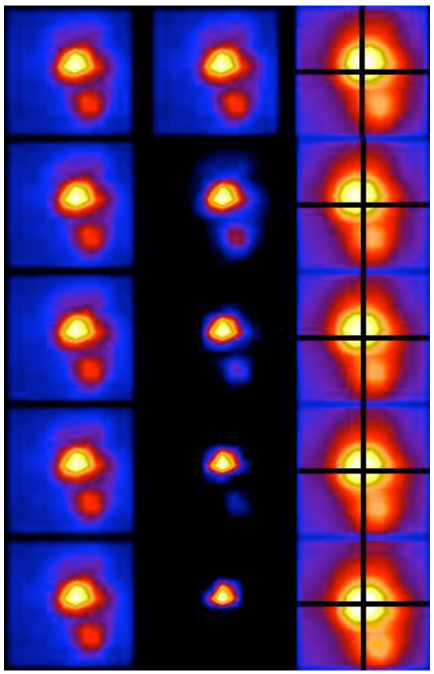
$$\mathrm{Var}(\hat{x})_n \propto \frac{\sigma_n^2}{8(m_0 - m_1)^2}$$

- Data from Vision AO system show this clearly in comparison to centroiding

	Noise prop	σ_x	σ_y
Correlation	1.58×10^{-5}	3.8×10^{-3}	4.23×10^{-3}
Centroider	212×10^{-5}	19.6×10^{-3}	20.9×10^{-3}

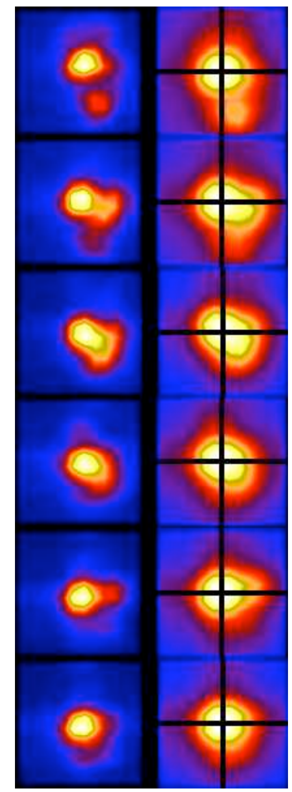
WFS CCD data provided by Abdul Awwal

Correlation works with spot deformities



Centroider (w/ thresholding)

- Just like background, spot features throw off the centroider
- Correlation (with a spot reference) is insensitive to these features



Correlation

WFS data provided by Kai LaFortune

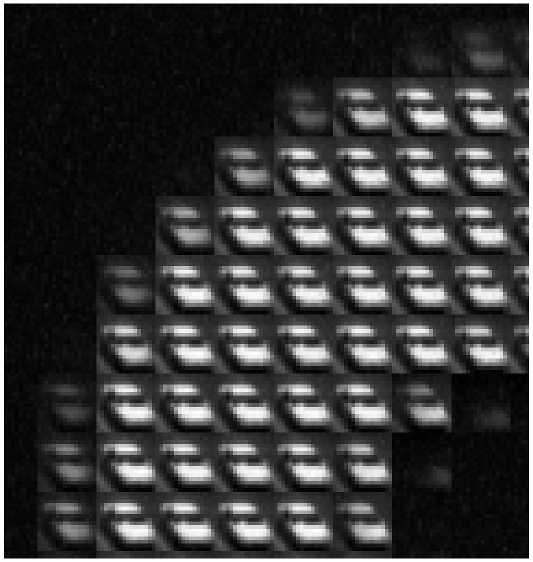
Summary: algorithm options

- What do you know about the point source?
  - how well do you know the spot profile?
  - do the spots deform or change size?
- What are your performance constraints?
  - are you computationally limited?
  - how much noise can you tolerate?

- The choice of best algorithm depends on your system

Scene-based wave-front sensing uses the scene, not a point source

WFS camera image of one quadrant of subapertures



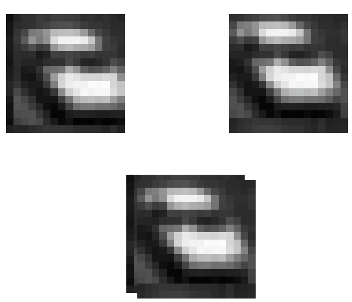
Camera image from C. Thompson and R. Sawvel

