Simulation of Adaptive Optics Systems

Marcos van Dam

CfAO

W.M. Keck Observatory

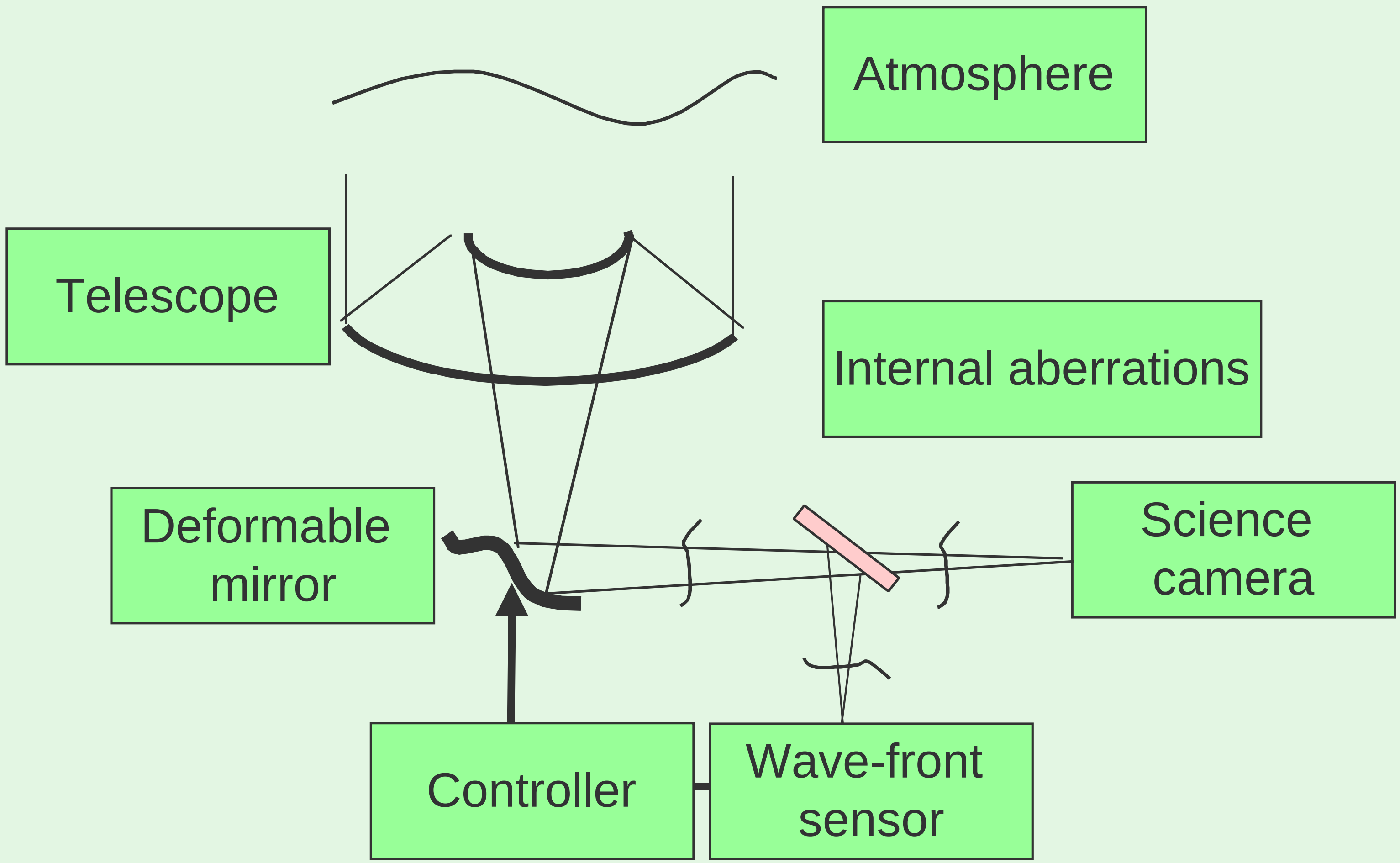
Why simulate?

- You don’t have an AO system but want to do AO research.
- Analyze performance of existing AO systems.
- Predict the performance of different algorithms and components.
- Explore parameter space when designing new systems.

How much does it cost?

- Single ground layer simulations require a desktop computer.
- Multi-conjugate adaptive optics with multiple guide stars and Fresnel propagation require a supercomputer.

