Aberration Theory

Geunyoung Yoon, Ph.D.
Assistant Professor
Department of Ophthalmology
Center for Visual Science
University of Rochester
Optics

- Quantum Optics
- Coherent Optics
- Diffractive Optics (Fourier Optics)
- Geometrical Optics (Aberration theory)
- Paraxial Optics (First Order Optics) (Gaussian Optics)
Outline

- What Is Wavefront?
  Huygens’s principle, Snell’s law, Fermat’s principle
  Paraxial (first order) approximation

- What Kind Of Wavefront Aberrations Are There?
  Monochromatic aberration (Seidel and wave aberrations)
  Chromatic aberration (Longitudinal, Transverse)

- Why Are These Aberrations Important?
  Relationship between aberrations and image quality
  (Pupil function, PSF, MTF, Image convolution…)

- How Can We Measure These Aberrations Of The Eye?
  Different types of wavefront sensors
Outline

- **What Is Wavefront?**
  Huygens’s principle, Snell’s law, Fermat’s principle
  Paraxial (first order) approximation

- **What Kind Of Wavefront Aberrations Are There?**
  Monochromatic aberration (Seidel and wave aberrations)
  Chromatic aberration (Longitudinal, Transverse)

- **Why Are These Aberrations Important?**
  Relationship between aberrations and image quality
  (Pupil function, PSF, MTF, Image convolution…)

- **How Can We Measure These Aberrations Of The Eye?**
  Different types of wavefront sensors
Outline

What Is Wavefront?
- Huygens’s principle, Snell’s law, Fermat’s principle
- Paraxial (first order) approximation

What Kind Of Wavefront Aberrations Are There?
- Monochromatic aberration (Seidel and wave aberrations)
- Chromatic aberration (Longitudinal, Transverse)

Why Are These Aberrations Important?
- Relationship between aberrations and image quality
  (Pupil function, PSF, MTF, Image convolution…)

How Can We Measure These Aberrations Of The Eye?
- Different types of wavefront sensors
Outline

➢ What Is Wavefront?
  Huygens’s principle, Snell’s law, Fermat’s principle
  Paraxial (first order) approximation

➢ What Kind Of Wavefront Aberrations Are There?
  Monochromatic aberration (Seidel and wave aberrations)
  Chromatic aberration (Longitudinal, Transverse)

➢ Why Are These Aberrations Important?
  Relationship between aberrations and image quality
  (Pupil function, PSF, MTF, Image convolution…)

➢ How Can We Measure These Aberrations Of The Eye?
  Different types of wavefront sensors
What Is Wavefront?

Huygens’s principle
Snell’s law
Fermat’s principle
Paraxial (first order) approximation
Wavefront vs Ray

“A wavefront is a surface over which an optical disturbance has a constant phase.”

Harmonic wave function

$$\psi(x, t) = A \sin(kx - \omega t)$$

- Plane wavefront
- Spherical wavefront
Wavefront vs Ray

“Rays are lines normal to the wavefronts at every point of intersection.”
Huygens’s Principle

“Every point on a primary wavefront serves as the source of spherical secondary wavelets, such that the primary wavefront at some later time is the envelope of these wavelets.”
Snell’s law

\[
\frac{n_i}{n_t} = \frac{\sin(\theta_t)}{\sin(\theta_i)}
\]

Reflection: \( \theta_i = \theta_r \)

Refraction: \( \theta_i > \theta_t \) when \( n_i < n_t \)
Fermat’s Principle

“The path actually taken by light in going from some point S to a point P is the shortest optical path length (OPL).”

\[ OPL = n_i \cdot \overline{SO} + n_t \cdot \overline{OP} \]
\[ = n_i \cdot \sqrt{h^2 + x^2} + n_t \cdot \sqrt{b^2 + (a-x)^2} \]

\[ \frac{dOPL}{dx} = 0 \text{ to minimize } OPL \]

\[ n_i \cdot \frac{x}{\sqrt{h^2 + x^2}} + n_t \cdot \frac{-(a-x)}{\sqrt{b^2 + (a-x)^2}} = 0 \]

\[ \frac{n_i}{n_t} = \frac{\sin(\theta_t)}{\sin(\theta_i)} \]
Paraxial Optics (First order optics)

\[ \frac{n_1 R (s_o + R) \sin \theta}{p_o} = \frac{n_2 R (s_i - R) \sin \theta}{p_i} \]

Approximation
\[ \sin \theta \approx \theta \]

\[ \frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} \]

Lens maker’s formula
Paraxial Optics (First order optics)

“The emerging wavefront segment corresponding to these paraxial rays is essentially spherical and will form a ”perfect” image at its center P.”
Third Order Optics

“The paraxial approximation, \( \sin \theta \approx \theta \), is somewhat unsatisfactory if rays from the periphery of a lens are considered.”

\[
\sin \theta = \theta - \frac{\theta^3}{3!}
\]

\[
\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R} + h^2 \left[ \frac{n_1}{2s_o} \left( \frac{1}{s_o} + \frac{1}{R} \right)^2 + \frac{n_2}{2s_i} \left( \frac{1}{R} - \frac{1}{s_i} \right)^2 \right]
\]
What Kinds Of Wavefront Aberrations Are There?