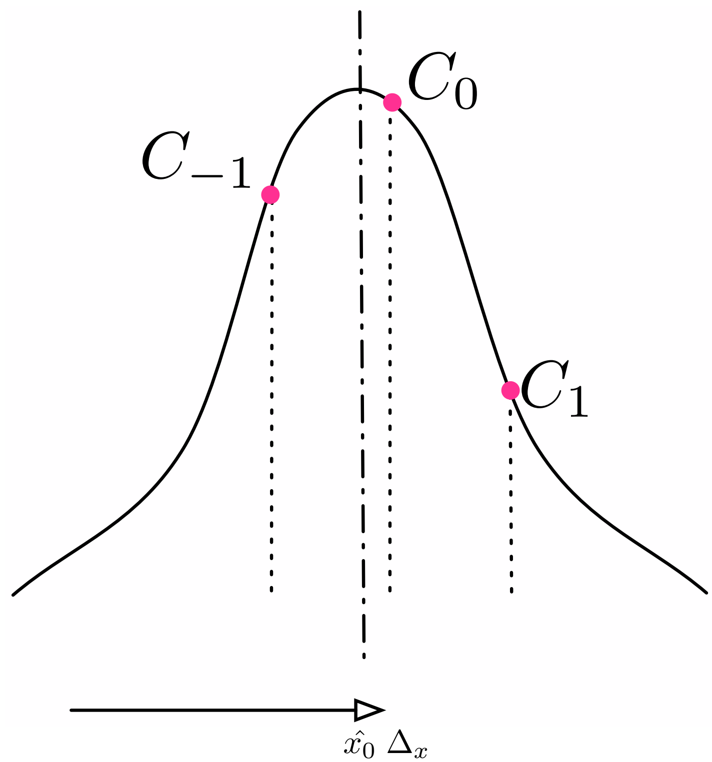
- Subimage of scene will shift just like a point source
- Need to field stop down

Estimate shift with a correlation-based algorithm

$$\hat{x_0} = \Delta_x + \frac{0.5(C_{-1} - C_1)}{C_{-1} + C_1 - 2C_0}$$

- Compute correlation to find minimum MSE between images





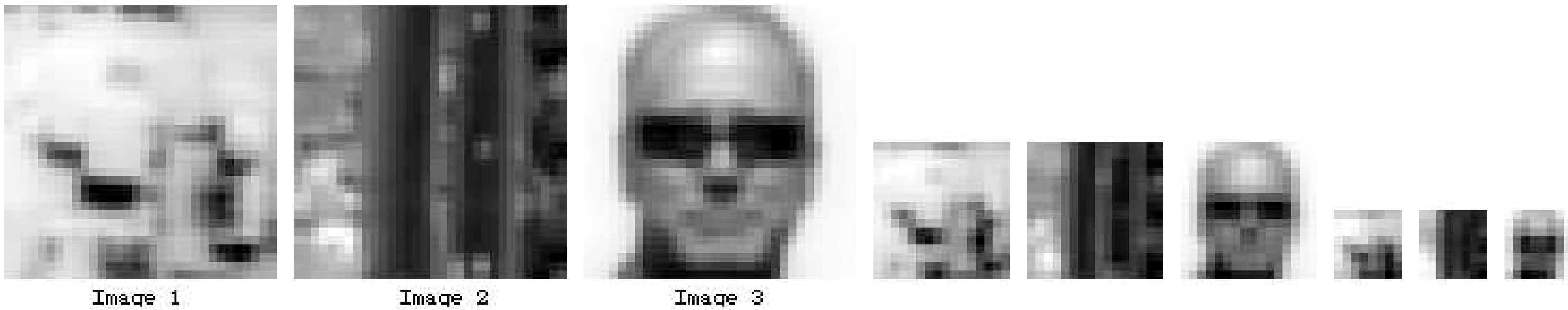
Performance can be rigorously analyzed

- Probabilistic analysis of scene quality based on image content
- In the zero-shift (closed-loop case) the error variance is