Adaptive Optics System



Introduction

- Atmosphere
- Telescope
- Imaging camera
- Tip/tilt and deformable mirrors
- Shack-Hartmann, curvature and pyramid sensors.
- Modeling and simulating dynamic behavior
- Corrected and uncorrected non-common-path aberrations.

Atmosphere

- Phase screens
  - Power spectral density methods
  - Covariance methods
- Time evolution: frozen flow
- Propagation to the telescope.

Phase screens: spatial power spectrum

- Use power spectral density, Φ, of Kolmogorov turbulence:
  - Easily extendible to other PSDs (e.g. von Karman spectrum).

$$\Phi_{\phi}(\kappa) = 0.023 r_0^{-5/3} \kappa^{-11/3}$$

- Generate complex independent, Gaussian, random numbers with zero mean and unit variance.
- Multiply by the square root of the PSD.
- Set PSD=0 at κ=0 (*i.e.*, set piston to 0).
- Take the discrete Fourier transform.
- The real component is a Kolmogorov phase screen.

(McGlamery)

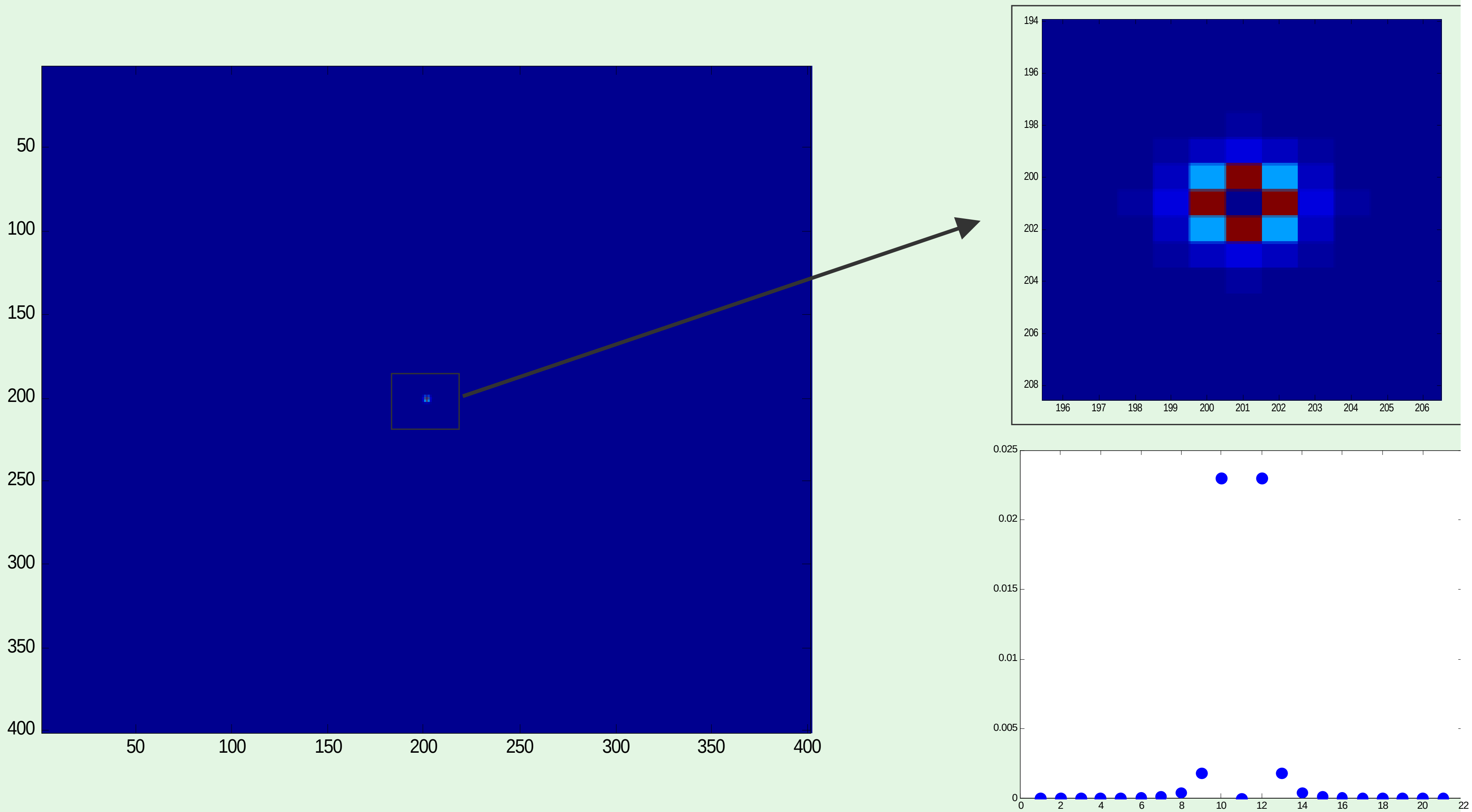
Phase screens: spatial power spectrum

Matlab code

```
sz=200; % size
% generate the power spectral density values
cx=(-sz:sz);
mx=(ones(2*sz+1,1)*cx).^2;
mr=sqrt(mx+transpose(mx));
psd=0.023*mr.^(-11/3);
psd(sz+1,sz+1)=0;
% generate the random numbers with Gaussian statistics
randomcoeffs=randn(2*sz+1)+i*randn(2*sz+1);
% phase screen!
phasescreen=real(fft2(fftshift(sqrt(psd).*randomcoeffs)));
```

Phase screens: spatial power spectrum

$$\Phi(\kappa) = 0.023 r_0^{-5/3} \kappa^{-11/3}$$

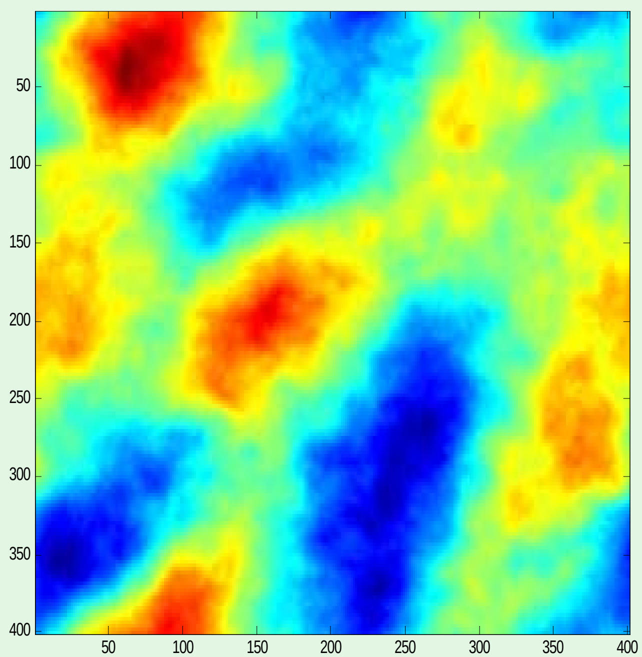


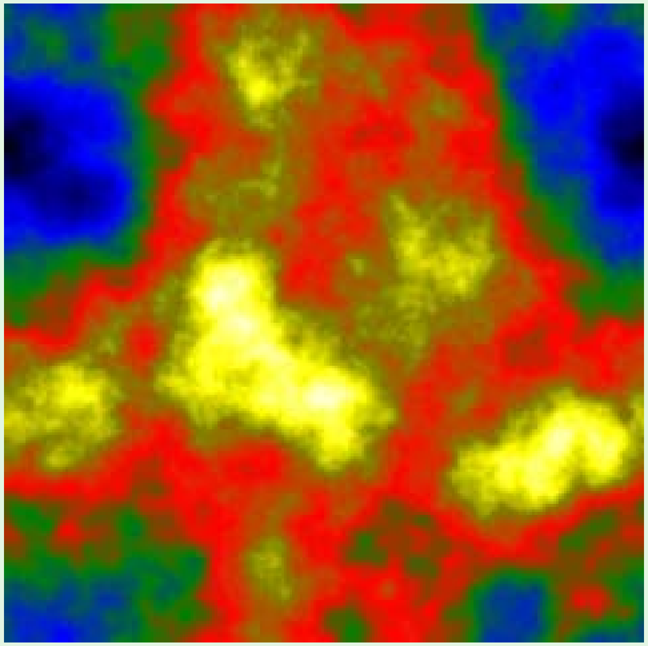
Phase screens: spatial power spectrum

- Problems
  - Phase screen periodic because discrete FT is periodic.
  - Low spatial frequencies are inadequately sampled
- Solutions
  - Use a small region of the screen OR
  - Add low order subharmonics (Johanssen and Gavel).

