

Local Wavefront Reconstruction: Simulations, Experiments and Predictions

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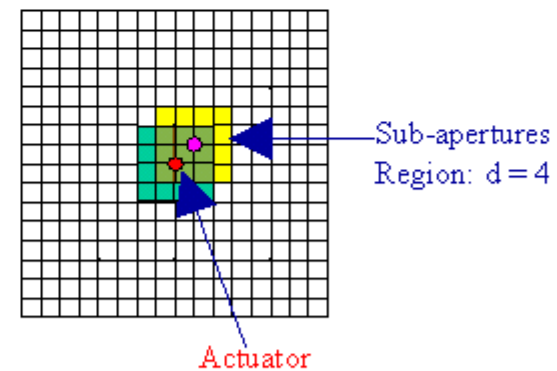
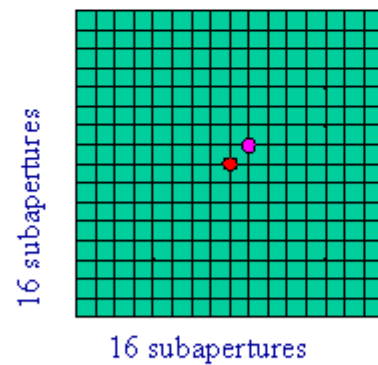
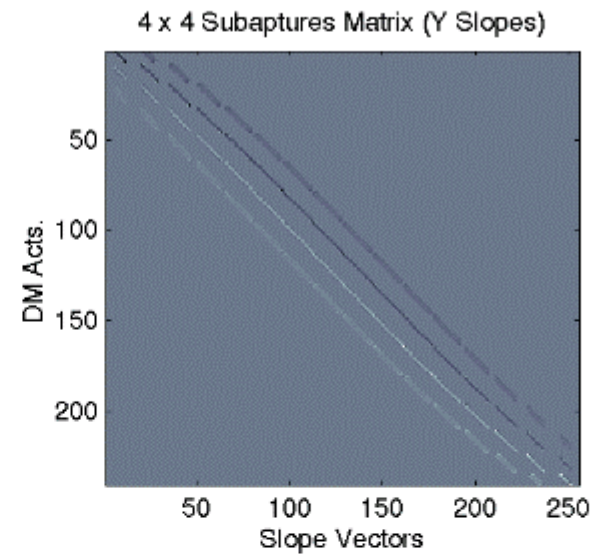
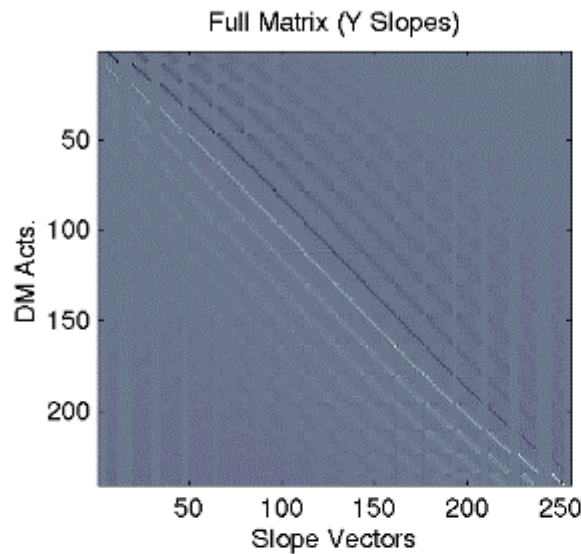
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November 10, 2002

Sparse Matrices: Rationale and Methodology



· The Matrix is fully populated
 · Reconstructor calculations for a $\sqrt{n \times n}$ SH sensors $\sim 2n^2$

· The Matrix has a banded structure
 · Reconstructor calculations for a $\sqrt{n \times n}$ SH sensors $\sim 2kn$, ($k = d \times d$)

· Need a smart indexing scheme

Sparse Matrices: Rationale and Methodology

Computation Challenge for WF Reconstructor:

- For wavefront sensor with \sqrt{n} actuator/sensor across the diameter of telescope the full reconstruction matrix A^+ has a size $\sim n$
- WFS reconstruction calculation $\Rightarrow \sim 2n^2$ multiplications and additions
- n is proportional to telescope diameter D^2 : big challenge for telescopes such as CELT