$$\sigma_x^2 = \frac{\sigma_1^2 - \sigma_{-1,1}^2}{8(m_0 - m_1)^2}$$

sharpness of autocorrelation

Scene performance depends on frequency content



$\sigma_x = 0.013$ $\sigma_y = 0.011$

$\sigma_x = 0.009$ $\sigma_y = 0.019$

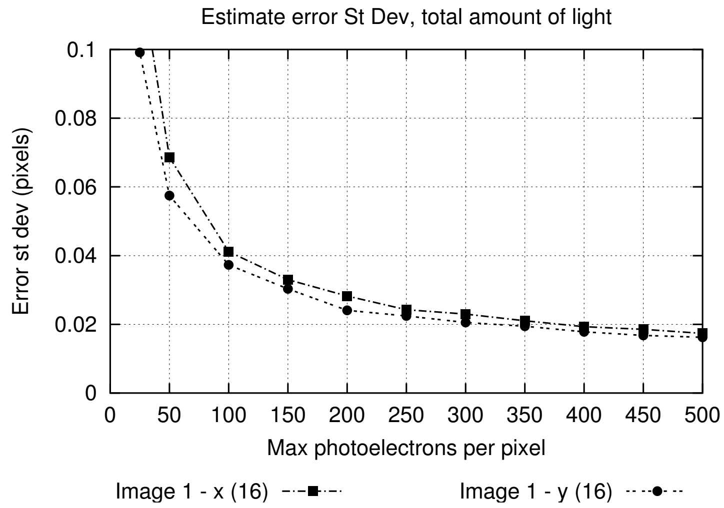
$\sigma_x = 0.013$ $\sigma_y = 0.008$

$\sigma_y = 0.087$

- Performance varies along x- or y-axis
- Need to find best field of view for expected scene scales

Scaling laws - exposure time

- In the case of changing exposure time, the error st. dev. follows as inverse power law in SNR

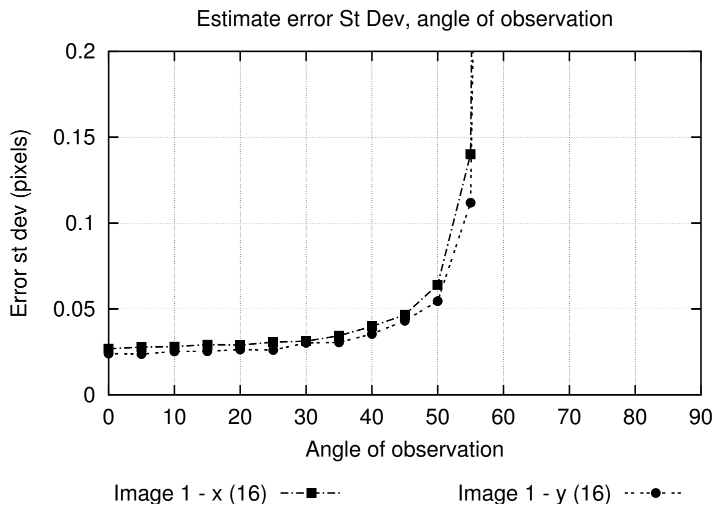


Same inverse power law in SNR as point sources

$$\sigma_x(f) = \frac{1}{\sqrt{f}} \frac{(\tilde{\sigma}_1^2 - \tilde{m}_1 - \tilde{\sigma}_{-1,1}^2)^{1/2}}{2\sqrt{2}(\tilde{m}_0 - \tilde{m}_1)}$$

Scaling laws - angle of observation

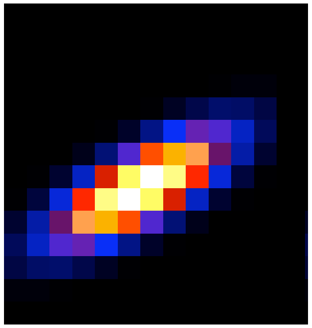
- Generate radiometric models for background levels
- Performance falls off with too much background



$$\sigma_x(b,f) = \frac{\left\{Nb^2 + 2bf^2(\tilde{m}_0 - \tilde{m}_2 + \tilde{t}f^{-1}) + f^3[\tilde{\sigma}_1^2 - (f-1)f^{-1}\tilde{m}_1 - \tilde{\sigma}_{-1,1}^2]\right\}^{1/2}}{2\sqrt{2}f^2(\tilde{m}_0 - \tilde{m}_1)}$$

Future problems: Laser Guide Stars

- The LGS ‘spot’ is elongated for off-axis suabpertures
- This elongation poses significant problems
  - how many pixels are necessary?
  - what is the best algorithm for slope estimation?
    - noise propagation concerns
    - bias and gain concerns



Sample 3x elongated spot

Papers to look up (not exhaustive)

- Shack-Hartmann WFS
  - van Dam & Lane, JOSA A 17, p1319-1324
  - Tyler & Fried, JOSA A 72, p804-808
  - Veran & Herriot, JOSA A 17, p1430-1439
  - dePater et al, Icarus 160, p359-374
- Solar AO
  - Rimmele et al, SPIE 4007, p218-231
  - Rimmele et al, SPIE 1542, p186-193
- Correlation/Scene-based WFS
  - Poyneer, Applied Optics in press
  - Poyneer, SPIE 5162 in press
- Spatial-filtering
  - Poyneer & Macintosh, submitted to JOSA A
  - Poyneer & Macintosh, SPIE 5169 in press
- Pyramids
  - Clare and Lane, SPIE 5169 in press
  - and many others...