Phase screens: spatial power spectrum

- Advantages
  - Very fast
  - Extendible to any other spatial power spectrum
  - Phase screens are periodic because discrete FT is periodic





Phase screens: Covariance methods

- Parameterize the continuous phase into orthogonal basis functions.
  - Phase at points in a grid (Wallner, Lane et al, Harding et al)
  - Average phase over pixels (van Dam and Lane)
  - Zernike polynomials (N. Roddier)
  - Karhunen-Loève functions
- Obtain the covariance matrix of the coefficients.

Phase screens: Covariance methods

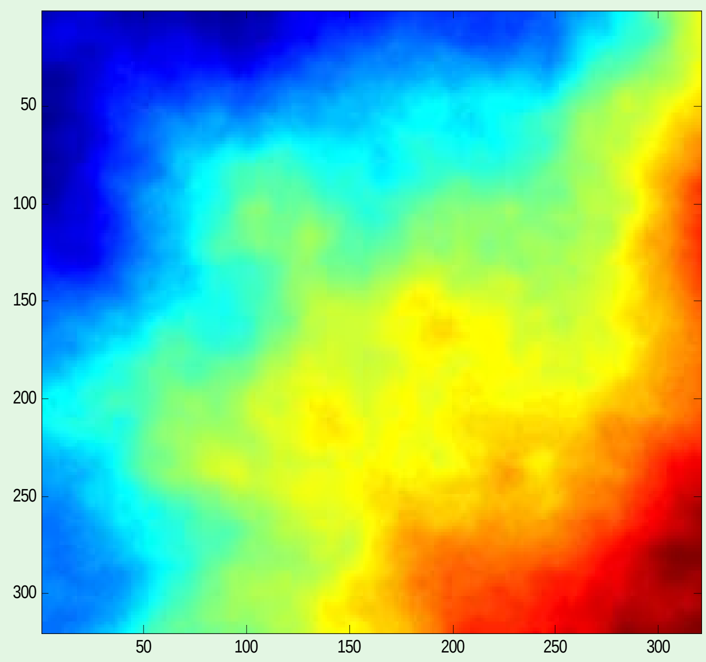
- e.g., Zernike polynomials (Noll)

	Z_2	Z_3	Z_4	Z_5	Z_6	Z_7	Z_8	Z_9	Z_{10}
Z_2	0.448						-0.0141		
Z_3		0.448				-0.0141			
Z_4			0.0232						
Z_5				0.0232					
Z_6					0.0232				
Z_7		-0.0141				0.0618			
Z_8	-0.0141						0.0618		
Z_9								0.0618	
Z_{10}									0.0618

Covariance of Zernike coefficients

Phase screens: Covariance methods

- Generate random Gaussian numbers with the right covariance to obtain the coefficients.
- Multiply the basis function by the random coefficients.
- Generates phase screens with exact statistics.
- Computing large covariance matrices is difficult.



General points to note

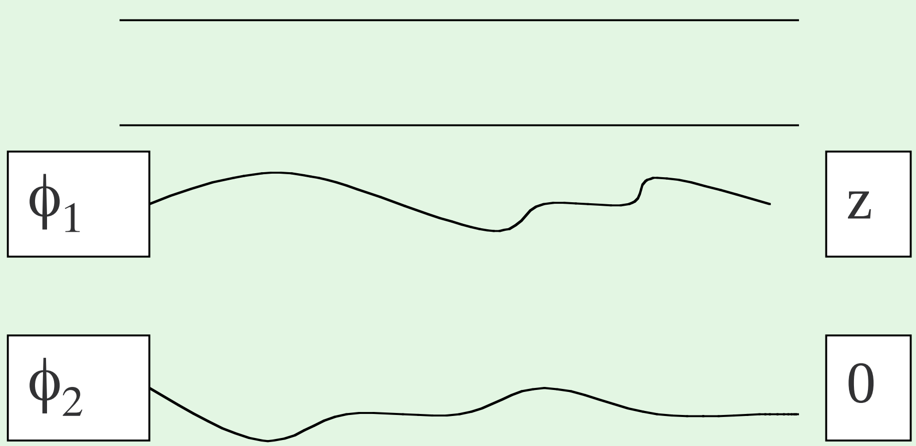
- Kolmogorov turbulence is self-similar. Phase screens are scaled by multiplying the phase by $(D/r_0)^{5/6}$.
- The phase is converted to a wavefront using $w = \phi\lambda/2\pi$.
- To compute discrete FTs, use the Fast Fourier Transform (FFT) which requires the number of points to be a power of two (e.g., 256, 1024).