Monochromatic aberration (Seidel and wave aberrations)

Chromatic aberration (Longitudinal, Transverse)
Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- Coma
- Astigmatism
- Field Curvature
- Distortion
Monochromatic aberrations (Seidel aberrations)

- **Spherical Aberration**
- **Coma**
- **Astigmatism**
- **Field Curvature**
- **Distortion**

![Diagram showing various aberrations and their effects on a lens](image-url)
Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- Coma
- Astigmatism
- Field Curvature
- Distortion
Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- Coma
- Astigmatism
- Field Curvature
- Distortion
Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- Coma
- Astigmatism
- Field Curvature
- Distortion
Monochromatic aberrations (Seidel aberrations)

- Spherical Aberration
- Coma
- Astigmatism
- Field Curvature
- Distortion

Diagram:
- Object
- Image
- Pin-cushion distortion
- Barrel distortion
Monochromatic aberrations (wave aberrations)

“The optical deviations of the wavefront from a reference plane or spherical wavefront.”
Wave aberrations (defocus)

Myopic (near sighted) eye

Perfect eye
Defocused wavefront
Wave aberrations (higher order)

Eye with higher order aberrations

- Perfect eye
- Aberrated wavefront
Wave Aberration of a Surface
Mathematical description of the aberration.
Zernike circle polynomials

\[ W(\_\_\_, \theta) = \sum C_n^m Z_n^m(\_\_\_, \theta) \]

Wavefront aberration
Zernike coefficient
Zernike polynomials (wavefront mode)

\[ Z_2^2(\_\_\_, \theta) = \_\_\_^2 \cos 2\theta \]

\[ m: \text{angular frequency} \]
\[ n: \text{radial order} \]
# Zernike polynomials

<table>
<thead>
<tr>
<th>$n = \text{order}$</th>
<th>$m = \text{frequency}$</th>
<th>$Z_n^m(\rho, \theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>$2\rho\sin\theta$</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>$2\rho\cos\theta$</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>$\sqrt{6}\rho^2\sin 2\theta$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$\sqrt{3}(2\rho^2-1)$</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>$\sqrt{6}\rho^2\cos 2\theta$</td>
</tr>
<tr>
<td>3</td>
<td>-3</td>
<td>$\sqrt{8}\rho^3\sin 3\theta$</td>
</tr>
<tr>
<td>3</td>
<td>-1</td>
<td>$\sqrt{8}(3\rho^3-2\rho)\sin \theta$</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>$\sqrt{8}(3\rho^3-2\rho)\cos \theta$</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$\sqrt{8}\rho^3\cos 3\theta$</td>
</tr>
<tr>
<td>4</td>
<td>-4</td>
<td>$\sqrt{10}\rho^4\sin 4\theta$</td>
</tr>
<tr>
<td>4</td>
<td>-2</td>
<td>$\sqrt{10}(4\rho^4-3\rho^2)\sin 2\theta$</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>$\sqrt{5}(6\rho^4-6\rho^2+1)$</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>$\sqrt{10}(4\rho^4-3\rho^2)\cos 2\theta$</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>$\sqrt{10}\rho^4\cos 4\theta$</td>
</tr>
<tr>
<td>5</td>
<td>-5</td>
<td>$\sqrt{12}\rho^5\sin 5\theta$</td>
</tr>
<tr>
<td>5</td>
<td>-3</td>
<td>$\sqrt{12}(5\rho^5-4\rho^3)\sin 3\theta$</td>
</tr>
<tr>
<td>5</td>
<td>-1</td>
<td>$\sqrt{12}(10\rho^5-12\rho^3+3\rho)\sin \theta$</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>$\sqrt{12}(10\rho^5-12\rho^3+3\rho)\cos \theta$</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>$\sqrt{12}(5\rho^5-4\rho^3)\cos 3\theta$</td>
</tr>
</tbody>
</table>
Wavefront mode for each Zernike polynomial

<table>
<thead>
<tr>
<th>n \ m</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td><img src="image9.png" alt="Image" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td><img src="image16.png" alt="Image" /></td>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
<td><img src="image19.png" alt="Image" /></td>
<td><img src="image20.png" alt="Image" /></td>
<td><img src="image21.png" alt="Image" /></td>
<td><img src="image22.png" alt="Image" /></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Wavefront aberration and Zernike coefficients

Wave aberration

Zernike coefficients

Zernike polynomial (mode)

Wave aberration

Zernike coefficients

Wave aberration

Zernike coefficients
Wavefront rms error

\[ rms = \sqrt{\sum (C_n^m)^2} \]

when \( rms \) is small.

\[ Strehl \approx 1 - \left( \frac{2\pi}{\lambda} \right)^2 \text{rms}^2 \]

Diffraction-limited (Strehl \( \geq 0.8 \))

