Tomographic reconstruction for multi-guiderstar adaptive optics

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Tomographic Reconstruction

**Fundamental questions for the architecture of AO for extremely large telescopes**

- Number of DMs, and their conjugate locations

- Number of actuators per DM
  Pareto-optimal solution (Gavel, DeKany, Baumann, Nelson)

- Number and placement of *laser* guide stars

- Brightness of guide stars

- Controller bandwidth
1. Measure wavefronts from guidestars at field angles $\theta_i$, $i=1\ldots\#gs$

$$\phi_i(x) = \int \Delta n(x - \theta_i z, z) \, dz$$

2. Estimate wavefront as it would appear coming from direction $\theta$ using a minimum variance estimator in the spatial frequency domain

$$\Phi(k_x) = \sum_{\#gs} g(k_x, \theta_i) \Phi_i(k_x)$$

3. Apply wavefront correction for science object at field angle $\theta$
Error analysis of Tokovinin’s algorithm

\[ \varepsilon(k_x, \theta) = \int P(k_x, z, \theta) \Delta n(k_x, z) dz - \sum_{l=0}^{#gs} g_l(k_x, \theta) \nu_l(k_x) \]

- **Wavefront error**: spatial frequency \( k_x \), science direction \( \theta \)
- Filtered integral of index variation, \( \Delta n \), along path
- Noise on each WFS, filtered by the estimator, \( g_k \)

\[ P(k_x, \theta, z) = \left[ e^{-ik_x \theta z} - \sum_{l=1}^{#gs} g_l(k_x, \theta) M(k_x) e^{-ik_x \theta z} \right] \]

- Select feedback gains \( g_l(k_x, \theta) \) to minimize the variance of wavefront error, for given \( C_n^2(z) = \langle \Delta n^2(z) \rangle \) and \( \sigma_n^2 = \langle \nu_f^2 \rangle \)

The algorithm \( g_l(k_x, \theta) \) assumes
- infinite aperture
- plane waves
Tomographic reconstruction error
(assumes infinite aperture and plane waves)

\[ \sigma^2 \approx \left( \frac{\Theta \delta}{r_0} \right)^{5/3} e(\theta) \]

- \( \Theta \) = constellation radius
- \( r_0 \) = transverse coherence distance (Fried’s parameter)
- \( \delta \) = effective layer thickness
- \( e \) = field-dependent factor (\( \leq 1 \) inside constellation)

150 nm rms: entire CELT AO error budget
Tomographic Reconstruction

Fourier transform interpretation of tomographic wavefront reconstruction

\[ \phi(x) = \int_{0}^{\infty} \Delta n(x - \theta z, z) dz \]

\[ \Phi(k_x) = \Delta N(k_x, -k_x \theta) \]

*Fourier slice theorem in tomography (Kak, 1988)*

- Each wavefront sensor measures the integral of index variation along the ray lines
- The line integral along z determines the \( k_z = 0 \) Fourier spatial frequency component
- Projections at several angles sample the \( k_x, k_y, k_z \) volume
Consequences of finite number and field of guide stars

- Wider spacing of guide stars gives less dense sampling in k-space. $\delta z \delta k_z \sim \text{constant}$ so thin $\delta z$ layers result in wide $\delta k_z$ lines. Wider $\delta k_z$ lines (thin layer of turbulence) means fewer samples (guide stars) are required.

- Unfortunately, this does not extend to multiple thin layers. E.g. for two thin $\delta z$ layers spaced apart by $\Delta z$, the k-space line is only $1/\Delta z$ wide, not $1/\delta z$. So it doesn’t change the situation much from turbulence over the entire $\Delta z$ region.

- Because of the angle of the sample lines in k-space, $k_z$ sampling interval increases with $k_x$, so higher $k_x$ spatial frequencies are less well measured. The cut of is near $k_x < 1/[\Delta z(\theta - \theta_{gs})]$
$k_x < 1/[(\Delta z (\theta - \theta_{gs}))]$ requirement spatial interpretation

$\theta \Delta z < r_0$
Back-projection algorithm
Kak, 1988

- Concepts are borrowed from tomography
- The Fourier-slice reconstruction algorithm is reformulated in terms of projection, filtered back-projection steps (mathematically equivalent to Fourier-slice in the infinite aperture plane wave case).
- Advantages are: finite aperture, “fan rays”, finite # of probe directions are brought out explicitly.
- Back propagation is mathematically equivalent to Tokovinin’s algorithm if we take into account $C_n^2(z)$ profile when back-propagating through the atmosphere. (Proven for ngs=1 case, “statistical addition” must be used for ngs>1)

Example tomographic reconstruction of an ellipsoidal target using back projections
Example NGS finite aperture reconstruction using tomographic back-projection*

\[ C_n^2 \] (Turbulence Strength) \[ \Delta n \] (Index variations)

Altitude

Projection

\[ C_n^2 \text{ weighted back-projection} \]

Uniformly weighted back-projection

5-guidestar reconstructions

*numerical implementation of Fourier-slice
note: Tokovinin & Viard’s algorithm is equivalent to a \( C_n^2 \) weighted back-projection
Tomographic Reconstruction

Cone (laser guide star) back-projections

\[ \Delta n \]

(Index variations)

3 laser guidestar reconstruction
5 laser guidestar reconstruction
5 natural star reconstruction
LGS tomography error predictions
30 m telescope, Mauna Kea 12-layer profile

- Predicts anisoplanatism, cone-effect (validated in single-guidestar simulations)
- Provides insight into number and placement of guidestars
- Back-projection gives ~100x improvement over uncorrected seeing – problem: need ~1000x
Tomographic Reconstruction

Simulation with a test-point $\Delta n$ demonstrates back-projection’s inherent error.

- **'Phantom'**
  - Single test point $\Delta n$ at 5 km

- **Reconstruction**
  - MK Cn2 profile-weighted back-projection

The diagram shows a comparison between the 'Phantom' and the MK Cn2 profile-weighted back-projection at 5 km and 25 km.