Propagation through the atmosphere

- Model turbulence as consisting of discrete layers at different heights between 0 and 20 km.
- In between the layers, there is no turbulence.
- There are several models for how many layers there are and what their height, wind speed and turbulence strength is.
- The model is site dependent.

Propagation through the atmosphere

- The complex amplitude at height z_+, $u(z_+)$, is 1.
- The complex amplitude at height z_-, $u(z_-)$, is $\exp[i\phi_1]$.
- The light propagates from one layer to the next.
- The phase of the turbulence at layer 2 is added: $u(0_+)=u(0_-)\exp[i\phi_2]$.
- How are the complex amplitudes at $u(z_-)$ and $u(0_+)$ related?



Fresnel diffraction

- The complex amplitude at distance z is related to the complex amplitude at 0 by:

$$u(\mathbf{x},0) \propto F[u(\mathbf{x},z)\exp[ik\mathbf{x}^2/(2z)]]$$

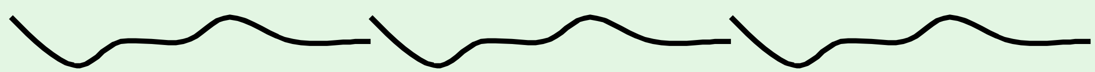
- If z is a few kilometers, this is very difficult to compute because the chirp varies too quickly. Instead, we can the fact that the FT of a chirp is another chirp to give:

$$u(\mathbf{x},0) \propto F^{-1}[F[u(\mathbf{x},z)]\exp[-iz\mathbf{x}^2/2k]]]$$

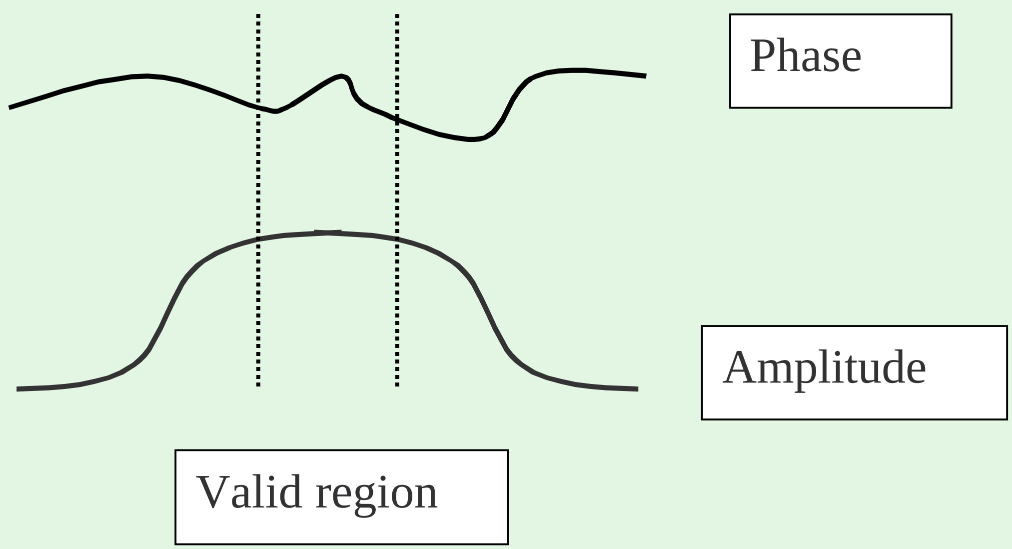
Chirp=exp[iαx^2]

Fresnel diffraction

- The discrete FT assumes periodicity.