Rayleigh’s \(_/4\) rule

\[ W_{p-v} \leq \frac{\lambda}{4} \]
Chromatic Aberration

Lensmaker’s formula: \[ \frac{1}{f} = (n - 1) \cdot \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

\( n = n(\lambda) \rightarrow f = f(\lambda) \) for polychromatic light

- **BK7 Schott glass**
- \( R_1, R_2 = 50\text{mm} \)
Longitudinal (axial) chromatic aberration (LCA)

White light

Short wavelength

Long wavelength

LCA
LCA of the human eye

Bennet and Rabbetts (1989)

Wavelength (nm)

~2.2D
Spectral sensitivity of the human eye

Optical effect of eye’s LCA on image quality $\approx 0.2$D defocus for monochromatic light
Transverse (lateral) chromatic aberration (TCA)
Why Are These Aberrations Important?

Relationship between aberrations and image quality
(Pupil function, Rms, PSF, SR, OTF, MTF, PTF, Image convolution…)
Image quality

How well can an optical system form an image?

We can improve image quality by correcting the aberrations.
Aberration vs Image quality

Pupil function (aberration) \[ P(x, y) = A(x, y) \exp\left(\frac{2\pi}{\lambda} W(x, y)\right) \]

Point Spread Function (PSF)

Optical Transfer Function (OTF)

Modulation Transfer Function (MTF)

Phase Transfer Function (PTF)

Strehl Ratio

Image convolution

autocorrelation

FT

FT
Point Spread Function (PSF)

\[ PSF = |FT(P(x, y))|^2 \]

The Point Spread Function, or PSF, is the image that an optical system forms of a point source.

The point source is the most fundamental object, and forms the basis for any complex object.
The PSF for a perfect optical system is the Airy disc, which is the Fraunhofer diffraction pattern for a circular pupil.
Point Spread Function vs. Pupil Size
Perfect Eye
Point Spread Function vs. Pupil Size

Typical Eye

1 mm  2 mm  3 mm  4 mm

5 mm  6 mm  7 mm
Strehl Ratio

diffraction-limited PSF (with no aberrations)

$H_{dl}$

actual PSF (with aberrations)

$H_{eye}$
Image Convolution

$$FT^{-1} \left[ FT(PSF(x,y)) \odot FT(O(x,y)) \right] = I(x,y)$$
Modulation Transfer Function (MTF)

\[ MTF(f_x, f_y) = \text{Re}[\text{FT}(\text{PSF}(x, y))] \]

The Modulation Transfer Function, or MTF, is a measure of the reduction in contrast from object to image.

The ratio of the image modulation to the object modulation at all spatial frequencies.
Modulation Transfer Function (MTF)

Object
100% contrast

Image
diffraction only

Image
diffraction +
0.25D defocus

MTF

Spatial Frequency
Optics Simulations
- Have fun!!! -

Fundamental Optics

http://webphysics.ph.msstate.edu/javamirror/java/light.htm

Wavefront Theory and Fourier Optics

http://www.optics.arizona.edu/jcwytant/math.htm
How Can We Measure These Aberrations Of The Eye?

Different types of wavefront sensors to measure ocular aberrations
Wavefront sensors for the eye

Subjective method

Ingoing light

Spatially Resolved Refractometer

Objective method

Ingoing light

Tcherning Laser Ray Tracing

Outcoming light

Shack-Hartmann
Measurement principle of wavefront sensors for the eye

Measurement of wavefront slope (1st derivative of wavefront) averaged over each subaperture on the pupil

\[
\frac{\partial W(x, y)}{\partial x} = k \cdot \Delta d_x \\
\frac{\partial W(x, y)}{\partial y} = k \cdot \Delta d_y
\]
Spatially Resolved Refractometer

Webb, Penney and Thompson (1992)

Subject adjusts the incident angle of light until retinal spot intersects reference spot.
Laser Ray Tracing

Navarro & Losada (1997), Molebny et al. (1997)

\[ \Delta d_x, \Delta d_y \]

Scanning different locations of the pupil

CCD

reference

CCD

reference
Tcherning Aberroscope

Tscherning (1894)

Dot pattern mask

\[ \Delta d_x, \Delta d_y \]

CCD
Shack-Hartmann wavefront sensor

Liang, Grimm, Goelz, and Bille (1994), Liang and Williams (1997)
Comparison of wavefront measurements using different wavefront sensors
Shack-Hartmann vs Spatially Resolved Refractometer (Objective vs Subjective methods)

Shack-Hartmann vs Laser Ray Tracing
(Outcoming vs Ingoing light)

FIGURE 3.
Contour plots of the wave aberration for the right eye of subject RN for H-S (left), LRT (center), and SRR (right) measurements. Left represents temporal side, and right represents nasal side. Pupil coordinates range from $-3.25$ mm to $3.25$ mm. Step between adjacent contour lines is $0.5$ $\mu$m. Tilt terms ($Z_1$ and $Z_2$) and defocus ($Z_4$) have been cancelled.
Thank you!