Sparse WF Reconstruction Matrix:

- A^+ has a diagonal band structure because each DM actuator is mostly influenced by the WFS slopes measured by the *nearest surrounding subapertures*. This is the base for our sparsification.
- Sparsification Method:
 - **Truncation:** (*a working approach*) Truncated a full reconstruction matrix corresponding to the set of sensors s that are within influence region size d of each actuator
 - **Localized least square (LLS):** (*a better approach*) Reconstruct the phase for each actuator using the nearest subapertures (within influence region size d) using the least square method (localized pseudo-inverse)
 - **Hierarchical controller:** (*the best approach*) Add extra layer(s) reconstruction on top of LLS to sense the global modes in a reduced (averaged over several subapertures) sensor data set
- The “sparsed” matrix will have $\sim 2kn$ non-zero components, with $k = d/d \ll n$ and k is a constant depends on the actuator influence region size not telescope size D
- The reconstruction calculations scale as $\sim n$ instead of n^2
- Hierarchic controller adds more calculations but scales only as $\sim n^{4/3}$

Sparse Matrices: Rationale and Methodology

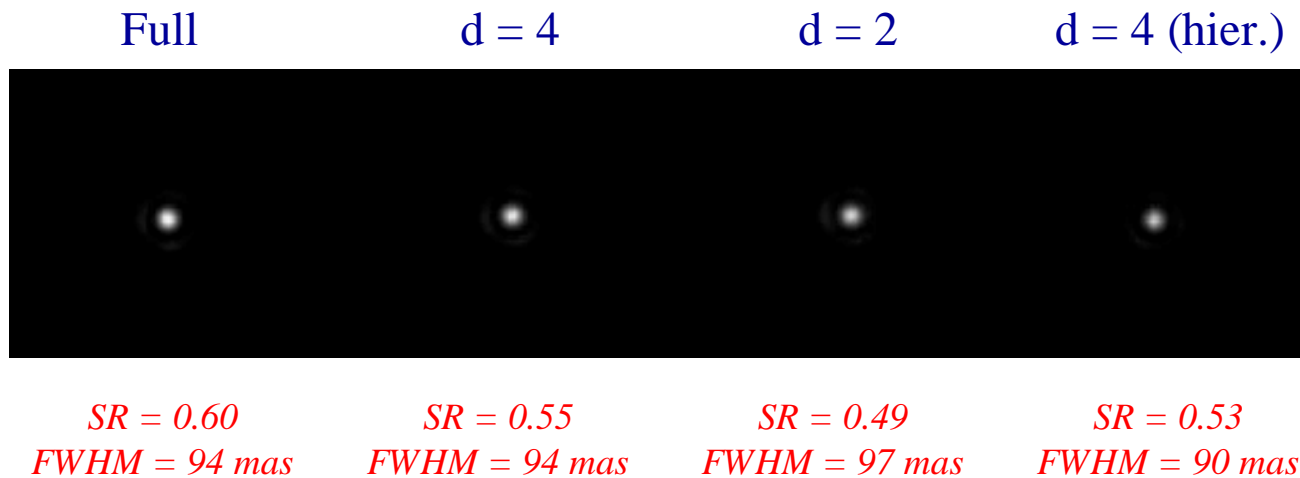
Sparse WF Reconstruction Matrix:

- The sparsed matrix (either truncation or LLS) behaves like a low pass filter which will always lose gains for global modes with spatial frequency $\nu < d/D$
- The sparse matrices may cause a higher gain for certain characteristic spatial frequency around $\nu = d/D$
- Hierarchic controller recovers most gains of the global modes without much increase of computation burden because the global modes are recovered on a much reduced data set in order of D/d

Experiment and Simulation on PALAO

- Experiment have been done for all three types of matrices on PALAO
 - Truncated sparse matrices with influence region size from $d = 2$ to $d = 12$
 - Localized least square (LLS) sparse matrices with influence region size from $d = 2$ to $d = 12$
 - Hierarchic controller based on a $d = 4$ LLS sparse matrix and a 4×4 global reconstructor
- Matrices maintain the full matrix format in the PALAO reconstructor => **did not** take the advantage of the computation saving but make the performance comparison easy
- Sparse matrix closed-loop wavefront sensor data is recorded at 100 Hz together with PHARO K-band images
- Simulations have been done for all three types of matrices using PALAO open loop data