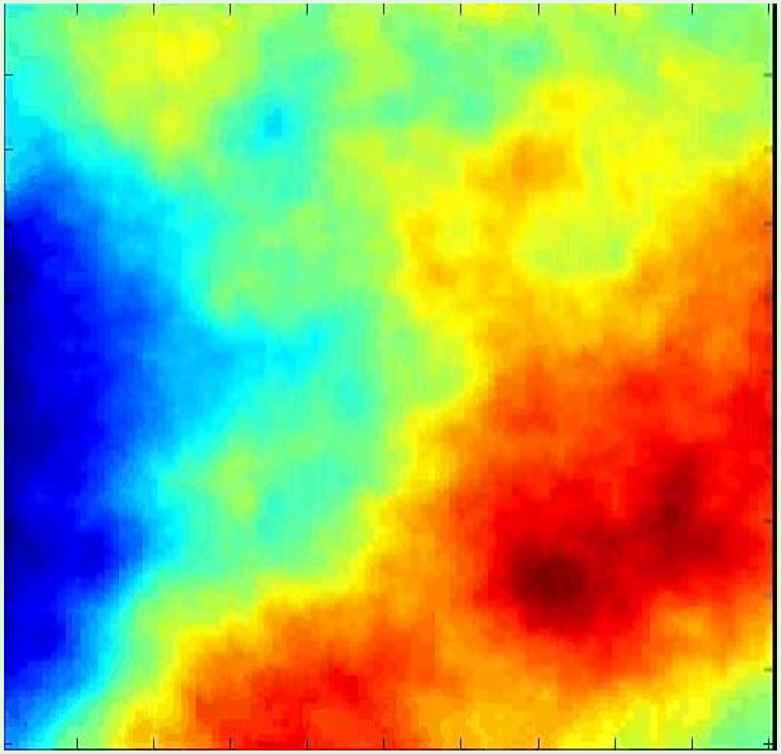


- Use a periodic phase screen (but won't have exact statistics) OR
- Window the complex amplitude:



Temporal evolution

- The atmosphere is “frozen” (Taylor hypothesis).
- Each layer of turbulence is blown by wind (a typical velocity is 10 m/s).
- If using a periodic phase screen, can wrap the screen around again and again.



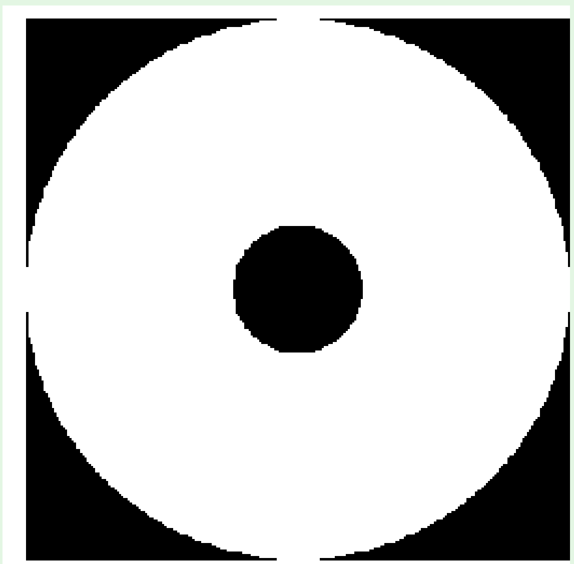
Summary of turbulence

- Have a small number of discrete turbulence layers at different heights.
- Each layer is moving with its own velocity and direction.
- The propagation between each layer is performed using Fresnel diffraction.
- The complex amplitude at the focal plane is

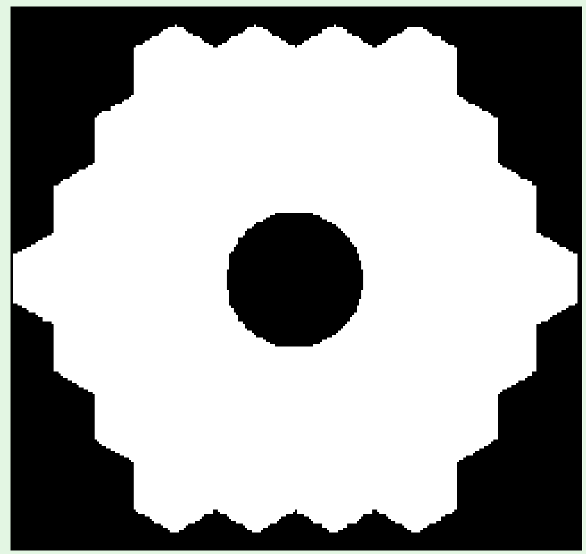
  $u(x) = a(x)\exp[i\phi(x)]$ where a(x) is the scintillation.
- Aberrations and corrective elements downstream add to the phase but do not affect a(x).

Telescope

- The telescope defines the entrance pupil.
- Multiply the complex amplitude by the pupil.



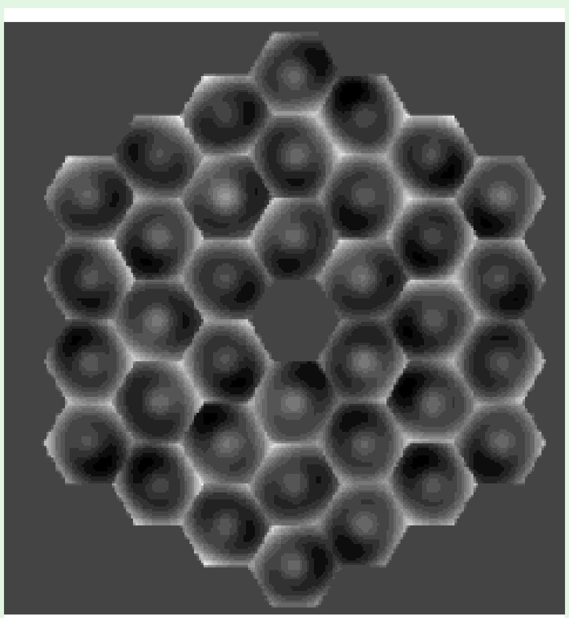
Circular primary mirror with secondary mirror obscuration



Keck Observatory entrance pupil

Telescope

- The telescope introduces vibrations.
- Mainly tip/tilt, but more complicated modes exist if the primary mirror is segmented.
- The surface of the primary mirror introduces common-path aberrations.
- These aberrations are sensed by both the WFS and the science camera, just like turbulence.



Wavefront aberrations on the Keck primary mirror.

Source: Lisa Poyneer

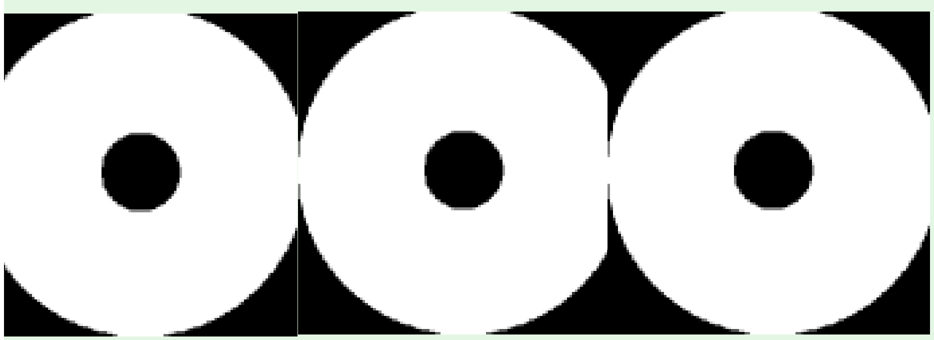
Imaging camera

- The Strehl ratio can be calculated via the Marechal approximation: $S=\exp[-\phi^2]$. Hence, the root-mean-squared (RMS) WF error defines the image quality.

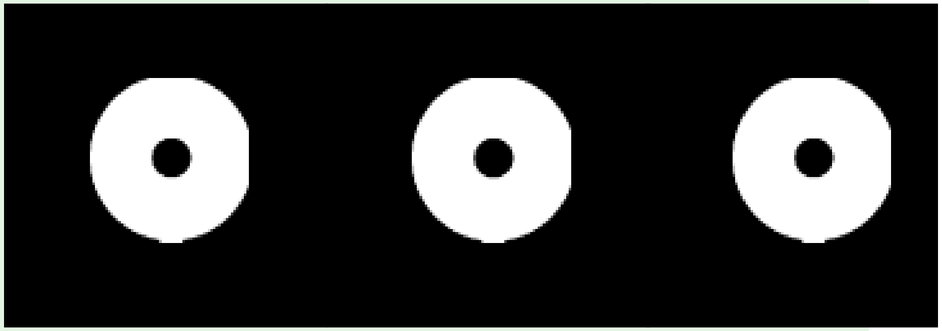
- The complex amplitude at the focal plane is a scaled FT of the complex amplitude at the pupil. Hence the intensity is given by $I(\xi) = \left| \int u(x) \exp\left[-i \frac{2\pi}{\lambda f} x\xi \right] dx \right|^2$

Imaging camera

- The numbers of pixels in pupil and focal planes are equal.
- The finer the spacing in the pupil, the wider the extent of the image.
- The complex amplitude at the pupil plane must be zero padded to mitigate the effect of FT periodicity.



No zero padding



With zero padding

WF Correction by parameterization

- One can parameterize the wave-front.
  - e.g., Zernikes, points at the corner (Fried) or in the middle of subapertures (Southwell).
  - Less computationally intensive, but neglects fitting error.