Full and Sparse Matrices Closed-loop K-band Images



- Bright star SAO64701: $M_V=5.9$ mag K0
- Reconstruction matrices used are indicated on top of each closed-loop images
- Estimated Strehls and FWHM sizes are indicated at the bottom of each images

Wavefront Error of Sparse Matrices

Experiment:

- Total wavefront error consists of system error and sparse matrix error

$$\sigma_{total}^2 = \sigma_{system}^2 + \sigma_{sparse}^2$$

- AO system error is calculated from the full matrix PHARO image Strehls

$$S \cup \exp \left[\frac{2\pi}{\lambda K} \sigma_{system} \right]$$

- Wavefront error due the sparse matrices are:

$$\sigma_{sparse}^2 = 4 \sum_j \frac{(a_j^{Full} - a_j^{Sparse})^2}{n_{act}}$$

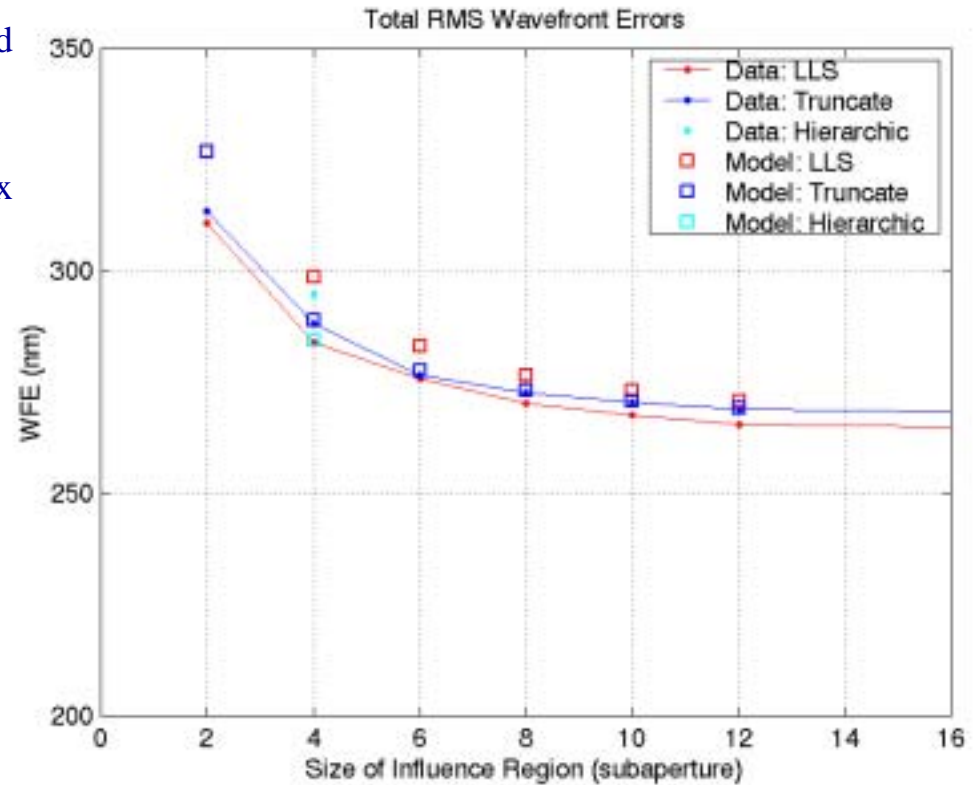
where a_j is the actuator command reconstructed from the recorded WFS data:

$$a^{Full} = A^{Full} s^{Sparse}$$

$$a^{Sparse} = A^{Sparse} s^{Sparse}$$

Model Simulation:

- Open loop data is used for model simulations
- Model simulates the closed-loop performance with the same sparse matrices
- Model uses the estimated AO system error of the full matrix to calibrate the model
- 100 Hz** recoded data lack of the excitation of higher frequency modes



← Matrix Sparseness

DM Actuator Residuals PSD: LLS Matrices

- *Upper panels:* The radially averaged PSD of the recorded closed-loop DM actuator residuals for different sparse matrices
- *Lower panels:* Same PSDs normalized by the PSD of full matrix case

